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MODEL STUDIES OF A DEVELOPING BOUNDARY LAYER OVER THE OCEAN

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ABSTRACT

The change in water vapour content of a so-called mixed boundary layer is studied by means of fairly simple models, of the type where a constant value of conservative quantities is assumed. It is found that near an ocean surface the relative humidity has an upper limit lower than 100% and, therefore, that the condensation level is at some height above the sea surface. Exceptions may occur in cases where the air-sea temperature difference is large.

A model of this type is integrated numerically. The model takes into account both the phase-change of the water substance and the influence of water vapour on the buoyancy. The result is compared to observations from the weather ship "M" in a special case.

Introduction

Laboratory experiments have demonstrated that when turbulence is generated mechanically near the surface of a resting, stratified liquid, a turbulent layer develops, and this layer gradually grows in thickness by entrainment of the undisturbed fluid. The turbulent layer is separated from the rest of the fluid by a more or less well-defined interface. Below this interface the fluid is virtually undisturbed. A similar development occurs when turbulence is generated by convection resulting from heating the liquid from below.

These laboratory experiments have direct applications to the atmosphere where turbulence may be generated by surface friction, or by heating when the earth's surface is warmer than the air. The development in time of such a turbulent or "mixed" layer has been studied by direct observations. It has also been simulated by models of different kinds. A special type of model has been constructed for the purpose of simulating specifically this type of boundary layer development. Ball (1960), Carson (1973) and others studied dry models, while Lilly (1968) constructed a cloud-topped model in order to study inversions over subtropical oceans.

These models have certain common features. Apart from turbulent fluctuations the potential temperature and water vapour mixing ratio is assumed to be constant with height through the mixed layer, although they usually vary in time. If clouds are formed, and the clouds are considered to be non-precipitating, the sum of the mixing ratios for vapour and liquid water is assumed to be constant. Also, in this case equivalent potential temperature or wet-bulb potential temperature may take the place of the potential temperature. The very stable layer, often found above the mixed layer, is simulated by a discontinuity in temperature and mixing ratio. As time goes on this inversion is lifted in relation to the overlaying air, leading to entrainment of warmer and usually dryer air into the boundary layer.

A vertically constant potential temperature and mixing ratio implies constant flux-convergence and accordingly a linearly varying flux. Therefore, if the fluxes are known at two heights, for instance at the bottom and top of the boundary layer, they are known everywhere inside the layer. However, the value at the top, which is connected with the aforementioned entrainment, is not easy to assess, and here certain dubious assumptions have to be made.

In a humid boundary layer the convection and the associated production of turbulent kinetic energy may also be provided by the water vapour, since an increase in water vapour content gives increased buoyancy of the air. In fact, as pointed out by Ball (1960), the flux of water vapour may be more important than the flux of temperature when mean tropical conditions are considered. Lilly (1968) specifically demonstrated that a cloud-topped boundary layer may be maintained with a downward sensible heat flux, although in this case most of the turbulent energy probably comes from release of latent heat in the cloud.

Lilly (1968) also points to another source of turbulent energy: if the wet-bulb temperature of the cloud-tops is higher than the wet-bulb temperature of the entraining air, a mixing may produce air which is colder and heavier than the surroundings. This process may lead to more rapid erosion of the air overlaying the boundary layer.

In the following we shall first consider a model for a boundary layer developing over an ocean with water surface temperature higher than the air. We shall use this model as the basis for a discussion of the rate at which water vapour is accumulated in the boundary layer. For the sake of simplicity we treat the vapour as a passive admixture with no influence on the air motion. As we already have mentioned, this is not necessarily the case in nature.

Later on we describe a more general model where we take into account the buoyancy of the moist air and the release of latent heat above the condensation level.

Dry model

The model boundary layer is characterized by a potential temperature, θ_o, and a water vapour mixing ratio, q_o, which are independent of height, but in general varies with time. The boundary layer is separated from the overlaying stable atmosphere by a discontinuity, $\theta_1 - \theta_o > 0$, in potential temperature. There is also a discontinuity in mixing ratio, $q_o - q_1$ (see Fig. 1).

Budget relations for such a model have been established by several authors, starting with Ball (1960). For the benefit of the reader we derive them below, with a brief description of their main content.

Following the analyses of Ogura and Phillips (1962), we may write the thermodynamic energy equation for this model as

$$\frac{\partial\theta_o}{\partial t} = -\frac{\partial}{\partial z}\overline{w'\theta'}$$

Since, according to the assumptions $\partial/\partial t\ (\partial\theta_o/\partial z) = 0$, it follows that $\overline{w'\theta'}$ must vary linearly with height, and especially that

$$h\frac{d\theta_o}{dt} = (\overline{w'\theta'})_o - (\overline{w'\theta'})_1 \tag{1}$$

where the two last terms are the turbulent fluxes at the bottom and top, respectively, and h is the boundary layer thickness.

