PARAMETRIZATION OF SUB-GRID SCALE GRAVITY WAVE MOMENTUM TRANSFER AND ITS INFLUENCE IN FORECAST/CLIMATE MODELS

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1. INTRODUCTION

There is general agreement today that gravity waves propagating upwards from the troposphere play a dominant role in shaping the circulation of the mesosphere, primarily through momentum transport. Curiously their importance in the troposphere and stratosphere is more controversial and it is fairly clear that their impact on the zonal circulation is secondary to that of synoptic scale systems. Whilst most features of the weather chart are successfully simulated by current numerical models there is growing evidence that orographic effects are being mishandled. Particularly in long term integrations (> ten days) there is a strong tendency towards zonality in the large-scale flow leading to unreasonable sea-level pressure patterns. (Mitchell and Bolton, 1982; Sadourny, 1982). The success of these 'climate' simulations is far more sensitive to sub-grid scale parameterization than short-range weather forecasts made with the same models.

Terms in the equations of motion which are locally small at any instant need not imply a negligible effect in the long term. It will be argued in this paper that it is the efficiency with which the earths orography extracts momentum from the overlying flow which is inadequately represented in numerical models and that this accounts for much of the observed systematic tendency towards zonality.

At first sight, the device of the sigma (σ)-coordinate transformation (Phillips, 1957) appears to dispense with possible problems at the lower boundary by subsuming the influence of varying terrain height into the adiabatic framework of the forecast model. However, as far as current numerical models (with horizontal resolution of about 150 km) are concerned much of dynamics of flow over mountain ranges is unresolvable and therefore should be regarded as irreversible physics. Even if these smaller scales were resolvable the notion that air adheres to the surface terrain (ie. $\dot{\sigma} = 0$ at $\sigma = 1$) is too restrictive precluding as it does the frequently observed separation effect. (Scorer, 1978). Until recently there have been few attempts to parametrize the effect of the unresolved scales of orographic height variation. Presumably it has been considered in the past that surface friction parametrization (via boundary layer similarity theory) and explicitly resolved pressure torque on major mountain ranges would satisfactorily account for the exchange of momentum between atmosphere and surface. Quantitative evidence that this is not the case will be provided later. Meanwhile we draw attention to the inefficiency of momentum transfer implied by conventional Monin-Obukhov similarity theory in stable conditions such as prevail over the continents in winter. By comparison the ability of orographically forced gravity waves to exert a substantial force on even the most gentle undulations in the landscape (through differences in pressure between the windward and lee slopes) is heightened. Under such circumstances, subjective experience suggests that cold air over Europe in winter tends to be too readily swept aside by Atlantic depressions in forecast models whereas in reality some resistance is often shown. 45 day integrations of the Meteorological Office 11-layer GCM under wintertime conditions frequently show the Atlantic stormtrack extending into Eastern Europe and sometimes as far as Siberia. Similar deficiencies exist over the United States where one might have expected the Rocky mountains to provide an effective barrier.

There is some question as to whether the explicit representation of mountain torque can be adequately handled in models. Time series of the global angular momentum, M (Hide, 1984) as derived from ECMWF initialized analyses show large changes taking place on a time scale of less than two weeks and independently supported by measurements of the length of the day. In an analysis of one such 'kick' in M which occurred in February 1980, I have estimated that the observed increase in M of ~0.3 x 10^{26}Kg.m^2s^{-1} during the space of ten days would require the speed of the entire Trade wind belt to increase by 50% during that period, if surface friction were to account for the angular momentum change. Such fluctuations in surface frictional stress are not observed, particularly in the global-mean sense, and it is clear that very large mountain torques occurred instead. Correct representation of these larger scale torques requires that synoptic systems are treated properly in the vicinity of the major mountain ranges.

In spite of recent interest in the role of gravity waves in the general circulation, much of the theory of flow over hills was laid down in the late Forties and Fifties by Queney and Scorer (e.g. Queney, 1947; Scorer, 1949, 1956). Since then, Sawyer (1959), Bretherton (1969) and Lilly (1972) amongst others have all recommended that some representation of sub-grid scale orographic wave drag needs is required in forecast/climate models. We will now proceed to examine some of the theoretical arguments which support this belief.

2. THEORY

A brief account will be given of the salient results from the theory of steady orographically forced gravity waves. Detailed descriptions of the linear theory can be found in standard texts (Holton (1975)) and Gill (1982)) and in the excellent reviews by Alaka (1960) and Smith (1979). For the sake of simplicity, the effects of the earths rotation will be ignored. (see Thorpe, 1985 for a discussion of the propagation characteristics of inertial-gravity waves).

