CONVECTIVE MOMENTUM TRANSPORT

M.W. Moncrieff
Atmospheric Physics Group, Physics Department
Imperial College
London, U.K.

Summary: Large scale momentum and vorticity budget studies are reviewed and oustanding problems identified. Existing methods for the parametrisation of dynamical transports are collated and commented on. The dynamical aspects of deep and shallow convection are reviewed with particular emphasis on the transport of momentum. A simple experiment using analytical cloud models to ascertain the role of momentum transport in large scale simulations is presented.

1. INTRODUCTION

The transport of momentum by convective and other subgridscale processes and its interaction with the large-scale flow is one of the most enigmatic, yet interesting, aspects of parametrisation. There exists no concensus of opinion on the importance of momentum or vorticity transports (collectively referred to as dynamical transports) in determining the structure, development and organisation of either mesoscale phenomena, such as convective complexes, cloud clusters and hurricanes, or the global-scale circulation patterns, or of how these transports should be represented.

The effects of momentum transport are extremely difficult to observe directly, particularly in the tropics where the weak large-scale pressure gradients and substantial local variability can lead to large sources of error in the momentum budget. Consequently, a direct appeal to observations to alleviate the uncertainty of the role of momentum transport is unreliable.

The parametrisation of subgridscale thermodynamical transports (heat and moisture) is essential because these maintain the large scale available potential energy and therefore represent energy producing scales of motion. Unfortunately, no such simple physical justification exists for the dynamical transports, and so their role is considerably more subtle.

It is known that the interaction of the physical processes (boundary layer, convective and radiative) has significant impact on large-scale models, and it is conceivable that equally strong interactions could exist between the dynamical and thermodynamical processes. For instance, the convective heating distribution and intensity can influence the position and amplitude of the Hadley circulation (thermal forcing), but this feature is also sensitive to dynamical transports, as demonstrated by Helfand (1979) using the GLAS model (mechanical forcing). Thus, the mechanical and thermal forcing of the large-scale tropical circulations is associated with the interaction between thermodynamical and dynamical subgridscale transports.

At first sight it is surprising that so little is known about the sensitivity of models to the internal transport of subgridscale momentum. Most of the experimentation has been associated with representing the stresses imposed on the lower boundary, but this should not be considered in isolation from the internal transports of momentum. However, the problem of relating the internal transports to the surface stresses has not yet received attention, probably because the nature of boundary layer transports in convectively disturbed conditions are not generally known. Convective processes are commonly considered as redistributing momentum in the vertical but certain organised (and mesoscale) systems can locally generate momentum. This aspect will be considered later in the context of deep convection. Consequently, the effect of momentum transport is a highly interactive, and thereby complicated, problem which justifies a cautious approach.

Recent experiments with the U.K. Meteorological Office model shows that the internal redistribution of momentum by subgridscale gravity waves can have significant effects on the model zonal flow in mid-latitudes as discussed by Shutts and Dickinson elsewhere in these proceedings. Although the effects of the gravity wave transports may not be fully understood, nevertheless the results indicate a marked model sensitivity; this suggests that *convective* momentum transport should be seriously considered. The total subgridscale transports are at any rate of comparable magnitude to the large-scale transports (Palmen & Newton, 1969).

This paper will review and comment on the wider aspects of convective momentum transport on various scales, but due to the present state of knowledge of the effects in large-scale models the conclusions are far from obvious.

2. LARGE-SCALE ASPECTS

In tropical regions where the horizontal pressure gradients are small, it is difficult to determine representative and accurate momentum budgets from observational data. Since pressure is eliminated in the vorticity equation, budget studies of vorticity are usually performed on tropical data sets. However, global models are universally formulated in terms of momentum equations so a momentum representation of convective transports is ultimately required. In this section the results of large-scale dynamical budget studies will be briefly reviewed.

2.1 Vorticity Budget Studies

In these studies the large-scale terms in the vorticity equation are determined, the equation having been subjected to spatial Reynolds averaging. Due to unresolved scales of motion (and observational errors) the budget equation does not balance and the sum of all the terms defines a residual Z, defined as

$$Z = \frac{\partial \zeta}{\partial t} + \nabla . (\zeta + f) \bar{\underset{\sim}{v}} + \underset{\sim}{k} . \nabla \wedge \omega \frac{\partial \bar{v}}{\partial p} \tag{1}$$

where ζ is the vertical component of relative vorticity. Since Z is considerably larger in convectively disturbed conditions, it is concluded that this residual, (an apparent vorticity source) is due to *convective* transports of vertical vorticity so that

$$Z \cong \left(\frac{\partial \zeta}{\partial t}\right)_c \cong -\nabla . \overline{\zeta' \underset{\sim}{v}'} - \underset{\sim}{k} . \nabla \wedge \overline{\omega' \frac{\partial \underset{\sim}{v}'}{\partial p}} \tag{2}$$

This is the general basis of vorticity budget studies.

One of the earliest vorticity budgets was performed by Riehl & Pearce (1969) using a Carribean data set. The conclusion was that the residuals obtained from these data were of the same order of magnitude as the resolvable (large-scale) terms in the vorticity equation.

A diagnostic study of the 200 mb vorticity balance in the tropics by Holton & Colton (1972) concluded that the vorticity generation by divergence could not (in a linearised analysis) be balanced by vorticity advection without a rapid damping of vorticity being imposed to reproduce the observed (time averaged) vorticity field. It was suggested that this damping (time scale ~1 day) was due to deep convection. In this analysis the circulation pattern was dominated by a large-scale divergent east-west and north-south components associated with the Asian summer monsoon, in which deep convection was prevalent. However, the quality and quantity of the data are questionable in this case.

Vorticity budgets over the Western Pacific have been obtained by Williams & Gray (1973), Reed & Johnston (1974), Ruprecht & Gray (1976), Hodur & Fein (1977) and Chu, Yanai & Sui (1981). Ruprecht & Gray also analysed data in the vicinity of the West Indies.

A number of vorticity budget studies have been made in the Eastern Tropical Atlantic (GATE region) particularly by Shapiro (1978), Stevens (1979), Reeves, Ropelewski & Hudlow (1979), Esbensen, Tollerud & Chu (1982) and Tollerud & Esbensen (1983).

Daggupaty & Sikka (1977) examined the vorticity budget associated with a monsoon low pressure system in the Bay of Bengal and Virgji (1982) considered the upper tropospheric budget over South America.

Consequently, a widely distributed series of studies in the tropics has been performed. Generalising, these indicate that the vertical profile of the vorticity residual is complicated but there exists an apparent source of positive vorticity in the upper troposphere and negative vorticity source in the lower troposphere. This is most marked in a time-mean profile but there is usually a considerable degree of time-dependence in the data. Although it is possible that these residuals are due to observational systematic errors, particularly in the upper troposphere, the general opinion is that convective transports are responsible, particularly those due to organised mesoscale patterns such as cloud clusters. Consequently, the main structures to explain are the upper level positive vorticity source and the low-level sink.