The turbulent flux at the top is connected to the entrainment of potentially warmer air from above into the boundary layer. In the absence of large-scale vertical velocity and radiation one gets

$$(\overline{w'\theta'})_1 = -(\theta_1 - \theta_o)\frac{dh}{dt} \tag{2}$$

Similar equations exist for the mixing ratio, i.e.

$$(3) \qquad h\frac{dq_o}{dt} = (\overline{w'q'})_o - (\overline{w'q'})_1$$

and

$$(4) \qquad (\overline{w'q'})_1 = (q_o - q_1)\frac{dh}{dt}$$

Equations (2) and (4) show that there is a connection between the flux of heat and humidity through the top. Indeed

$$(5) \qquad (\overline{w'q'})_1 = -k(\overline{w'\theta'})_1$$

where

$$(6) \qquad k = \frac{q_o - q_1}{\theta_1 - \theta_o}$$

For the surface fluxes one may use the bulk formulae which we shall write as

$$(7) \qquad (\overline{w'\theta'})_o = C\,(\theta_w - \theta_o)$$

$$(8) \qquad (\overline{w'q'})_o = C\,(q_s(\theta_w) - q_o)$$

where θ_w is the sea surface temperature and $q_s(\theta_w)$ is the saturation mixing ratio at this temperature. Also, it is assumed that the air pressure at the sea surface is 1000mb, so that θ_o is effectively the air temperature at the lower boundary. The use of the same constant C in both equations is based on the assumption that eddy diffusivity for heat and moisture is the same.

Equations (1), (2) and (3) determine the time evolution of θ_o, q_o and h, provided the other unknown quantities can be eliminated. We may substitute for the surface fluxes from (7) and (8), but are still left with one unknown, namely $(\overline{w'\theta'})_1$, the turbulent flux of heat at the top of the boundary layer. Physically this flux is tied to the erosion of the overlaying non-turbulent atmosphere and presumably connected to the density of turbulent kinetic energy.

The turbulent energy equation, relevant to the present situation, may be written as

$$\frac{\partial E}{\partial t} = - \frac{\partial}{\partial z} (\overline{w'E'}) + \frac{g}{\theta_o} \overline{w'\theta'} - \varepsilon$$

where $E=\frac{1}{2}(u'^2+v'^2+w'^2)$. The two last terms are the production and dissipation of turbulent energy, while the second term may be thought of as representing the eddy transfer in vertical direction. Ball (1960) assumes that both the first and the last terms are small compared to any of the remaining ones. Then

$$(\overline{w'\theta'})_1 = - (\overline{w'\theta'})_o$$

and this relation closes the system of equation.

Ball's hypothesis has been questioned by Deardorff, Willis and Lilly (1968) on the basis of laboratory experiments, and several authors have later used the assumption

(9) $$(\overline{w'\theta'})_1 = - A(\overline{w'\theta'})_o$$

where A typically has the value 0.1.

It may be shown that an assumption equivalent to (9) is that a prescribed fraction, R, of the total turbulent energy production, P^+, is transformed into potential energy in the upper part of the boundary layer where the heat flux is negative.

If this "negative production" is called P^-, we therefore have

$$P^- \doteq - R P^+ . \tag{10}$$

On the other hand, since the production, being proportional to the heat flux, varies linearly through the boundary layer, one may easily verify that

$$\frac{P^-}{P^+} = \left[\frac{(\overline{w'\theta'})_1}{(\overline{w'\theta'})_0} \right]^2$$

and, therefore, $A = R^{\frac{1}{2}}$.

In the following we shall use (10) as our closure assumption. However, as mentioned by Deardorff et al. (1968), the actual value of R is not well known, and it may also vary in time as the boundary layer develops.

Since we have left out of consideration the influence of water vapour on the production of turbulent energy, the model is basically applicable to cases with significant air-sea temperature difference, especially in cold climates, and where no release of latent heat takes place.

The relative humidity and condensation level

Since dry air is mixed into the boundary layer from above, at the same time as water vapour evaporates from the sea surface, it is not obvious that the mixing ratio will increase. Furthermore, even when the mixing ratio increases, the relative humidity may decrease, since the temperature increases at the same time.

In order to study the relation between these effects, it is instructive to develop an equation for the relative humidity, U_o at the sea level. To this end we rewrite (8) in the following way, assuming that the air-sea temperature difference is small

$$(11) \qquad (\overline{w'q'})_o = C\,\varepsilon p_o^{-1}\left[e_s(\theta_w) - U_o e_s(\theta_o)\right]$$

$$\approx C\varepsilon p_o^{-1}\left[(1-U_o)e_s(\theta_o) + e_s'(\theta_o)(\theta_w-\theta_o)\right]$$

where $e_s(\theta_o)$ is the saturation water vapour pressure at temperature θ_o and $e_s'(\theta_o)$ its derivative, while $\varepsilon = 0.622$. Furthermore, $q_o = U_o \varepsilon p_o^{-1} e_s(\theta_o)$ giving

$$\frac{dq_o}{dt} = \varepsilon p_o^{-1} e_s(\theta_o)\frac{dU_o}{dt} + U_o \varepsilon p_o^{-1} e_s'(\theta_o)\,\frac{d\theta_o}{dt}$$

and inserting from (1), (3), (5) and (7) one gets after some manipulation

$$(12) \qquad \frac{h}{C}\,\frac{dU_o}{dt} = (1-U_o)\left[1 + \frac{\varepsilon L}{R\theta_o^2}\,(1+A)\,(\theta_w-\theta_o)\right]$$

$$- A\frac{\varepsilon L}{R\theta_o^2}\,(\theta_w-\theta_o) \quad - \frac{Ak}{q_s(\theta_o)}\,(\theta_w-\theta_o)$$

Here also the Clausius-Clapeyron equation has been used in the form $e_s'(\theta_o) = \varepsilon L R^{-1}\theta_o^{-2} e_s(\theta_o)$.