Linearizing the Boussinesq equations of motion about a basic state atmosphere of constant buoyancy frequency, N and height dependent wind profile, U(z) it can be shown that a steady sinusoidal disturbance in vertical velocity, w(x,z) of horizontal wavelength ($^{2\pi}/_{k}$) is governed by:

$$\frac{\partial^2 w}{\partial z^2} + \left(\frac{N^2}{U^2} - k^2 - \frac{1}{U}\frac{d^2U}{dz^2} - \frac{1}{UHo}\cdot\frac{dU}{dz} - \frac{1}{4Ho^2} \right) w = 0 \qquad (2.1)$$

where $w(x,z) = w(z)\exp[ikx + {}^{z}/_{2H_o}]$ and H_o is the exponential scale height of the undisturbed density field. Generally speaking, the terms involving Ho in eqn. (2.1) are unimportant and the contents of the square bracket are usually written as:

$$l^2 - k^2$$

where l^2 is the Scorer parameter equal to $\frac{N^2}{\bar{U}^2} - \frac{1}{\bar{U}} \cdot \frac{d^2\bar{U}}{dz^2}$

When l^2 is independent of height and $l^2 > k^2$ solutions to (2.1) correspond to plane sinusoidal wave motion with vertical wavelength $2\pi/\nu$ where

$$\nu = (l^2 - k^2)^{1/2}$$

If $k^2 > l^2$, w decays with height and disturbance energy is not free to radiate away from its source. Taking typical values for N($10^{-2}s^{-1}$) and U (10 ms^{-1}), it is found that $l^2 \sim 10^{-6}$ m^{-2} so that if the horizontal wavelength ($2\pi/k$) is greater than about 15 km, $l^2 \gg k^2$ and $\nu \sim l$. Solutions of eqn. (2.1) corresponding to pure upward radiation of disturbance energy are then given by:

$$w(x,z) = \underline{\text{Real}} \; [\; A \exp[i(kx+lz) + z/2H_o] \;] \qquad (2.2)$$

where A is an arbitrary complex amplitude coefficient. Using this equation together with the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - \frac{w}{Ho} = 0 \qquad (2.3)$$

it is easy to show that the vertical momentum flux $\tau = \rho_o \overline{uw}$ (where ρ_o is the basic undisturbed density and the overbar denotes a horizontal average) is given by:

$$\tau = - \frac{1}{2} \cdot \frac{1}{k} \rho_s |A|^2 \qquad (2.4)$$

with ρ_s denoting the surface density.

The amplitude constant is determined from the (linearized) lower boundary condition:

$$w = U(z=0) \frac{\partial h}{\partial x} \quad \text{at } z = 0$$

if the flow is required to follow the terrain height $h(x)$ at the surface. From eqn. (2.2) this implies that:

$$A = U(z=0).\, ik\, h_m$$

where $h(x) = h_m \cos kx$ so that:

$$\tau = -\frac{1}{2}\, l\, \rho_s\, U^2 k\, h_m^2 \quad \text{at } z = 0$$

Moreover when the curvature of the vertical profile of U can be neglected in the definition of l^2, the vertical momentum flux becomes:

$$\tau = -\frac{1}{2}\, \rho_s k N\, U. h_m^2 \quad \text{at } z = 0 \qquad (2.5)$$

Applying the same procedure to relate the stress at any level to the vertical displacement of streamlines δh it is straightforward to show that:

$$\tau = -\frac{1}{2}\, .k\rho_o UN\, \delta h^2 \qquad (2.6)$$

Strictly, the linearized analysis leading to (2.6) is only accurate when U and N are constant and when the horizontal wavelength is sufficiently long for the waves to be hydrostatic (ie $l^2 \gg k^2$). It can however be shown that the differential equation (2.1) together with the continuity equation imply that $d\tau/dz = 0$ for any profile of U and N except at a critical line where U = 0. (see Eliassen and Palm, 1961). In view of this constraint, (2.6) will later be used to infer the height variation of δh^2 when $d\tau/dz = 0$, even if U and N are functions of height.

Substituting some 'typical' values into eqn. (2.5).

eg. $\rho_s = 1.3\ Kgm^{-3}$, $k = 2\pi/(20\ km)$, $N = 10^{-2}s^{-1}$, $U = 10\ ms^{-1}$ and $h_m = 200m$, we find that the surface stress $\tau = 0.8\ N\ m^{-2}$ which is very large by comparison with normally quoted values of surface friction ($\sim 0.1\ Nm^{-2}$).

For an isolated bell-shaped mountain ridge of the form:

$$h = \frac{h_m\ a^2}{(a^2+x^2)}$$

it can be shown that when the mountain half-width a is such that $al \gg 1$ (the hydrostatic limit) the total drag per unit length is equal to $\frac{\pi}{4}.\rho_s NUh_m^2$. The mean surface stress averaged over a distance 4a across the mountain (representative of the total horizontal scale of the ridge) is:

$\frac{\pi}{16a}.\rho_s NUh_m^2$ and should be compared with eqn. (2.5). Using the same values as before except with a = 5 km, the expression above gives a mean surface stress of $\sim 0.2\ Nm^{-2}$, which seems more reasonable than the value for the sinsuoidal mountain. What is immediately clear from these simple calculations is that even a modest sized hill can locally exert a drag on an airstream comparable with if not greater than surface friction.

Inclusion of Coriolis forces into the linear analysis for the bell-shaped hill causes a reduction of the wave drag by an amount dependent on the non-dimensional parameter $^{af}/_U$ where f is the Coriolis parameter (Smith, 1979a). When $^{af}/_U = 1$ corresponding to a mountain half-width of $\sim$ 100 km. the net drag force is reduced to about a third of its non-rotating value. On the other hand, numerical simulation (eg Peltier and Clark, 1979) shows that linear theory usually underestimates the surface wave drag and in their study of downslope windstorms, resonant amplification led to a tripling of the drag over its linear value. Given that the mean land-averaged sub-grid scale orographic variance appropriate to the current operational forecast model at the Meteorological Office is of the order of $(200m)^2$ it evident that a parametrization based on linear theory is likely to make a big impact on the spin-down time of synoptic eddies.