2.2 Momentum Budget Studies

As in the case of vorticity, an apparent momentum source $\underset{\sim}{X}$ can be defined as

$$\underset{\sim}{X} = \frac{\partial \overline{\underset{\sim}{v}}}{\partial t} + (\overline{\underset{\sim}{v}}.\nabla)\overline{\underset{\sim}{v}} + \overline{\omega}\,\frac{\partial \overline{v}}{\partial p} + f\underset{\sim}{k}\wedge\overline{\underset{\sim}{v}} + \nabla\overline{\phi} \qquad (3)$$

where $\underset{\sim}{v} = (u,v)$ and $\overline{\phi}$ is the geopotential. Provided that the residuals are due to subgridscale eddies $\underset{\sim}{X} \cong \left(\frac{\partial \underset{\sim}{v}}{\partial t}\right)_c$ and

$$\left(\frac{\partial \underset{\sim}{v}}{\partial t}\right)_c = -\,(\nabla.\overline{\underset{\sim}{v}')\underset{\sim}{v}'} - \frac{\partial}{\partial p}\,(\overline{\omega'\underset{\sim}{v}'}) \qquad (4)$$

defines the subgridscale momentum sources, which in disturbed conditions is assumed to be predominately due to convective transports.

However, since momentum budgets obtained directly from data may not be meaningful, very few studies have been performed. Global angular momentum budget studies by Palmen & Newton (1969) concluded that the vertical transport of momentum by subgridscale processes is necessary to maintain the time and zonally averaged angular momentum balance. This result was also concluded from Hantel & Hacker (1978), while Houze (1973) estimated that the convective momentum transport is of the same order of magnitude as that due to large-scale processes.

Likewise, very few momentum budget studies have been performed over limited areas. Stevens (1979) calculated momentum budgets due to easterly wave activity in the GATE area. Lee (1984) determined the residuals in the tangential momentum budget in developing tropical cyclones and showed that there are positive residuals in the upper and lower troposphere with negative anomalies in mid-troposphere.

Certain general circulation models suggest that convective momentum transport has significant effects on the Hadley circulation as in Stone, Quirk & Somerville (1974), and Helfand (1979).

Despite the evidence shown in a variety of tropical studies, the view that convective dynamic transport is a significant factor in influencing the general circulation of the tropics is not unanimous, and Thompson & Hartmann (1979) and Sardesmukh & Held (1984) dispute the significance. The latter showed that in the GFDL model a reasonable upper tropospheric flow could be obtained without appealing to subgridscale momentum transport, contradicting the observational results. Consequently, the role of subgridscale momentum transport and its role in the large scale dynamics remains an enigma.

2.3 Momentum Transports Derived from Vorticity Budgets

The above summarises the effect of subgridscale dynamical transports in separately determining vorticity and momentum budgets. To the author's knowledge only one study, Sui & Yanai (personal communication), attempts to derive momentum transports from the more reliable vorticity analysis. Using the GATE phase III data sets,eddies with a horizontal space scale of lesser than 200 km cannot be resolved explicitly by these data, and these residuals are therefore subgridscale. (This also includes the lower end of the mesoscale range). These studies again show that large positive vorticity residuals exist in the upper troposphere during convectively disturbed periods, and that these residuals are correlated with $\partial\bar{\zeta}/\partial p$. Detailed case studies of four major convective events were examined, two squall line type cloud clusters and two non-squall clusters. The former was associated with an enhanced low-level easterly jet.

The momentum residuals can be derived from the vorticity residuals by assuming that

$$\left(\frac{\partial \zeta}{\partial t}\right)_c = \underset{\sim}{k} \cdot \nabla \wedge \left(\frac{\partial \bar{\underset{\sim}{v}}}{\partial t}\right)_c \tag{5}$$

where $\underset{\sim}{v} = (u,v)$ is the horizontal velocity. Splitting $\bar{\underset{\sim}{v}}$ into irrotational and non-divergent components

$$\bar{\underset{\sim}{v}} = \bar{\underset{\sim}{v}}_\psi + \bar{\underset{\sim}{v}}_\chi \tag{6}$$

where $\bar{v}_\psi = \underset{\sim}{k} \wedge \nabla\psi$ is the non-divergent (rotational) part of the velocity and $\bar{\underset{\sim}{v}}_\chi = \nabla\chi$ is the irrotational (divergent) part. Then Eq (5) can be written as a Poisson equation,

$$\nabla^2 \left(\frac{\partial \psi}{\partial t}\right) = Z \tag{7}$$

which can be solved subject to appropriate boundary conditions. Hence the rotational component is,

$$\left(\frac{\partial \bar{v}_\psi}{\partial t}\right)_c = \underset{\sim}{k} \wedge \nabla \left(\frac{\partial \psi}{\partial t}\right)_c$$

Sui & Yanai conclude that the vorticity and momentum residuals indicate three distinct aspects.

(a) The upper tropospheric flow is retarded by the convective activity and this can be visualised as representing a vortex doublet.

(b) The mid-tropospheric flow, and to a lesser extent the upper level flow, show strong correlations between $\partial\bar{\zeta}/\partial p$ and Z suggesting that convectively induced subsidence is dominant.

(c) The low-level flow is probably affected by shallow convective and boundary layer eddies.

3. DYNAMICAL ASPECTS OF ORGANISED CONVECTION

Budget studies give little information regarding the nature of the eddies which effect the dynamical transports. These are more subtle than the thermodynamical counterparts because distinct regimes of convection in different environmental shears can have markedly different dynamical transports, even though the thermodynamical transports are broadly similar. Consequently, the *organisation* of convection by the large-scale shear is central in influencing the nature and amplitude of the dynamical transports.

If the convection was simply a stochastic process then it would be valid to represent the transports by a form of "K" theory, based on momentum diffusion principles. However, one of the distinctive features of convection in the earth's atmosphere is its highly organised nature, with deep convection in particular occurring in a series of well-isolated events. The dynamical transports and the mean flow interaction are thereby complex and difficult to represent in terms of mean flow variables, and of a different form from stochastic, fully turbulent features. As a result, a detailed understanding of the processes which effect organisation and scale in convective processes is necessary.

3.1 Scales of Convective Organisation

Convection is organised on a variety of interactive scales, which considerably complicates a representation of the transports. It is convenient to divide the different types of organisation into four categories, on a scale principle, as schematically shown in Fig. (1).

By definition, the mesoscale types of organisation have scales greater than the grid scales of global models but are still not well resolved. In terms of dynamical structure, the large scale *vertical shear* is of fundamental importance, and large-scale convergence is known to be important in maintaining the systems. It is therefore instructive to review the main roles of the vertical shear in influencing the convective structure by appealing to dynamical models.