The physical reality behind the terms on the right hand side of (12) is clear. The first term contains the two opposite effects of evaporation and heating, while the remaining terms represent the influence of the entraining warm and dry air. The equation shows that for a sufficiently large value of U_o (U_o close to 1), $dU_o/dt < 0$ so that the relative humidity then will decrease. On the other hand, if U_o is below a certain limit, determined by the value of the other variables, the relative humidity increases. In the cases we shall be primarily concerned with, the air initially tends to be quite dry. Therefore, this analysis indicates that there is an upper limit to the relative humidity at the sea surface, as long as the air is being constantly heated. However, we find it necessary once more to stress that cooling by long-wave radiation, which has been left out of consideration, may change this picture.

An expression for the mentioned limit of U_o is easily derived from (12) by equating the right hand side to zero. In terms of the "saturation deficit" one gets

$$(1-U_o)_{lim} = \frac{A\left[\frac{\varepsilon L}{R\theta_o^2} + \frac{k}{q_s(\theta_o)}\right](\theta_w-\theta_o)}{1 + (1+A)\frac{\varepsilon L}{R\theta_o^2}(\theta_w-\theta_o)}$$

Further simplifications may be obtained if only the limiting value for $A \to 0$ is considered. Then, according to (2) and (9) also $(\theta_1-\theta_o) \to 0$. However, since in the limit

$$\frac{d\theta_o}{dt} = \left(\frac{\partial\theta}{\partial z}\right)_1 \frac{dh}{dt} = \Gamma_1 \frac{dh}{dt}$$

one gets

$$\left(\frac{A}{\theta_1-\theta_o}\right) \to \frac{1}{h\Gamma_1}$$

as $\quad A \to 0$

and the asymptotic value in this case becomes

$$(13)\quad (1-U_o)_{lim} = \frac{\theta_w-\theta_o}{1+\frac{\varepsilon L}{R\theta_o^2}(\theta_w-\theta_o)} \cdot \frac{q_o-q_1}{\Gamma_1 h q_s(\theta_o)}$$

Since a characteristic value of $\varepsilon LR^{-1}\theta_o^{-2}$ is 7, eq. (13) shows that the limit is independent of the air-sea temperature difference, if this difference is not very small.

As an example we insert the numerical values

$\theta_w-\theta_o= 5K$, $q_o-q_1= 10^{-3}$, $q_s(\theta_o)= 7.7x10^{-3}$,

$\Gamma_1= 5 \times 10^{-3}Km^{-1}$ and $h = 10^3m$,

and get

$$(1-U_o)_{lim} = 0.096.$$

We may evaluate the speed at which this limit is approached. To this end let us assume for a moment that U_o varies much more rapidly than the other variables in (12), so that they may be considered as constants. Then for $A = 0$, one may solve a differential equation for $(1-U_o)$:

$$\frac{d}{dt}(1-U_o)+\frac{C}{h}\left(1+\frac{\varepsilon L}{R\theta_o^2}(\theta_w-\theta_o)\right)(1-U_o) = \frac{C}{h}\frac{(q_o-q_1)(\theta_w-\theta_o)}{\Gamma_1\, h\, q_s(\theta_o)}$$

with constant coefficients, and the solution shows that $(1-U_o)$ approaches exponentially the asymptotic value (13). One may take as a characteristic time scale for this variation the expression

$$t_c = \frac{h}{c}\left(1+\frac{\varepsilon L}{R\theta_0^2}(\theta_w-\theta_o)\right)^{-1}$$

derived from the exponent of the solution.

For $C = C_H|V| = 1.3 \times 10^{-3} \times 10\ ms^{-1}$, and the other parameters as above, one gets $t_c = 16$ hrs. Since this time is so long we cannot have much trust in the assumptions upon which it is based, namely the constancy of the other variables. It also indicates that a solution of the initial value problem derived from the previous equations, in most cases will show larger values than the value derived from (13). This is demonstrated by the example integration to be described later.

It is easy to derive a simplified formula for the height of the condensation level, h_c. To this end we first develop an expression for the vertical derivative of q_s, assuming that θ is constant:

$$\left(\frac{\partial q_s}{\partial z}\right)_\theta = \frac{\varepsilon g e_s(T)}{RpT} \left(1 - \frac{\varepsilon L}{cpT}\right)$$

where

$$T = \left(\frac{p}{p_o}\right)^\kappa$$

From this an expression for U at any height in the boundary layer follows:

$$U = \frac{q_o}{q_{so} + z.\left(\frac{\partial q_s}{\partial z}\right)^*} = \frac{U_o}{1 + \frac{1}{q_{so}}\left(\frac{\partial q_s}{\partial z}\right)^* .z}$$

where the asterisk indicates a specific value inside the boundary layer. Inserting from above we get

$$U \simeq \frac{U_o}{1 - \left(\frac{\varepsilon L}{C_p\theta_o} - 1\right).\frac{z}{H_o}}$$

where the approximation consists in assuming

$$\frac{\theta_o}{T^*}\,\frac{p_o}{p^*}\,\frac{e_s(T^*)}{e_s(\theta_o)}\left(\frac{\varepsilon L}{C_p\theta_o}\,\frac{\theta_o}{T^*} - 1\right) = \left(\frac{\varepsilon L}{C_p\theta_o} - 1\right)$$

and where $H_o = \frac{R\theta_o}{g}$. We then get the condensation level as

$$(14) \qquad h_c = (1-U_o)\,\frac{H_o}{\frac{\varepsilon L}{C_p\theta_o} - 1}$$

Since most of the simplifying assumptions we have made, in order to derive this formula, have the effect of reducing the condensation level, we may consider it as some sort of a lower limit to the possible values of h_c.

With the numerical values of $(1-U_o)_{min}$ derived above one gets $h_c = 177$m.