So far it has been assumed that the main source of unresolved mountain torque is associated with inertial-gravity wave radiation. Quasi-geostrophic and semi-geostrophic theories which assume the mountain surface to be isentropic predict a symmetrical steady pressure response about a symmetrical mountain and therefore no drag in the direction of the ambient wind. The latter theory breaks down when the aspect ratio of the mountain approaches N/f whatever the basic speed of the airstream since an isentropic boundary cannot be sustained. However, solutions to the semi-geostrophic equations are still obtaintable by the pure Lagrangian method described in Cullen et al (1986). These show that for steep mountains (slope > N/f) such as the Alps, semi-geostrophic theory predicts upstream 'damming' of cold air which is subsequently released as a 'weir' across to the lee side. A geostrophically supported cold air wedge (Fig. 1) on the windward side of such an Alpine ridge would cause a net force to be exerted though not necessarily accompanied by a radiating gravity wave.

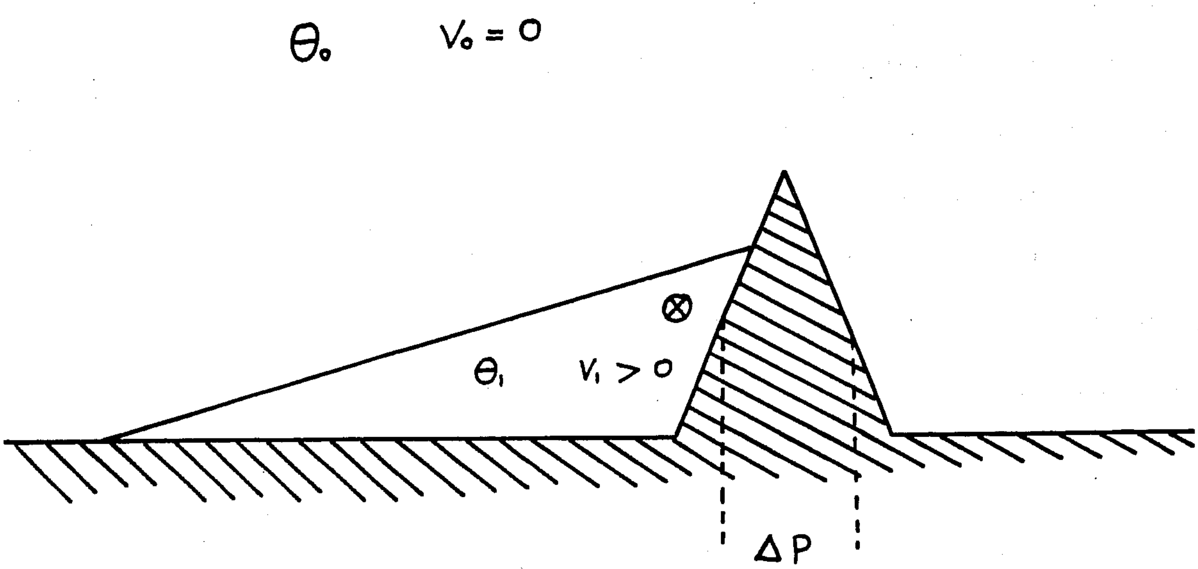

Fig. 1. Schematic profile of cold air dammed against a two-dimensional mountain ridge. Cold air with potential temperature θ_1 lies beneath a motionless isentropic atmosphere such that $\theta_0 > \theta_1$. The sloping interface is balanced by a velocity difference $V_1 - V_0$ between the two air masses so that V_1 is into the picture.

Another less frequently discussed aspect of pressure drag on orography is that arising from the geostrophically balanced component of the pressure field associated with the undisturbed wind field. As noted by Smith (1979a), in a uniform pressure gradient ∇p, Archimedes Law demands a net 'lift force' F on the mountain given by:

$$\underline{F} = -\nabla p . \text{ (volume of the mountain)}$$

or $$\underline{F} = (\rho_0 f \underline{k} \wedge \underline{V}_g) \text{ (volume " " ")}$$

where $\underline{V}_g$ is the geostrophic wind vector and $\underline{k}$ is the unit vector pointing vertically upwards. Although in a unidirectional airstream this force does no work (since it is at right angles to the wind) it must affect the local momentum balance. Since it is typically so much larger than the component of drag in the direction of the flow it is of some importance to understand how this cross-stream momentum is extracted from the flow and to see if numerical models handle the process correctly. (Smith, 1979b). The use of an enhanced orography (Wallace et al, 1983) in numerical models will lead to a spurious increase in the lift force though the effect of this is unclear.

3. OBSERVATIONS OF SURFACE PRESSURE DRAG AND VERTICAL MOMENTUM FLUX

Observational evidence that mesoscale orographic features constitute an important agency for momentum extraction has accumulated over many years. A comprehensive review is provided in Palmer et al (1986): here we state some of the principal conclusions.