3.2 Role of the Vertical Shear in Determining the Dynamical Transports

The vertical shear has two main roles in parametrisation. First, it strongly influences the convective dynamics, particularly the 'shape' of the convection and thereby the *direct* dynamical transport and, second, the profile of the environmental subsidence momentum transport is a function of the shear profile, an *indirect* effect. For various reasons, it is convenient to consider deep (precipitating) and shallow (non-precipitating) convection separately.

3.2.1 Deep Convection

The vertical shear has long been known to strongly influence the form of mid-latitude cumulonimbus convection, as many observational studies have shown (Browning & Ludlam (1962), Newton (1966)), while tropical

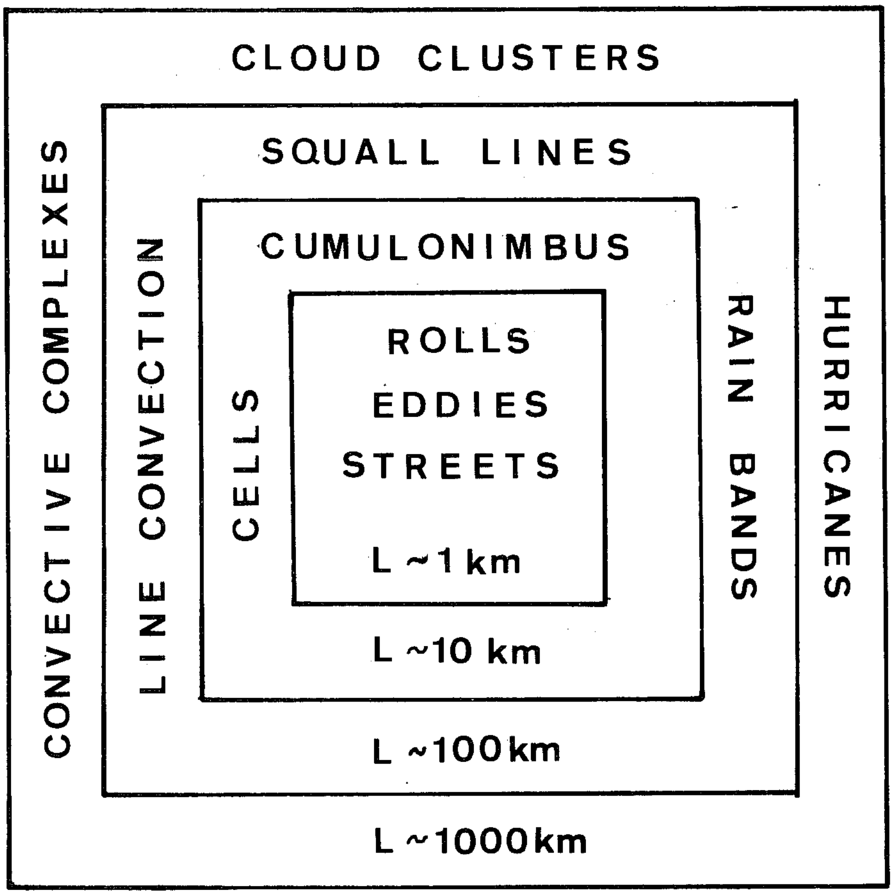


Fig. 1 Scales of convective organisation

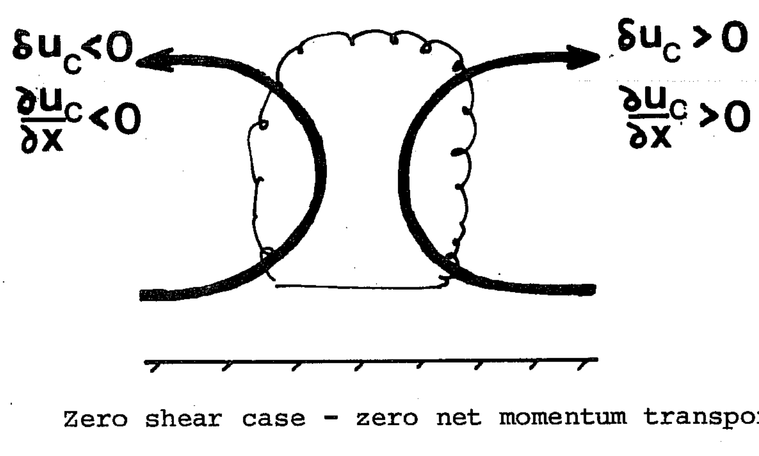


Fig. 2 Zero shear case - zero net momentum transport

cumulonimbus and squall lines in particular are well correlated with jet-like profiles (Betts, Grover & Moncrieff (1976), Fernandez (1980), Bolton (1984)).

Numerical models have confirmed the importance of the vertical shear (Moncrieff & Miller (1976), Klemp & Williamson (1978), Weisman & Klemp (1982), Thorpe, Miller & Moncrieff (1982)).

Theoretical models by the author and colleagues have dynamically quantified the role of the shear and the different regimes of convection which can exist. (Moncrieff & Green 1972, Moncrieff & Miller 1976, Moncrieff 1978, 1981, Thorpe, Miller & Moncrieff 1982). (These theoretical models are in fact relevant to both cloud and mesoscale organisation). The simplest type of model representing convection in small vertical shear (classical model) has been used by Miller & Moncrieff (1983) as a dynamical cloud model in a deep convection parametrisation scheme. The more complex models reviewed in Moncrieff (1981) are relevant if the large-scale vertical shear is significant or has a particular profile. These shear flow models are of particular relevance to the transport of momentum and it is useful to summarise the most important aspects. (It should also be noted that these models are two-dimensional but are useful in demonstrating important principles).

The apparent momentum source can be represented as a sum of detrainment and subsidence fluxes, following the analogy of Ooyama (1971), so considering the x-component of momentum in pressure coordinates

$$\left(\frac{\partial \bar{u}}{\partial t}\right)_c \cong \frac{a}{A}\left[(\delta u)_c \frac{\partial u_c}{\partial x} - \omega_c \frac{\partial \bar{u}}{\partial p}\right] \tag{9}$$

where $u_c(p)$ is the net outflow momentum per unit mass, ω_c is the cloud p-velocity, a/A is the fractional cloud area. δu_c, $\partial u_c/\partial p$ and ω_c can in principle be derived from dynamical cloud models. It is convenient to choose special cases, and for simplicity only updraught fluxes are considered. Eq (9) shows that if $\partial\bar{u}/\partial p$ is non-zero then the last term (subsidence) induces momentum transport, and the first term (outflow fluxes) is non-zero only if $\partial\bar{u}/\partial p \neq 0$ because otherwise equal amounts of positive and negative outflow momentum are generated, as in Fig.(2).