The derivation of (12) rests on the assumption that the air-sea temperature difference is small. However, if this temperature difference is large we may find it necessary to include another term in the series expansion of $(\overline{w'q'})_o$ in (11). Then (12) will contain the additional term

$$\frac{e_s''(\theta_o)}{2e_s(\theta_o)}\,(\theta_w-\theta_o)^2$$

on the right hand side. Since this term is always positive, it has the effect of increasing U_o. This is the reason why such phenomena as arctic sea smoke can develop under conditions similar to those described by the model.

Moist model

In this section we develop a model for a mixed boundary layer where the upper part contains a cloud (strato-cumulus). Accordingly, it will be necessary to consider heating (cooling) associated with phase changes of the water substance. We shall also take into account the influence of the water vapour content on the buoyancy of the air. In its basic concepts the model is similar to Lilly's (1968). However, we have tried to use a more general closure assumption. Also, we have neglected any large-scale vertical velocity, and radiation from the cloud tops. The modifications are partly dictated by the circumstance that we intend to use the model basically for studying a rapidly developing boundary layer over the ocean.

As already mentioned, the appropriate conservative quantities in this case are the equivalent potential temperature, θ_e , and the mixing ratio of the water substance to the dry air. We define θ_e by the differential equation

$$(20) \qquad \frac{d\theta_e}{\theta_e} = \frac{d\theta}{\theta} + \frac{Ldq}{C_p T}$$

where above the condensation level q is identical to q_s.
Apart from the small differences produced by the turbulent eddies, θ_e will be assumed to be vertically constant through the whole boundary layer. Below the condensation level θ also has a constant value, θ_o , virtually the air temperature at the ocean surface. Then, with sufficient accuracy

$$(21) \qquad \theta_e = \theta_o \left(1 + \frac{Lq_o}{C_p \theta_o}\right)$$

where q_o is the constant value of the water vapour mixing ratio below the condensation level.

In the cloud top the water substance consists partly of vapour and partly of liquid (frozen) water, and the constant mixing ratio is

(22) $q_s + \ell = q_o$

where ℓ is the mixing ratio of liquid water (ice).

For small fluctuations we get from (20)

(23) $\frac{\theta'_e}{\theta_e} = \frac{\theta'}{\theta} + \frac{Lq'}{C_p T}$

where $q' = q'_s$ above the condensation level.

We now proceed to develop an expression for the production of kinetic energy, taking into account both the content of water vapour and of liquid water. Within the present level of approximation we may write the buoyancy term in the equations of motion as

(24) $-\frac{1}{\rho}\rho' = \frac{1}{\theta_o}\theta' + \delta q'$

below the cloud base, and

(25) $-\frac{1}{\rho}\rho' = \frac{1}{\theta}\theta' + \delta q'_s - \ell'$

above. Here

$\delta = (1-\varepsilon)/\varepsilon = 0.608$

In the expressions on the right hand side we now introduce the conservative quantities θ_e and $q_s+\ell = q_o$, using (23), and get

$$(26) \qquad \frac{\theta_e'}{\theta_e} + (\delta - \frac{L}{C_o\theta_o})\, q'$$

and

$$(27) \qquad \frac{\theta_e'}{\theta_e} - (\frac{L}{C_pT} - \delta)\, q_s' - \ell'$$

$$= \frac{\theta_e'}{\theta_e} - (\frac{L}{C_pT} - (1+\delta))q_s' - (q_s' + \ell')$$

respectively.

Now at constant pressure

$$q_s' = \varepsilon p^{-1} \frac{de_s(T)}{dT}\, T'$$

This, together with (23), gives

$$(28) \qquad \frac{\theta_e'}{\theta_e} = \frac{\theta'}{\theta}(1+\alpha)$$

where, in Lilly's notation

$$\alpha = \frac{L}{C_p}\, a$$

$$a = \varepsilon p^{-1} \frac{de_s(T)}{dT} = \frac{\varepsilon^2 L}{RT^2} \frac{e_s(T)}{p}$$

When this is introduced above, we get the buoyancy term in the cloud layer expressed as

$$(29) \qquad \frac{1+a(1+\delta)\theta_e}{1+\alpha} \frac{\theta_e'}{\theta_e} - (q_s' + \ell')$$

The production of turbulent energy by the buoyancy is then given by the two following expressions, valid below and above the condensation level, respectively

$$\frac{g}{\theta_e}\overline{w'\theta_e'} + g(\delta - \frac{L}{C_p\theta_o})\overline{w'q'}$$

$$\frac{1+a(1+\delta)\theta_e}{1+\alpha} \quad \frac{g}{\theta_e} \overline{w'\theta_e'} - g \overline{w'(\ell'+q_s')}$$

We shall treat a and α as constants. Then, since the fluxes of the conservative quantities vary linearly with height, the production terms we have just written, will also do so. Because of this, the fluxes and the production terms are known at any level as functions of the boundary values $(\overline{w'\theta_e'})_o$, $(\overline{w'q'})_o$, $(\overline{w'\theta_e'})_1$ and $\overline{w'(q_s'+\ell')_1}$, and we may compute the total production by vertical integration.

Equation (5) and (6) is applicable also in this case, if $\theta_{e1} - \theta_e$ is substituted for $\theta_1 - \theta_o$. Also, we assume that

(30) $\quad (\overline{w'\theta_e'})_o = C\,(\theta_{ew} - \theta_e)$

where $\theta_{ew} = \theta_w(1 + \frac{Lq_sw}{c_p\theta_w})$.