Direct measurements of the upward momentum flux, $\rho_0 \overline{u'w'}$ have been made in aircraft flights by Lilly and Kennedy (1973), Lilly (1978), Lilly et al (1982), Brown (1983), Hoinka (1984, 1985). In all cases, the momentum flux was found to be predominantly downward and exhibited a marked variation in amplitude between studies. (see Table 1) as one might expect given different mountain ranges and synoptic conditions.

1'

Study	No. of measurements	Mountain Range	$\rho_0 \overline{u'w'}$ (Nm^{-2})
Lilly and Kennedy (1973)	1	Rockies	-0.6
Lilly (1978)	1	"	-1.2
Lilly et al (1982)	20	"	-0.08
Brown (1983)	5	British Isles	-0.15
Hoinka (1984)	1	Pyrenees	-0.7
Hoinka (1985)	1	Alps	-0.3

Table 1. Aircraft measurements of the vertical momentum flux.

The 'moderate amplitude wave' reported by Lilly and Kennedy (1973) had a horizontal wavelength of about 50 km and was accompanied by a region of severe turbulence between 15 and 20 km. Furthermore, the vertical momentum flux was roughly independent of height up to the base of the turbulent layer whereupon the stress rapidly decreased to zero. The very large momentum flux quoted in Lilly (1978) was found in a severe downslope windstorm at the Colorado Front range of the Rockies. At the height of this storm, Lilly estimated from surface pressure measurements that the net drag exerted was between $^1/_4$ and $^1/_2$ of the total surface friction force (assuming 0.1 Nm^{-2}) integrated around the earth at 40°N! Even over much less rugged terrain in the British Isles, Brown (1983) found substantial downward momentum fluxes whose occurrence apparently fitted in with the predictions of linear theory given the vertical profile of wind and static stability. Hoinka (1984, 1985) also finds very large vertical momentum fluxes over the Alps and Pyrenees which, in agreement with Lilly and Kennedy (1973), appear to be absorbed in layers characterised by turbulence.

Few direct measurements of surface pressure drag on mesoscale orography exist. However, Smith (1978) describes a field experiment in which a set of microbarographs were placed across the Blue Ridge mountain of the Central Appalachians (width ~ 2 km, height ~ 300 m) in order to evaluate the net drag force. Typical values of stress averaged over the area of the ridge were 1 Nm^{-2} -considerably greater than the vertical momentum flux commonly measured from aircraft. Hoinka (1985) also found a discrepancy between the measured vertical momentum flux $\rho_0\overline{u'w'}$ and a surface pressure drag estimate ranging from 1.6 to an enormous 6.7 Nm^{-2}. This is probably accounted for by the additional term $\rho_0 f\overline{v'\theta'}/\theta_z$ in the general expression for the vertical momentum flux in a rotating system. (Eliassen and Palm (1961) ;v' is the disturbance wind component in the y-direction and θ' is the potential temperature perturbation). Below the height of the mountain peaks, the cold air damming effect will lead to large values of v' and θ' with negative correlation (Fig. 1).