(a) Steering-Level Model with $\partial\bar{u}/\partial p$ = Constant

In this case there are contributions from both subsidence and outflow fluxes, as summarised in Fig. (3a). It was shown by Moncrieff (1978) that

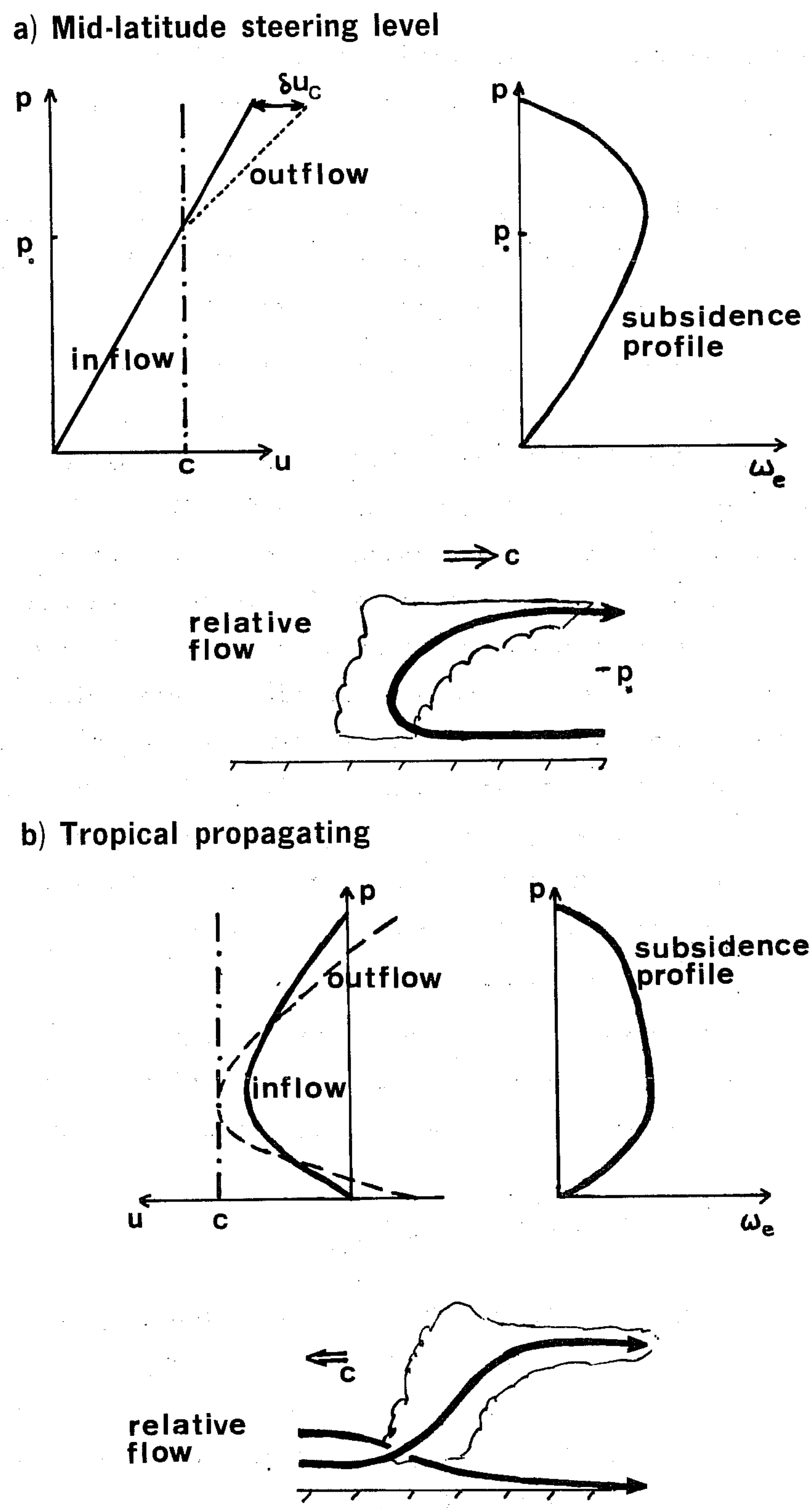
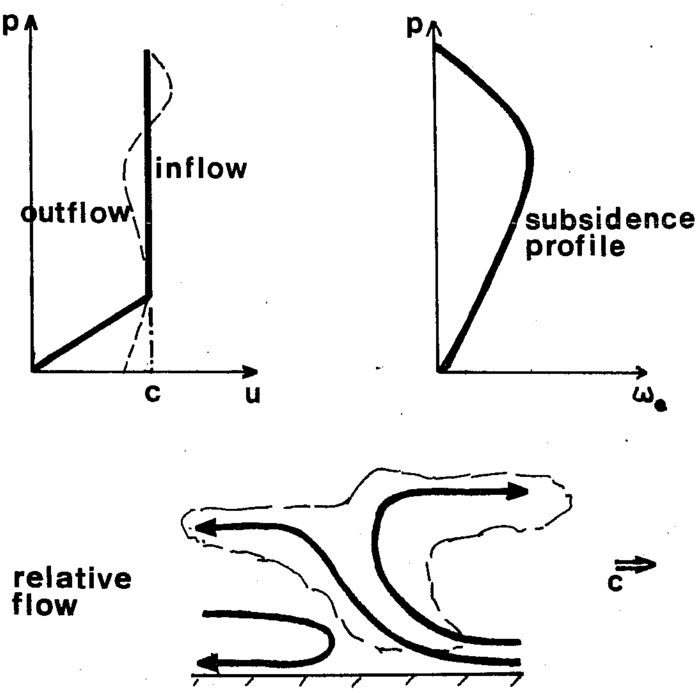


Fig.3 Organised convective transport regimes:

(a) midlatitude steering-level (in constant shear)
(b) tropical propagating (in jet profile)
(c) midlatitude squall line (in large low-level shear profile)
(d) tropical propagating jump (in double jet profile)