This follows from (7), (8) and (21). However, we still need a closure assumption and, as before, we assume that (10) is satisfied where now P^+ is the total production and P^- the total destruction by the buoyancy, both layers taken together. From this relation it is possible to deduce an equation by which $(\overline{w'\theta_e'})_1$ may be determined from the surface fluxes. However, this equation is quite complicated, and we therefore propose an iterative procedure, which may be programmed quite easily on a computer. Also, this seems to be a natural thing to do, since the other equations for the problem also have to be solved numerically.

It is convenient to write the production in the following form

(31) $\quad P_o(1-\zeta_o) + P_{co}\,\zeta_o \quad$ for $z < h_c$

(32) $\quad P_{c1}(1-\zeta_1) + P_1\,\zeta_1 \quad$ for $z > h_c$

where P_o and P_{co} is the production at the bottom and top of the cloudfree layer, where P_{c1} and P_1 apply for the cloud layer. Furthermore, ζ_o and ζ_1 are scaled heights, i.e.

$$\zeta_o = \frac{z}{h_c}$$

$$\zeta_1 = \frac{z-h_c}{h-h_c}$$

Formulas for the P's in (31) and (32) are derived from the expressions for the production terms above, taking into account the linear variation with height. They are

$$(33) \qquad P_o = \frac{g}{\theta_e}(\overline{w'\theta_e'})_o + g(\delta - \frac{L}{c_p\theta_o})(\overline{w'q'})_o$$

$$(34) \qquad P_1 = \{\frac{1+a(1+\delta)\theta_e}{1+\alpha}\frac{g}{\theta_e} + gk\}(\overline{w'\theta_e'})_1$$

$$(35) \qquad P_{co} = P_o(1-\frac{h_c}{h}) + \{\frac{g}{\theta_e} - gk(\delta - \frac{L}{c_p\theta_o})\}\frac{h_c}{h}(\overline{w'\theta_e'})_1$$

$$(36) \qquad P_{c1} = \{\frac{1+a(1+\delta)\theta_e}{1+\alpha}\frac{g}{\theta_e}(\overline{w'\theta_e'})_o - g(w'q')_o\}(1-\frac{h_c}{h}) + P_1\frac{h_c}{h}.$$

The iterations start with a guess value of $(\overline{w'\theta_e'})_1$ from which the above four P's are computed. Algorithms for calculations of the positive and negative value of the vertically integrated production are easily derived using the procedure we outline below for the lower part:

If P_o and P_{co} have the same sign, the integral is simply

$$(37) \qquad \tfrac{1}{2}(P_o + P_{co})\, h_c$$

If the signs are different, the integral consists of two parts of different signs, i.e.

$$(38) \qquad \tfrac{1}{2}\frac{P_o^2}{P_o - P_{co}}\, h_c$$

and

$$(39) \qquad \frac{1}{2} \frac{P_{co}^2}{P_{co} - P_o} h_c$$

We then compute $-P^-/P^+$ and, if necessary, repeat with an adjusted value of the flux. Note that $P_1 < 0$ and, therefore, an increase in the absolute value of $\overline{(w'\theta_e')}_1$ usually makes the ratio larger.

Having thus derived a value for $(\overline{w'\theta_e'})_1$, we can compute a time-step of the variables θ_e and q_o from

$$(40) \qquad h\frac{d\theta_e}{dt} = (\overline{w'\theta_e'})_o - (\overline{w'\theta_e'})_1$$

$$(41) \qquad h\,\frac{dq_o}{dt} = (\overline{w'q'})_o + k(\overline{w'\theta_e'})_1$$

$$(42) \qquad (\theta_{e1} - \theta_e)\,\frac{dh}{dt} = -(\overline{w'\theta_e'})_1$$

while h_c may be derived from (14).

If we also want to take into account a large-scale vertical velocity, w_1, at the inversion level, and consider the effect of a net outgoing radiative flux, F, from the cloud top, equation (42) takes the form

$$(43) \qquad (\theta_{e1} - \theta_e)\,(\frac{dh}{dt} - w_1) = -(\overline{w'\theta_e'})_1 + \frac{F}{c_p\rho}$$

The equation follows more or less intuitively from an energy balance consideration, but may also be derived more rigorously, following Lilly's (1968) procedure.

An example

As a demonstration of the performance of the model we have made a simulation of a specific meteorological case. Some relevant observations are given in Fig. 3, containing radiosonde data from Örlandet on the coast of Norway, for March 15th 1978, 12 GMT and from the weather ship "M" 12 hrs. later. These observing stations are about 400 km apart. Making due allowances for observational errors, especially with regard to the humidity, we shall assume that the mixed layer is 710m thick at the coast and about 1700 m at the weather ship. Also, for the weather ship, the condensation level is probably somewhere around 700m. Simultaneous satellite pictures show that clouds start to form about 100 km from the coast.

Cold air from the continent enters the Norwegian Sea and blows towards the weather ship, and the idea with the experiment was to simulate the changes which a column of air undergoes when it is heated over the ocean. Because of lack of observations, the wind field over the ocean is difficult to assess. However, one may to some extent make use of the following considerations.

According to (30) and (8), both $(\overline{w'\theta_e'})_o$ and $(\overline{w'q'})_o$ is proportional to C. The same must then be true for the value of $(w'\theta_e')_1$ derived from (10), since all the expressions (33)-(36) are homogeneous in $(\overline{w'\theta_e'})_o$, $(\overline{w'q'})_o$ and $(\overline{w'\theta_e'})_1$. It then follows that the right hand sides of the prognostic equations (40), (41) and (42) also is proportional to C. On the other hand we may write

$$\frac{d}{dt} = \frac{ds}{dt}\,\frac{d}{ds} = v\,\frac{d}{ds}$$

where s is the distance along the trajectory and v is the wind speed. If we also assume that C is proportional to the wind speed, s may be used as independent variable instead of t, and v drops out of the equations. Therefore, we may find the value of the variables along the trajectories without knowing the speed at which the column moves.