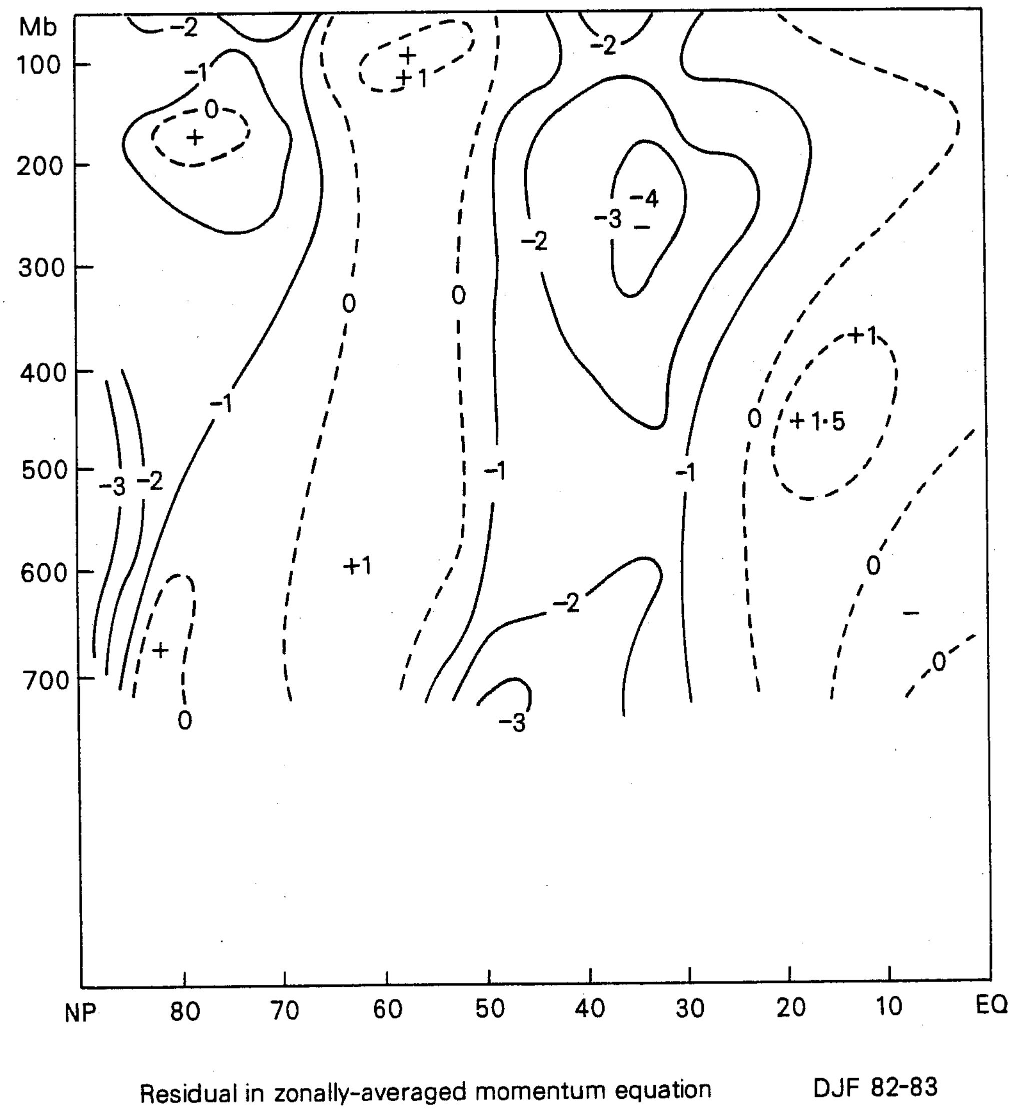

Fig. 2. The residual term in a zonal-mean momentum budget for December, January and February '82 - '83. Units: ms^{-1}/day.

Whilst these individual studies do seem to confirm the local importance of gravity wave drag as a sink of momentum, we need some assurance that this is still true in an average sense over land areas. If these orographically forced gravity waves were to break (dissipate) in the troposphere or lower stratosphere, it might be possible to measure their net effect through a zonal-mean momentum budget. Nurmi (1983) has carried out such a calculation for February 1979 using data from the first special observing period of FGGE. The residual term was found to be quite substantial and implied a deceleration of the westerlies of about 2 ms^{-1}/ day near 200 mb. A similar calculation was carried out by the author (see Palmer et al, 1986) using uninitialized ECMWF analyses for the period Dec 1 1982 to Feb 28 1983. Fig 2 shows the resulting latitude/height cross-section of the residual term in the momentum equation (zonal component). A clear pattern emerges with a mid-latitude region of deceleration peaking near the tropopause and towards the surface (the budget was not extended to 1000 mb since much of the data then used would have been extrapolated). Unfortunately, it is impossible to put error bars on these calculated values and at worst they may be a figment of the ECMWF analysis system. Nevertheless, other studies using rawinsonde data directly have found indications of an upper level drag force (Holopaisen and Lau (1980) and energy sink (Kung, 1967) similar to that implied by the budget studies described here.

A better approach to analysing the momentum budget from real data is to integrate vertically so that the Coriolis torque vanishes (due to mass continuity). The net vertically-integrated horizontal momentum flux convergence is then balanced, in the time-mean, by the surface stress.

Swinbank (1985) has made a detailed calculation of the zonally-averaged mountain torque and surface friction during January and June 1979 using FGGE data and compared them with the total surface stress evaluated indirectly from the net horizontal momentum flux convergence. Friction torque is calculated using the boundary layer scheme in the Meteorological Office GCM. The sum of the friction and mountain torque in January 1979 was found to be smaller than the inferred total surface stress (due to horizontal momentum flux convergence) by a factor of two. In contrast, little discrepancy was found in June 1979 consistent with the notion that systematic westerly bias of numerical models is a winter phenomenon. Swinbank concluded that surface friction was possibly underestimated though it was more likely that unresolved mountain torque accounted for the difference.

The problem of measuring pressure drag using routine synoptic data is not hard to appreciate. Smith (1982) summarizes observations of pressure drag for several mesoscale mountain ranges such as the Alps, the New Zealand Alps, Iceland and the Colorado Front Range. These show that pressure differences of the order of 10 mb across mountain ranges of width 100-200 km are by no means uncommon. The implied sink of momentum is equivalent to a nominal surface friction stress of 0.1 Nm^{-2} acting over a region 100 times the horizontal area occupied by the mountain range, though it is only barely resolvable in current numerical models. Failure to parametrize these mesoscale momentum sinks could well explain the tendency towards excessive westerly flow in numerical models/GCM's and fits in with Swinbanks observation of a 'missing torque' in the Northern Hemisphere winter.

4. A PARAMETRIZATION SCHEME FOR SUB-GRID SCALE GRAVITY WAVE DRAG

There are two components to the gravity wave parametrization scheme to be described:

(i) a surface stress formula (τ_s)

(ii) an algorithm to determine $\frac{d\tau}{dz}$

The surface stress is calculated from eqn. (2.5) using the appropriate sub-grid scale orographic variance instead of $^1/_2 h_m{}^2$ and a 'tuned' value for the constant k. By using this expression for τ_s, it has been assumed that the waves are hydrostatic and propagate vertically without reflection. Trapping of energy beneath layers of small Scorer parameter l (ie. lee wave effects) is ignored.