c) Mid-latitude squall line



d) Tropical propagating jump

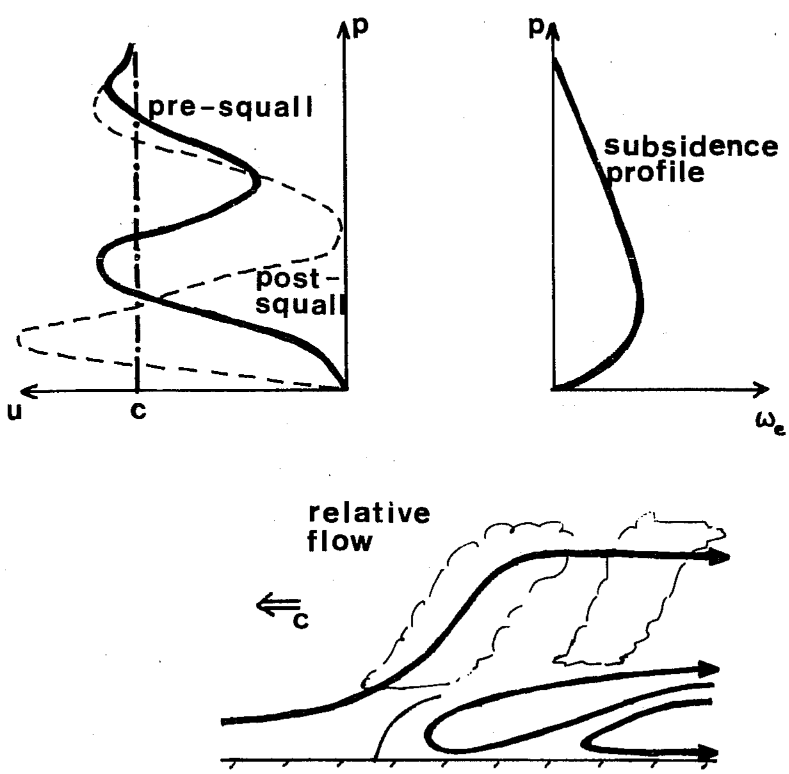


Fig.3 (Contd.)

the cloud is tilted downshear and so momentum is transported counter-gradient. The outflow term therefore enhances the flow, while the subsidence term advects the profile downwards, the shape depending on the variation of $\bar{\omega}$ with pressure.

(b) Propagating and Overturning Model

This model developed by Moncrieff & Miller (1976) has a different direct effect on the environment, and is relevant for tropical convection because it propagates relative to the flow everywhere in the troposphere, as is observed to be the case for tropical squall lines which tend to be associated with jet-like profiles.

The predominant feature of this case is that westerly momentum is injected in upper levels which has the effect of decreasing the easterlies, but easterly momentum is injected into mid levels, as shown in Fig. (3b). The effect of subsidence induced momentum is to decrease easterly momentum in upper levels and increase it in low levels.

(c) Jump Model (Large Shear in Low Levels)

This type of model was produced in Thorpe, Miller & Moncrieff (1982), and is different from case (b) because the parcels do not overturn but 'jump' up everywhere. The main effect on the cloud scale is to 'block' the low level flow and, as can be seen, the subsidence will only be significant in the low levels, as shown in Fig. (3c).

(d) Jump Model (Jet-Like Profiles)

This model has been applied to West African Squall Lines (Dudhia, Moncrieff & So, 1985) and is therefore relevant to squall line cloud clusters in GATE. In a similar fashion to case (b), the upper levels easterlies are depleted by the direct effect of this type of convection, as shown in Fig. (3d).

These models show that the nature of the vertical shear is very important in two respects. First, in determining the type of convection and thereby the outflow modification and, second, in influencing the important subsidence induced momentum transport. Clearly, it is quite misleading to make generalisations or form composites in different shear conditions. The above also implies that the effect of momentum transport can in principle be geographically determined for different shear regimes.

The propagating and jump models have two different sources of outflow momentum, namely momentum transport effected by the divergence of the correlations between the horizontal and vertical eddy components and momentum generated from the pressure gradient. The latter, unlike the former, does not integrate to zero over the convective layer; it thereby provides a net source of momentum to the convective grid column, and thereby is associated with convectively induced horizontal pressure gradients.

The above considerations show that the momentum can either be *enhanced or depleted depending on the type of convection which exists*. Moreover, the effect of environmental subsidence is generally the dominant process and this is directly a function of the vertical shear.

It is interesting to note that the jump and propagating cases can provide an explanation of why Sui & Yanai's budget results showed a depletion of the upper level easterly momentum, the large scale shear is broadly similar to case (d) which injects westerly momentum into upper levels.

3.2.2 Shallow Convection

The role of convection in the boundary layer and lower troposphere is generally accepted as influencing the thermodynamic profiles, and in this sense its representation is important. Experience has shown that both deep and shallow convection are necessary. The particular role of *organised* shallow convection in determining the momentum balance of the lower troposphere is not yet clear. The budget studies reviewed above show that significant vorticity transports exist in the lower troposphere, but it is not possible to determine if the organised shallow convective eddies (cloud streets, Ekman cloud rolls, cellular convection, and large eddies in general) have a significant effect which is not included in present boundary layer stress parametrisation schemes. The stress parametrisation in the boundary layer is based on fully turbulent flow principles without explicit representation of 'large eddies' either of the mechanical or convectively driven type. The vertical shear is again of fundamental importance in influencing eddy structure. The nature of the boundary layer dynamical transports in convectively disturbed conditions is not well known; the effect of downdraughts, gust fronts, etc. in the wake of deep convective activity is an outstanding problem.

It is useful to review and classify different types of shallow convection in terms of their dynamical structure, since this is closely associated with the transport properties. It is convenient to classify the (linearised) models of small-scale convection into inviscid and viscid types.

3.2.3 Inviscid Types

(a) Stable Stratification

Gravity waves are the simplest and most well known phenomena in stably stratified conditions. The long gravity waves in particular can transmit energy up to great heights, where the amplitude increases like $(\rho/\rho_s)^{-\frac{1}{2}}$. The absorption of gravity wave energy due to critical layers and shear was a topic developed by Bretherton (1969). The subject of gravity wave drag is now receiving fresh interest (Shutts, 1985, Dickinson, 1985) from the parametrisation viewpoint. However, gravity waves represent only one mechanism of small-scale transport. These gravity wave drag experiments indicate that large-scale models are sensitive to subgridscale dynamical transports, and for this reason the momentum transport by subgridscale effects is likely to be a problem of considerable future interest.

In the presence of rotation, stably stratified flow can become symmetrically unstable. This is most relevant in frontal regions where the instability may manifest itself as rainbands, but as considered by Bennetts & Hoskins (1979), this requires the presence of moisture.

(b) Unstable Stratification

In unstably stratified conditions the structure of the convective instability is governed by a Richardson Number. However, in inviscid flow no finite space scale exists and this is a fundamental problem; the ratio of the vertical to horizontal wavelength of the fastest growing modes is infinite. The case of two-dimensional disturbances in a constant shear flow is the only exception but, as shown by Kuo (1963), three-dimensional disturbances have an orientation parallel to the wind shear with a much faster growth rate than the two-dimensional counterpart. The perpendicular component in this case has no finite space scale as above. Consequently, the inviscid dynamical structure as deduced from linearised theory cannot be satisfactory.

3.2.4 Viscid Types

For atmospheric application eddy viscosity is appropriate since molecular viscosity is only effective on timescales many orders of magnitude greater than observed.

(a) Stable Stratification

In the presence of viscosity, gravity waves are damped and although energy is dissipated to the mean flow, no additional gravity modes appear.