As initial data for the model we used h = 710m, θ_e = 270K and q_o = 0.008. The sea surface temperature was assumed to be 5°C.

The result of a model integration, shown on Fig. 4, is in reasonable agreement with the observations from the weather ship. The value of R was chosen to be 0.01 in this case. The strength of the temperature discontinuity at the top of the mixed layer varied strongly with distance, but showed a value of 1.2° at 400 km. This value seems to be a bit too small. A change of R to 0.1 gave 2.8°. However, the boundary layer thickness then grew to a seemingly unrealistic value (2152m).

On the basis of the observations (Fig. 3) one may also try to verify the model assumption of a constant value of the conservative quantities. This assumption seems to be reasonably well satisfied for θ_e. However, the same cannot be said for the mixing ratio which at the weather ship decreases from 2.4×10^{-3} to 0.7×10^{-3} from bottom to top. This is partly due to the fact that in Fig. 3 the amount of liquid and frozen water has not been taken into account. Besides, there most certainly are errors in the humidity measurements. In fact, the relative humidity is measured to be below 84 p.c. even in the upper part of the boundary layer. However, this is in contradiction with the dense cloud cover observed from satellites.

Finally, one must expect a decrease of the mean values of the conservative quantities in the direction of the turbulent fluxes. Taking into account that there is a downward flux of equivalent potential temperature in the upper part of the mixed layer, one must expect θ_e to increase upwards in this region. However, this is not the case for the mixing ratio, since the air which entrains the boundary layer from above is usually much dryer and, therefore, the mixing ratio will decrease monotonously from bottom to top.

Conclusions

In this paper we have studied a simplified theoretical model of a developing boundary layer over the ocean. It has been possible to demonstrate that in most cases the relative humidity close to the ocean surface is unlikely to exceed an upper limit less than 100 per cent. Accordingly, the cloud base, if any, must be well above the ocean, usually at least 100-200m. In fact, a numerical integration of a more complete model gave values around 800m for a specific case. Exceptions from this rule may be found in cases when the air-sea temperature difference is extremely large.

The numerical model is easy to modify, and a systematic use will presumably give valuable insight into the dynamics of the planetary boundary layer in different weather situations. Also, a systematic comparison with observations could give useful information about the size of the surface fluxes and the somewhat uncertain closure assumption.

Another interesting thing which we have barely touched upon, is the question of what is likely to happen when the equivalent potential temperature above the capping inversion is lower than in the boundary layer. Lilly (1968) concluded that a stationary boundary layer cannot exist in this case. In the non-steady case we study, the effect is different but may be equally important. We have previously mentioned that it presumably leads to increased turbulent energy and increased erosion of the non-turbulent air above the boundary layer. Satellite pictures show that the clouds, while in most cases taking the form of strato-cumulus, sometimes have the structure of open cells. One may even observe a downstream change from one type to the other. This phenomenon is often considered as a reflection of the increase in boundary layer thickness. However, one may speculate that the open cell structure is associated with the abovementioned configuration of equivalent potential temperature.

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Captions

Fig. 1: Structure of a "dry" model (water vapour considered as a passive admixture).

Fig. 2: Structure of a "wet" cloud-topped model.

Fig. 3: Equivalent potential temperature and water vapour mixing ratio computed from radiosonde data from Orlandet and the weather ship "M".

Fig. 4: Results of a model integration. Numbers on abscissa represent distance along a trajectory. The curves show computed values of boundary layer thickness, height of condensation level, equivalent potential temperature and mixing ratio.

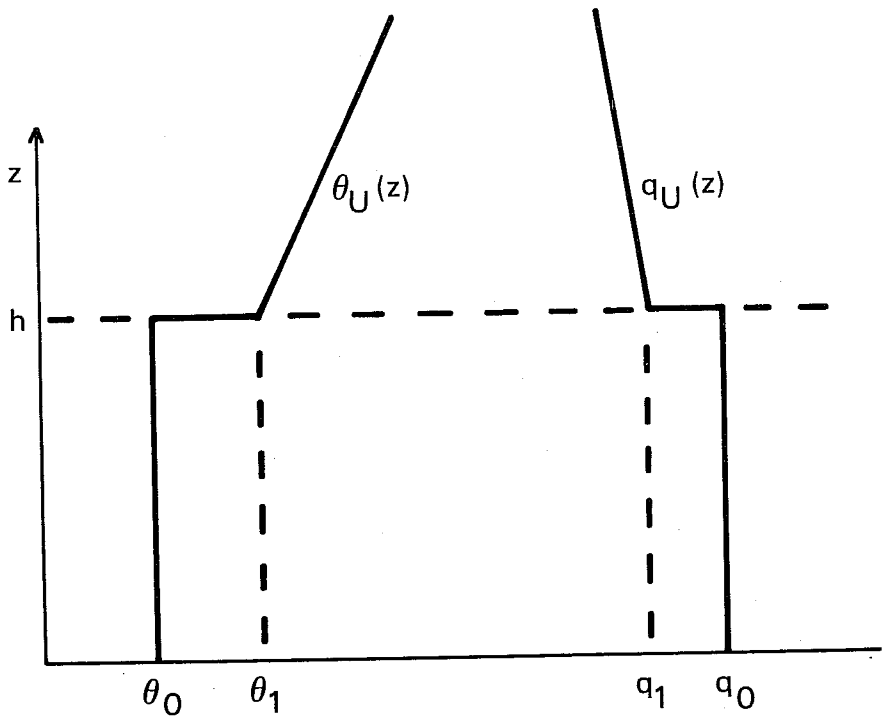

Fig. 1 Structure of a "dry" model (water vapour considered as a passive admixture).