As stated earlier the horizontally-averaged wave stress τ at any level is constant with height in the absence of transience, dissipation or critical layers. Dissipation in the form of clear air turbulence will be assumed to set in when the wave amplitude becomes large enough for instability to occur. In common with other theories of gravity wave breaking and momentum deposition (Lindzen, 1981) it will be assumed that the amplitude of the wave in these regions of instability adjusts (decreases) until marginal stability is attained. Unlike these theories, marginal instability is defined as the point where a local measure of the Richardson number equals $^1/_4$ rather than the point where linear theory predicts convective instability. In practice, the difference is probably not important.

Vertically propagating gravity waves amplify as $\rho_o^{-1/2}$ as they reach progressively less dense regions and ultimately achieve an amplitude for which the local Richardson number falls below a value of $^1/_4$. Theory suggests however that this need not be a region where the undisturbed flow has a small Richardson number - if anything the contrary is implied. Indeed, Scorer (1969) suggests that it is the tilting of highly stable layers which leads to the turbulence experienced by aircraft and glider pilots.

A rather elegant result from the theory of steady, two-dimensional stratified flow due originally to Long (1953) can be used to formulate a 'wave Richardson number' which will form the basis of our parametrisation of gravity wave breakdown. Scorer (1969) also uses this approach in his penetrative account of billow mechanics. The vorticity and continuity equations in the x-z plane for steady, two-dimensional flow are:

$$\frac{D\eta}{Dt} + \eta \, \mathrm{Div} \, \underline{V} + g \frac{\partial \phi}{\partial x} = 0 \qquad (4.1)$$

and $$\frac{D\rho}{Dt} + \rho \, \mathrm{Div} \, \underline{V} = 0 \qquad (4.2)$$

where $\eta = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$, $\underline{V} = u\underline{i} + w\underline{k}$, $\frac{D}{Dt} = u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z}$

and ϕ = log (potential temperature). (other symbols have their usual meaning). Eliminating Div $\underline{V}$ between 4.1 and 4.2 gives:

$$\rho \frac{D}{Dt} (^\eta/_\rho) + g \frac{\partial \phi}{\partial x} = 0 \qquad (4.3)$$

and since in a steady state eqn. (4.1) is equivalent to Div $(\rho \underline{V}) = 0$ a mass streamfunction may be introduced so that:

$$\rho u = -\frac{\partial \psi}{\partial z} \quad \text{and} \quad \rho w = \frac{\partial \psi}{\partial x}$$

For adiabatic flow ϕ must be a function of ψ alone and so letting $\phi(x,z) = \Phi(\psi)$, eqn. (4.3) becomes:

$$\rho \frac{D}{Dt} \left(\eta/_\rho\right) + g \frac{d\Phi}{d\psi} \cdot \frac{\partial \psi}{\partial x} = 0$$

or

$$\rho \cdot \frac{D}{Dt} \left(\eta/_\rho\right) + gw \cdot \frac{d\Phi}{d\psi} = 0 \qquad (4.4)$$

But if $h(x,\psi')$ is the height of a particular streamline ψ' then $w = \frac{Dh}{Dt}$ and:

$$\frac{D}{Dt} \eta/_\rho + gh \cdot \frac{d\Phi}{d\psi} = 0 \qquad (4.5)$$

since $\frac{d\Phi}{d\psi}$ is constant on a streamline.

If the subscript zero denotes undisturbed upstream values then it is easily shown that:

$$\eta(x,z) = \frac{\rho}{\rho_o} \left(\eta_o + g(h - h_o) \frac{B_o}{U_o} \right) \qquad (4.6)$$

where $B_o = \frac{d\phi_o}{dz}$

The physical content of this equation is certainly worth absorbing since it does not assume anything about the form of the disturbance (eg sinusoidal) or basic flow. It gives us a formula for the horizontal component of the vorticity vector in terms of the vertical displacement of a streamline from its undisturbed height together with known 'basic state' parameters. Inspection of the second term in brackets on the RHS of eqn. (4.6) reveals that vorticity is created by an amount proportional to the static stability and inversely proportional to the upstream wind speed. Physically, this expresses the fact that a given streamline displacement leads to greater buoyancy forces for larger static stability and that these are experienced by an air parcel for a time inversely proportional to the horizontal wind speed. Vorticity is generated through the horizontal variation of the buoyancy force which for hydrostatic waves (of small aspect ratio) is manifested as vertical wind shear. The factor ρ_1/ρ_o is important only if the streamline displacement is comparable with the density scale height and can, for the purposes of our scheme, be neglected.