In a rotating flow the dynamics are controlled by the Taylor Number $T_a = v_g/(K_m\Omega)^{\frac{1}{2}}$ where v_g is the geostrophic wind in the free atmosphere, K_m is the viscosity coefficient and Ω represents the rotation. The most important large eddies are Faller vortices or Ekman instability eddies, which are aligned at an angle to the geostrophic wind direction in the free atmosphere at about 10° in counterclockwise sense. These eddies transport momentum from the mean flow, and have an associated vorticity transport which can affect the Ekman pumping. According to Lilly (1966) the Ekman instability exists for $T_a > 100$, while for $T_a < 100$ parallel instability orientated clockwise from the geostrophic wind is the preferred type. These large eddies are relevant to hurricane boundary layer at large distances from the eye, in nocturnal boundary layers and dry forced convection in the subcloud layer particularly over the oceans.

(b) Unstable Stratification

Cellular convection is the main type of convection in an unsheared basic flow (Rayleigh-type instability). The dynamics of this motion are not fully understood. There is little doubt that this type of convection is a momentum mixing process controlled by the Rayleigh Number $R_a = g/\theta$ $\partial\theta/_{\partial z}/K_M K_H$. However, in a shear flow there exists a preferred orientation of the convection at a small angle to the geostrophic wind. The direction of momentum transport is such as to cause momentum mixing parallel to the band axis but upgradient transport transverse to the bands.

Consequently, there are many types of large eddies involved in subgridscale momentum transport and the process is generally anisotropic. These are most prevalent in the boundary layer. It is likely that the drag due to these eddies will dominate gravity wave drag in the lower atmosphere but at very high levels the latter is probably more important.

4. THE PROBLEM OF THE SCALE OF CONVECTIVE INFLUENCE

One of the most fundamental problems which arises in the interaction between convection and the mean flow is the scale of the response of the environment to the scale of the convective forcing (this is closely associated with the aspect ratio defined as the ratio of the horizontal to the vertical scale). Inviscid, linearised theory is unhelpful because the aspect ratio is infinitely small; viscid theory gives a finite aspect ratio but is usually too small.

In parametrisation, the ratio of the horizontal area of the updraught to the grid area (fractional area) is an important parameter. It arises directly in the Kuo scheme, indirectly in all mass flux schemes and in "soft" adjustment methods disguised as an adjustment time scale. Present parametrisation schemes are applied in a "one-dimensional" fashion in the sense that the effect of convection in one grid box is isolated from neighbouring boxes. Thus, in mass flux schemes convective mass flux is compensated in each individual grid box, which clearly poses problems in high resolution models, because the subsidence is constrained to take place over an area which can be physically too small.

The dynamical problem of an isolated heat source in a large domain was addressed by Eliassen (1959). (This problem can also be solved by considering a point source in a Green's function approach). The appropriate horizontal scale length can be shown to be the Rossby radius $\sqrt{gB}\,H/f$ which is of the order 1000 km. Consequently, in the case of an isolated heat source in a rotating fluid the appropriate descent scale is the Rossby radius.

The problem of momentum injection at a source involves geostrophic adjustment. Again, the appropriate scale of response is controlled by the Rossby radius. For parametrisation purposes, it can be assumed that the momentum and thermal adjustment scales are the same.

5. MOMENTUM PARAMETRISATION METHODS

Following the analogy of thermodynamic parametrisation based on the work of Ooyama (1971), the apparent momentum source can be written for a single cloud in pressure coordinates as

$$\left(\frac{\partial \underset{\sim}{v}}{\partial t}\right)_c = \frac{a}{A} \left[(\underset{\sim}{v}_c - \bar{\underset{\sim}{v}})(\text{div}\ \underset{\sim}{v})_c - \omega_c \frac{\partial \underset{\sim}{v}}{\partial p}\right] \tag{10}$$

where $\underset{\sim}{v} = (u,v)$ and a/A is the fractional cloud area so the cloud mass flux is $M_c = a\omega_c/A$. If a cloud ensemble is taken then this expression is summed over the cloud spectrum.

In the Schneider & Lindzen (1976) scheme, it was assumed that the cloud momentum $\underset{\sim}{v}_c$ is conserved and that $\underset{\sim}{v}_c = (\bar{\underset{\sim}{v}})_{\text{cloud base}}$ in which case

$$\left(\frac{\partial \underset{\sim}{v}}{\partial t}\right)_c \cong \frac{\partial}{\partial p} \left[M_c\ (\underset{\sim}{v}_c - \bar{\underset{\sim}{v}})\right] \tag{11}$$

where M_c is the mass flux at cloud base. The assumption that the cloud momentum is conserved is only justified if zero net work is done by the pressure field. In steady two-dimensional flow the quantity

$$\underset{\sim}{V} = \underset{\sim}{v} - \int_{z_o}^{z} \frac{\partial}{\partial \psi} \left(\frac{\delta p}{\rho}\right) dz \tag{12}$$

is conserved by parcels, where $z - z_o$ is the displacement from an inflow level and δp is the pressure deviation from a static mean state. Momentum is therefore only conserved if the net work done by the pressure field is zero. If V is parametrised then a more dynamically respectable scheme would result. Shapiro and Stevens (1980) used a similar scheme in a study of the dynamical transports associated with the GATE composite wave data set.

If a two-dimensional cloud model is used then,

$$\left(\frac{\partial \underset{\sim}{v}}{\partial t}\right)_c = \frac{a}{A} \left[(u_c - \bar{\underset{\sim}{v}}) \left(\frac{\partial u}{\partial x}\right)_c - \omega_c \frac{\partial \underset{\sim}{v}}{\partial p}\right] \tag{13}$$

Since the cloud model can give u_c, ω_c and $(\partial u/\partial x)_c$, $(\partial \underset{\sim}{v}/\partial t)_c$ can be found directly.

Helfand (1979) used the GLAS general circulation model and found sensitivity to the momentum transport by deep convection, particularly in its effect on the meridional circulation. The Hadley circulation was strengthened, which reduced the mean meridional temperature gradient in the tropics. This led to an enhanced baroclinic activity in mid-latitude because the meridional temperature gradient in that region was increased.

6. VORTICITY PARAMETRISATION METHODS

Convective, vorticity transports used in the dynamical studies of Reed and Johnston (1974) and Shapiro (1978) were parametrised by using

$$\left(\frac{\partial \zeta}{\partial t}\right)_c = M_c \frac{\partial}{\partial p} (\zeta_c - \bar{\zeta}) \tag{14}$$

where $\bar{\zeta}$ is the average cloud vorticity.