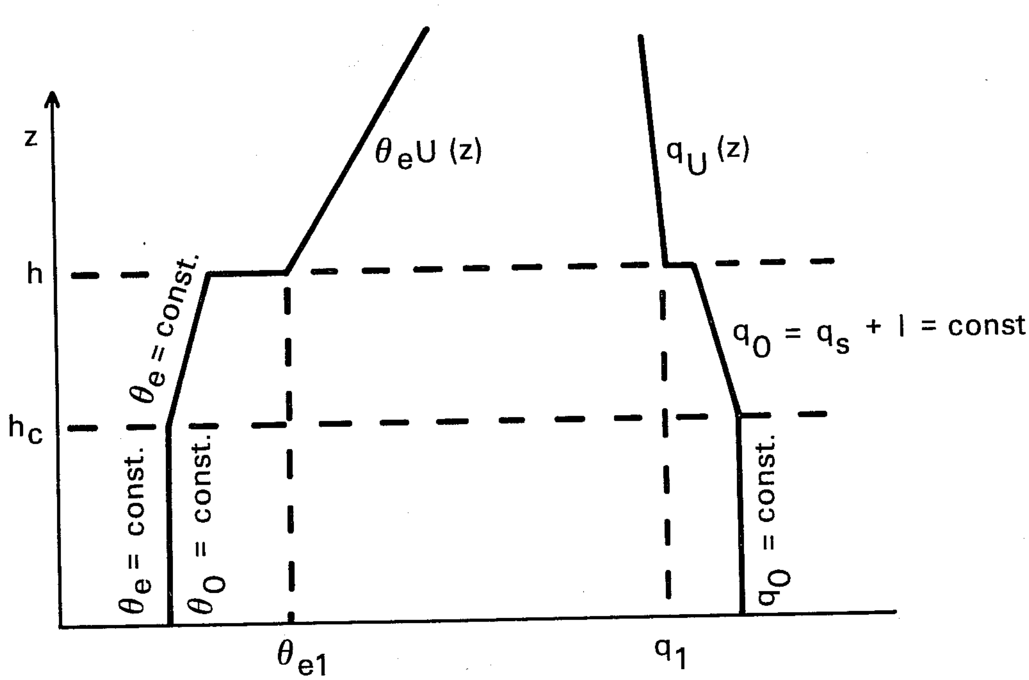

Fig. 2 Structure of a "wet" cloud-topped model.