A wave Richardson number $\tilde{Ri}$ can now be defined by:

$$\tilde{R}_i = \frac{gB}{\eta^2} = \frac{N^2}{\left[\,|\eta_o| + \left|\frac{N_o^2}{U_o}\right| \delta h\,\right]^2} \qquad (4.7)$$

where $\delta h = h - ho$. ALthough the denominator involves only the vertical displacement and basic state variables, the numerator requires a local measure of the static stability. To account for the change in static stability caused by vertical displacements of amplitude $|\delta h|$ it is convenient to assume that they occur over a depth scale $|U_o|/N_o$ (the fundamental vertical length scale of this system of equations) so that the minimum realizable value of N (N_{min}) would be given by:

$$N^2_{min} = N_o^2 \left(1 - \frac{N_o|\delta h|}{U_o} \right)$$

Using this expression for the numerator in eqn. (4.7) leads to an equation for the lowest expected value of the Richardson number within some disturbance of amplitude $|\delta h|$ ie.

$$\tilde{R}i = \frac{N_o^2 \left(1 - N_o \frac{|\delta h|}{Uo} \right)}{\left[|n_o| + \frac{N_o^2}{U_o} |\delta h| \right]^2}$$

This may be written in non-dimensional form by setting $N_o \frac{|\delta h|}{|U_o|} = |\delta h*|$ so that:

$$\tilde{R}_i = R_i \frac{(1 - |\delta h*|)}{(1 + R_i^{1/2} |\delta h*|)^2} \qquad (4.8)$$

where Ri equals the basic state Richardson number. This wave Richardson number embodies Lindzen's convective overturning criterion (numerator small) and Scorer's billow instability mechanism (denominator large) so that for sufficiently large $|\delta h*|$, $\tilde{R}_i$ becomes less than $^1/_4$.

The final stage in the parametrization scheme is to link $|\delta h*|$ to the wave stress τ via equation (2.6) giving:

$$\tau = - \frac{1}{2} k \rho_o \frac{\bar{U}_o^3}{N_o} |\delta h*|^2 \qquad (4.9)$$

Following Lindzen we employ a 'saturation' hypothesis whereby if $\tilde{R}i$ is found to be $< {}^1/_4$, the displacement amplitude is reset to a saturation value $|\delta h*|_{sat}$ such that $\tilde{R}_i = {}^1/_4$. One can, in principle therefore determine the vertical distribution of τ within the region $\tilde{R}_i < {}^1/_4$ using eqn. (4.9) and the saturation hypothesis. Outside of these regions the stress must be independent of height in accordance with the Eliassen-Palm theorem.

Consider now the algorithm for applying the scheme to any gridpoint column of a numerical model. Firstly calculate the surface stress τ_s using:

$$\underline{\tau}_s = \rho_s \, k \, N_o \, U_o \times \text{(orographic variance)}$$

To be consistent with theory, U_o and N_o should be mean values over a depth scale comparable with the standard deviation of the orography. Since our parametrization is based on a two-dimensional model the values of U_o will refer to the component of the vector wind in the direction of the surface stress. The calculated surface stress is carried over to the next level at which eqns. (4.8, 4.9) are used to determine $\tilde{R}_i$. If $\tilde{R}_i \geq 1/4$ no stress is absorbed and the process is repeated at the next level up, otherwise eqn (4.8) is solved for $|\delta h*|_{sat}$ with $\tilde{R}_i = 1/4$. Eqn. (4.9) then implies a new value of $\underline{\tau}$ which is also carried over to the next level and the process repeated. In this way a vertical stress profile may be constructed and any remaining stress may be absorbed into the uppermost model layer.

A literal interpretation of the scheme suggests that wavebreaking is very sensitive to the value of U_o. This is easily seen for the Lindzen convective overturning limit of the scheme for which saturation of the wave field requires $|\delta h*| = 1$. The saturation stress is then proportional to U_o^3 and inversely proportional to N_o from eqn. (4.9). It is shown in Palmer et al (1986) that for surface stresses of 0.1 Nm^{-2}, waves will typically break in the lower troposphere when $U_o < 5$ ms^{-1} and in the lower stratosphere when $U_o < 15$ ms^{-1}. Middle and upper tropospheric winds tend to be too strong for wave breaking normally. Fig. 3 shows the vertical distribution of stress calculated for an idealized profile of wind and static stability and an orographic variance of $(400m)^2$.