More recent studies by Shapiro and Stevens (1980), Cho and Cheng (1980), Yanai, Sui and Chu (1982) use a form which included entrainment, detrainment, twisting and stretching of vortex tubes,

$$\left(\frac{\partial \zeta}{\partial t}\right)_c = \frac{\partial u_c}{\partial x} (\zeta - \zeta_c) - M_c \frac{\partial \bar{\zeta}}{\partial p} + \bar{\eta} \frac{\partial M_c}{\partial p} - \underset{\sim}{k} . \nabla M_c \wedge \frac{\partial \bar{\underline{v}}}{\partial p} \tag{15}$$

where $\eta = \zeta + f$ is the absolute vorticity. The terms on the right hand side are outflow fluxes, subsidence fluxes, stretching of absolute vorticity and twisting of mean flow vorticity by the subsidence, respectively.

A basic unresolved problem is to ascertain whether momentum or vorticity parametrisation is the most fundamental, and to show that the methods are mutually consistent. However, since large-scale models use momentum equations for practical purposes a momentum parametrisation is required.

7. THE USE OF ORGANISED CONVECTION MODELS

It is important that the thermodynamical and dynamical transports are self-consistent otherwise systematic errors could be induced. For instance, thermodynamical transports can act to intensify the Hadley circulation, while the mechanical effects of the dynamical transports can either intensify or deplete the Hadley circulation, depending on the type of dynamical transports.

The use of fully non-linear dynamical convection models can ensure that these fluxes are self-consistent. The author's analytical models have been used in Miller & Moncrieff (1983) for thermodynamic transports. These models have recently been made more suitable for parametrisation through the use of pressure (and sigma) coordinates by Moncrieff (1985). Moreover recently subsidence profiles have been generalised by using parcel lapse rates which are functions of individual trajectories.

A simple dynamical and thermodynamical parametrisation scheme can be devised if it is assumed that the subsidence fluxes dominate the outflow fluxes, in which case

$$\left(\frac{\partial \emptyset}{\partial t}\right)_c = \dot{\sigma}_e \frac{\partial \bar{\emptyset}}{\partial \sigma} \tag{16}$$

where $\emptyset$ = u, v, q or T.

Using the mass flux profile for the steering-level model and following the counterparts of the formulae developed in Miller & Moncrieff and modified in Moncrieff (1985) to sigma coordinates,

$$\dot{\sigma}_e(\sigma) = -\int_{\sigma=1}^{\sigma} \left(\frac{\partial u_e}{\partial x}\right) d\sigma = \begin{cases} \frac{\sqrt{A}}{2L_R} \int_{\sigma_*}^{\sigma} u_o d\sigma_o & 1 \geqq \sigma \geqq \sigma_* \\ \frac{\sqrt{A}}{2L_R} \left[\int_{1}^{\sigma_*} u_o d\sigma_o - \int_{\sigma_*}^{\sigma} u_1 d\sigma \right] & \sigma_* \geqq \sigma \geqq \sigma_T \end{cases} \tag{17}$$

where $L_R = \sqrt{g\ B}\ H/f$, σ_T is the cloud top, A is the grid area. σ_* is given by

$$\sigma_* = (1 + \beta\sigma_T)/(1 + \beta) \ ; \ \beta = \tfrac{1}{2}(1 + \sqrt{1 + 4R}) \tag{18}$$

where $R = CAPE/\tfrac{1}{2}(\Delta u)^2$ is a Richardson Number, so $\Delta u = (2CAPE/R)^{\frac{1}{2}}$ and defining $A = \Delta u/H$, the inflow and outflow speeds were shown by Moncrieff (1985) to be given by the expressions

$$u_o(\sigma_o) = -A(\sigma_o - \sigma_*) \tag{19}$$

$$u_1(\sigma_1) = A\beta^2/(\sigma_* - \sigma_1) \tag{20}$$

with $$\sigma_1(\sigma_o) = \sigma_o - (1 + \beta)(\sigma_o - \sigma_*)/\beta \tag{21}$$

The value of CAPE, the convective available potential energy, and the cloud top (σ_T) are determined by conventional methods from grid-scale data as in Miller & Moncrieff (1983). The value of R can either by specified or calculated from CAPE and the grid-scale vertical shear.

This scheme can be made more complicated by

(a) including the outflow fluxes

$$(u_c - \bar{\underset{\sim}{v}}) \left(\frac{\partial u}{\partial x}\right)_c$$

(b) including modified forms for $\dot{\sigma}_e$ using different cloud models.

The thermodynamical and dynamical transports can therefore be separately assessed in various degrees of complexity and the individual contributions to the outflow and subsidence fluxes determined. The different types of dynamical cloud model affect the scheme in two ways. First, by changing the vertical profile of $\dot{\sigma}_e$ and second by changing the outflow fluxes. The former is the dominant process but the latter is probably a subtle effect because it can represent either downgradient or upgradient transports.

8. CONCLUSIONS

Basic aspects of the parametrisation of dynamical quantities in large-scale models have been reviewed, and it is clear that a number of outstanding problems exist. The large-scale observational budget studies indicate that vorticity sources and sinks are present. However, a dynamical mechanism for producing these anomalies has yet to be established. Since the mean production of vorticity in a finite volume of fluid is highly constrained and determined by boundary fluxes, it is difficult to establish a dynamical mechanism for the *internal* transports. Moreover, certain large scale simulations seem not to require dynamic transports to achieve realistic atmosphere structure. It is important that this paradox should be resolved.

The effect of stationary gravity wave dynamical transports on the global circulation shows that at least some small scale dynamical processes are important. Since gravity wave and convective transports are related this indicates that fundamental problems have to be addressed to resolve the enigma of dynamical transports. The consistency of momentum, vorticity and thermodynamic parametrisation is an area which has yet to be established.

Over recent years a large amount of effort has been expended in obtaining realistic thermodynamical parametrisation schemes, yet no existing operational model contains a specification of momentum transport. This disparity is understandable but unsatisfactory and it is considered that until the effects of dynamical transports are properly understood, the representation of physical effects in large scale models cannot be completely assessed.

9. REFERENCES

Bennetts, D.A. and B.J. Hoskins, 1979: Conditional symetric instability - a possible explanation for frontal rainbands. Quart. J. R. Met. Soc. 105, 945-962.

Betts, A.K., R.W. Grover and M.W. Moncrieff, 1976: Structure and motion of tropical squall lines over Venezuela. Quart. J. R. Met. Soc. 102, 395-404.

Bretherton, F.P., 1969: Momentum transport by gravity waves. Quart. J. R. Met. Soc. 95, 213-243.

Browning, K.A. and F.H. Ludlam, 1962: Airflow in convective storms. Quart. J. R. Met. Soc. 88, 117-135.

Chu, J.H., M. Yanai and C.H. Sui, 1981: Effects of cumulus convection on the vorticity field in the tropics Part I, The large scale budget. J. Meteor. Soc. Japan, 59, 535-546.