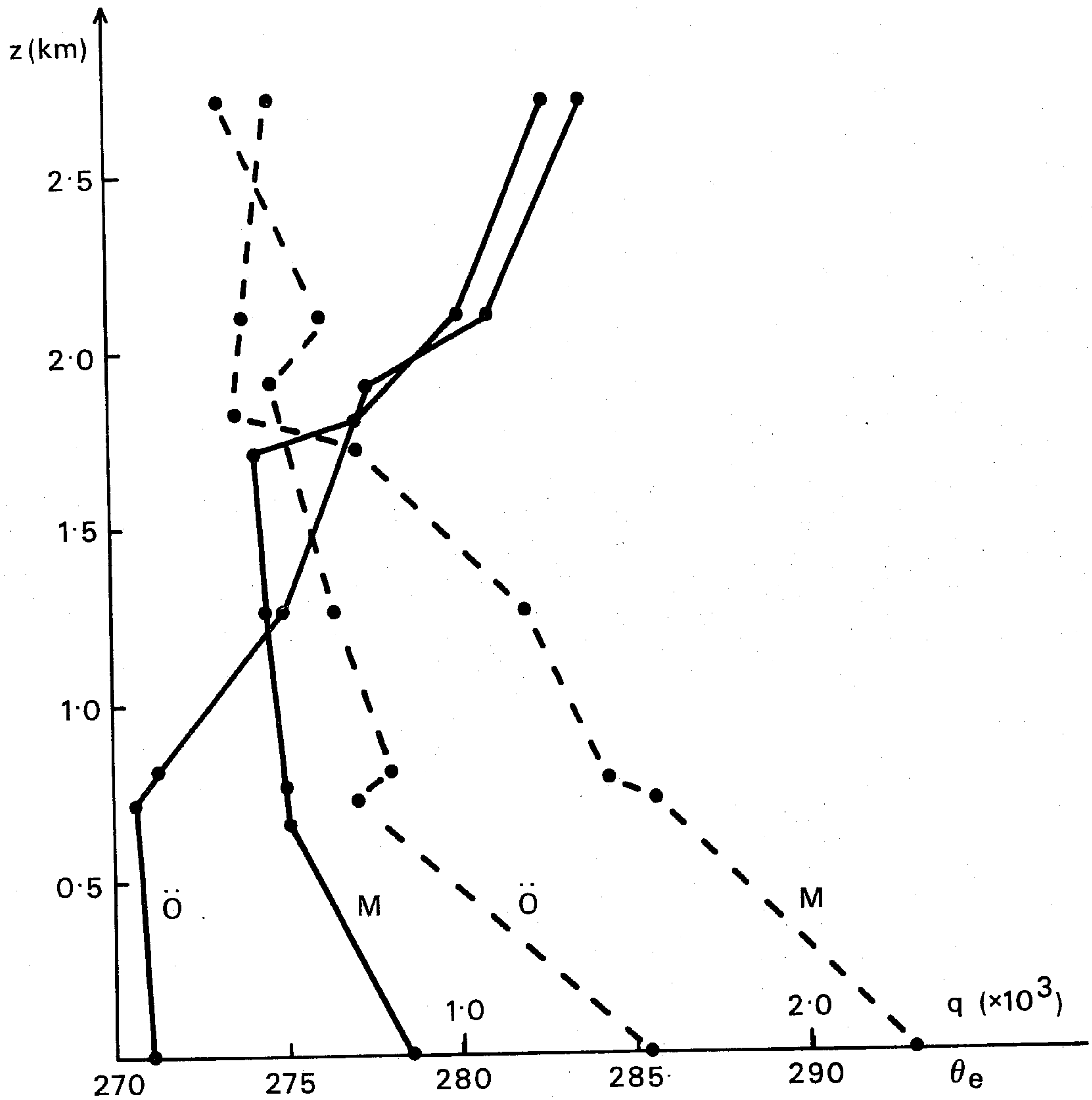

Fig. 3 Equivalent potential temperature and water vapour mixing ratio computed from radiosonde data from Orlandet and the weather ship "M".

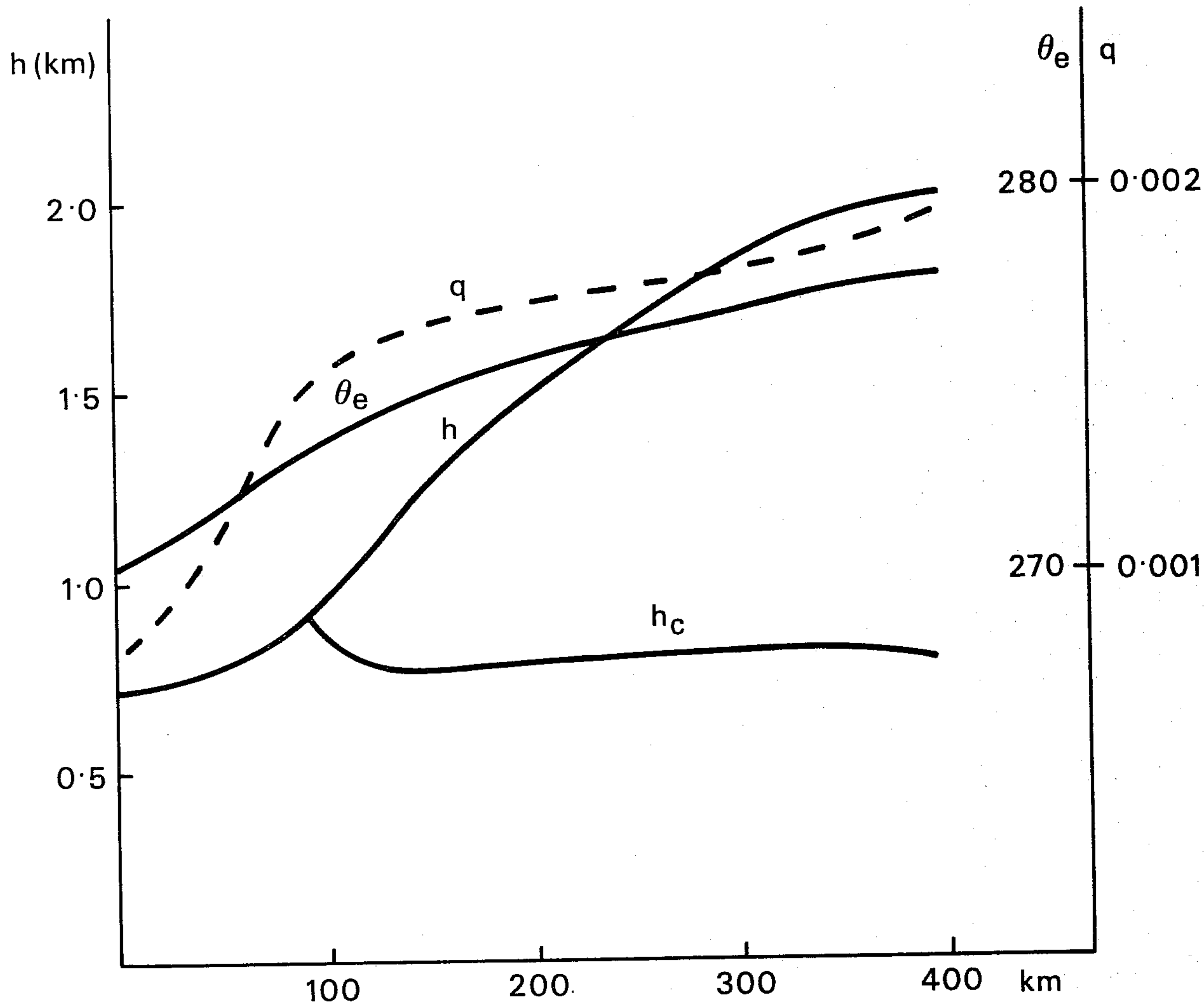

Fig. 4 Results of a model integration. Numbers on abscissa represent distance along a trajectory. The curves show computed values of boundary layer thickness, height of condensation level, equivalent potential temperature and mixing ratio.

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