Daggupaty, S.M. and D.R. Sikka, 1977: On the vorticity budget and vertical velocity distribution associated with the life cycles of monsoon depressions. J. Atmos. Sci., 34, 773-792.

Dudhia, J., M.W. Moncrieff and D.K. So, 1985: The two-dimensional dynamics of West African squall lines (manuscript).

Eliassen, A., 1959: On the formation of fronts in the atmosphere. The Atmosphere and Sea in Motion. Rockefeller Institute Press/Oxford University Press.

Esbensen, S.K., E.I. Tollerud and J.H. Chu, 1982: Cloud-cluster-scale circulations and vorticity budget of synoptic-scale waves over the eastern Atlantic ITCZ. Mon. Wea. Rev., 110, 1677-1692.

Fernandez, W., 1980: Environmental conditions and structure of some types of convective mesosystems observed over Venezuela. Arch. Meteor. Geoph. Biokl. Series A, 29, 248-267.

Hantel, M. and J.M. Hacker, 1978: On the vertical eddy transports in the in the northern atmosphere, Part 2. Vertical eddy momentum transport for summer and winter. J. Geophys. Res., 83, 1305-1318.

Helfand, H.M., 1979: The effect of cumulus friction on the simulation of the January circulation by the GLAS model of the general circulation. J. Atmos. Sci., 36, 1827-1843.

Hodur, R.M. and J.S. Fein, 1977: A vorticity budget over the Marshall Islands during the spring and summer months. Mon. Wea. Rev., 105, 1521-1526.

Holton, J.R. and D.E. Colton, 1972: A diagnostic study of the vorticity balance at 200 mb in the tropics during the northern summer. J. Atmos. Sci. 29, 1124-1128.

Klemp, J.B. and R.B. Wilhelmson, 1978: The simulation of three dimensional convective storm dynamics. J. Atmos. Sci., 35, 1070-1086.

Kuo, H.L., 1963: Perturbation of plane couette flow on stratified fluids and the origin of cloud streets. Phys. Fluids, 6, 165-183.

Lee, C.S., 1984: The bulk effects of cumulus momentum transport in tropical cyclones. J. Atmos. Sci., 41, 590-603.

Lilly, D.K., 1966: On the instability of Ekman boundary flow. J. Atmos. Sci., 23, 481-494.

Miller, M.J. and M.W. Moncrieff, 1983: The use and implementation of dynamical cloud models in a parametrisation scheme for deep convection. Proceedings of Workshop on Convection in Large-scale Numerical Models, ECMWF.

Moncrieff, M.W., 1978: The dynamical structure of two-dimensional steady convection in constant vertical shear. Quart. J. R. Met. Soc., 104, 543-567.

Moncrieff, M.W., 1981: A theory of organised steady convection and its transport properties. Quart. J. R. Met. Soc., 107, 29-50.

Moncrieff, M.W., 1985: Steady convection in pressure coordinates. Quart. J. R. Met. Soc., 111, 857-866.

Moncrieff, M.W. and J.S.A. Green, 1972: The propagation and transfer properties of steady convective overturning. Quart. J. R. Met. Soc., 98, 336-352.

Moncrieff, M.W. and M.J. Miller, 1976: The dynamics and simulation of tropical cumulonimbus and squall lines. Quart. J. R. Met. Soc., 102, 373-384.

Newton, C.W., 1966: Circulations in large sheared cumulonimbus. Tellus 18, 699-712.

Ooyama, K., 1971: A theory of the parametrisation of cumulus convection. J. Met. Soc. Japan, 49, 744-756.

Palmer, E. and C.W. Newton, 1969: Atmospheric circulation systems. New York Academic Press, 603 pp.

Reed, R.J. and R.H. Johnson, 1974: The vorticity budget of synoptic-scale wave disturbances in the tropical western Pacific. J. Atmos. Sci., 31, 1784-1790.

Reeves, R.W., C.F. Ropelewski and M.D. Hudlow, 1979: Relationship between large-scale motion and convective precipitation during GATE. Mon. Wea. Rev. 107, 1154-1168.

Riehl, H. and R.P. Pearce, 1968: Studies of interaction between synoptic and mesoscale weather elements in the tropics. Atmospheric Science Paper 126, Dept. Atmos. Sci., Colorado State University, Fort Collins, Colorado 80523.

Ruprecht, E. and W.M. Gray, 1976: Analysis of satellite observed tropical cloud clusters 1. Wind and dynamic fields. Tellus 28, 391-415.

Sardeshmukh, P.D. and I.M. Held, 1984: The vorticity balance in the tropical upper troposphere of a general circulation model. J. Atmos. Sci., 41, 768-778.

Schneider, E.K. and R.S. Lindzen, 1976: A discussion of the parametrisation of momentum exchange by cumulus convection. J. Geophys. Res., 81, 3158-3160.

Shapiro, L.J., 1978: The vorticity budget of a composite African tropical wave disturbance. Mon. Wea. Rev., 106, 806-817.

Shapiro, L.J. and D.E. Stevens, 1980: Parametrisation of convective effects on the momentum and vorticity budgets of synoptic-scale Atltantic tropical waves. Mon. Wea. Rev., 108, 1816-1826.

Stevens, D.E., 1979: Vorticity, momentum and divergence budgets of synoptic-scale wave disturbances in the tropical eastern Atlantic. Mon. Wea. Rev., 107, 535-550.

Stone, P.H., W.J. Quirk and R.C.Somerville, 1974: The effect of small-scale vertical mixing of horizontal momentum in a general circulation model. Mon. Wea. Rev., 102, 765-771.

Thompson, S.L. and D.L. Hartmann, 1979: Cumulus Fiction: Estimated influence on the tropical mean meridional circulation. J. Atmos. Sci., 36, 2022-2026.

Thorpe, A.J., M.J. Miller and M.W. Moncrieff, 1982: Two dimensional convection in non-constant shear: A model of mid-latitude squall lines. Quart. J. R. Met. Soc., 108, 739-762.

Tollerud, E.I. and S.K. Esbensen, 1983: An observational study of the upper-tropospheric vorticity fields in GATE cloud clusters. Mon. Wea. Rev., 111, 2161-2175.

Virgji, H, 1982: An estimate of the summertime tropospheric vorticity budget over South America. Mon. Wea. Rev., 110, 217-224.

Weisman, M.L. and J.B. Klemp, 1982: The dependence of numerically simulated convective storms on the vertical wind shear and buoyancy. Mon. Wea. Rev., 110, 504-520.

Williams, K.T. and W.M. Gray, 1973: Statistical analysis of satellite-observed trade wind cloud clusters in the western north Pacific. Tellus 25, 313-336.

Yanai, M., C.H. Sui and J.H. Chu, 1982: Effects of cumulus convection on the vorticity field in the tropics Part II: Interpretation. J. Meteor. Soc. Japan, 60, Special Issue 411-424.