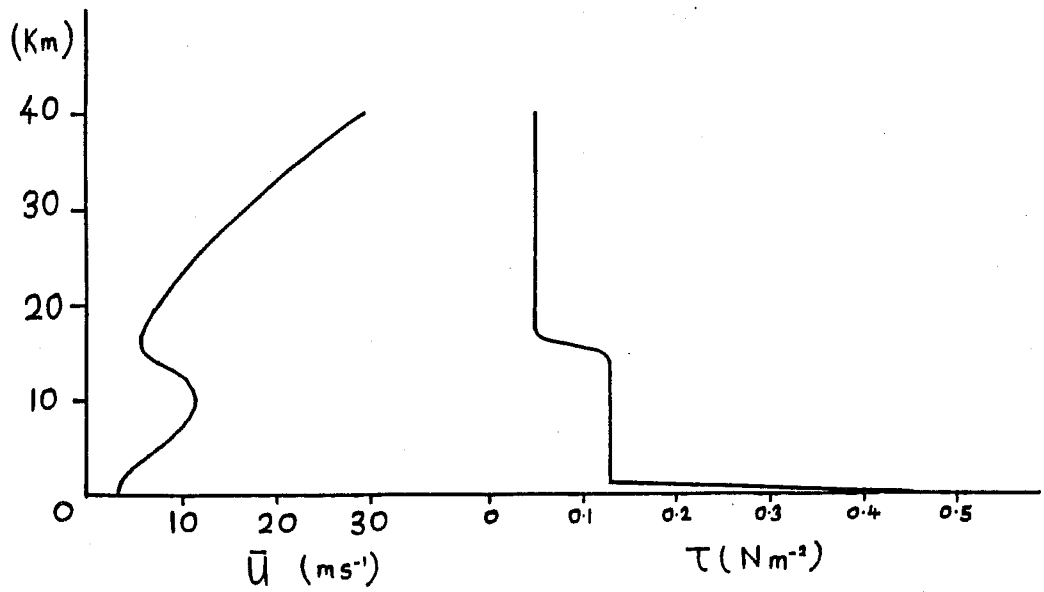

Fig. 3. Wind profile and associated stress as calculated using the Richardson number based gravity wave breaking scheme. The static stability was assumed to vary linearly between point values of $2 \times 10^{-5} m^{-1}$ at z=0, $1 \times 10^{-5} m^{-1}$ at 10 km and $4 \times 10^{-5} m^{-1}$ at 12 km and above. The orographic height variance was taken to be $(400m)^2$.

Absorption of wave energy is found in the boundary layer and in the lower stratosphere. When a critical level ($U_0 = 0$) is encountered $|\delta h|$ tends to zero and the scheme demands that $\tau \to 0$. In practice, U_0 is never exactly zero at any level in a numerical model and the condition $\tau = 0$ must be enforced above the layer in which U_0 changes sign.

Details of the effect of this parametrization scheme on long term integrations of a numerical model may be found in Palmer et al (1986). A cruder formulation, utlizing the same surface stress expression but assuming a fixed profile of $^{d\tau}/_{dz}$, has been used operationally at the British Meteorological Office since December 1984. Its performance has been documented by Kitchen and Dickinson (1986) and Dickinson (1985). The main findings of our study of the impact made by the Richardson number scheme on long term integrations are summarised here.

The most obvious improvement made in the circulation pattern resulting from a 90 day integration in a 'perpetual January' mode was evident in the zonal-mean cross-section of zonal wind and temperature. An overall reduction of the speed of the westerlies is found north of 40°N by typically ~ 2-3 ms^{-1} but with much larger values above 200 mb where the waves break. Between 40°N and 20°N there is net increase in westerly component of momentum due to enhanced input from the subtropical trade winds. The combined effect is to produce a southward displacement of the subtropical jetstream to about 30°N, a reduction in the surface winds and a substantially weaker polar night jetstream. All three effects are beneficial. The principal improvements at sea-level are a slowing down of the excessive westerly flow across Eastern Europe, Russia and the United States.

5. DISCUSSION

The existence of quasi-stationary orographically forced gravity waves which extend upwards into the stratosphere is beyond doubt. As far back as the early 1950's, Ludlam had made careful visual observations of orographic cirrus and lenticular clouds which seemed to persist in certain positions in the sky (Ludlam, 1952). He was able to deduce that upper tropospheric air had been lifted a distance at least twice the height of the hills disturbing the flow. By any geographical standard, English hills/downs are of modest stature (a few hundreds of metres) yet as we have seen in section 2 the total drag exerted on them by a fresh breeze (Beaufort scale 5) may at least be comparable with surface frictional drag. In more rugged terrain the potential for extremely large momentum sinks exists. However it is quite possible that the large-scale circulation responds to strong local sinks of momentum by reducing the surface wind speed and thereby the surface stress also. Over large areas of the continent in winter, gravity wave activity may only be important in transient cyclonic episodes and the very high effective drag coefficient (due to pressure forces acting on hills in conditions of high static stability) might ensure that surface winds are normally light there. The southward displacement of the Siberian High by westerly winds is a common problem in climate models which may be caused by too little momentum exchange with the surface.

A natural question to arise is, 'Why do the current generation of models appear to be particularly prone to this error?' Early climate simulations (eg. Mintz, 1965; Kasahara et al, 1973) gave time-mean sea-level pressure patterns which are at least comparable with those of current high resolution models and frequently better. On the contrary, todays high resolution models give much better short-term forecasts and are capable of anticipating major developments several days in advance.

The speed of the middle latitude westerlies at low levels is essentially determined by the height-integrated poleward momentum transport by large-scale eddies and the efficiency of momentum exchange with the surface. Green (1981) has speculated that model baroclinic waves might be too efficient at transporting momentum polewards so that a larger surface wind would be required to dispense with the momentum. Palmer et al (1986) show that this is not the case and that their modelled poleward momentum transport rate compares well with observation. However, if one examines published momentum transports in some of the early climate models (eg. Kasahara et al, 1973)) there is evidence of gross underestimation by a factor of two or three. This is likely to result from the sensitivity of modelled momentum transport to horizontal resolution (eg Wellck et al, 1971).

These facts point directly to a failure in the modelled surface stress/wind relationship in agreement with Swinbank (1985). Early model successes in simulating the observed Northern hemisphere winter distribution of zonal-mean winds seem to have resulted from the cancellation of two modelling errors - the underestimation of poleward momentum transport due to insufficient resolution and too small an effective drag coefficient in the modelled surface stress/wind relationship. It has been supposed here (and in Palmer et al 1986) that the extra surface stress required to correct the momentum budget is associated with radiating gravity waves which may deposit momentum at some distance from their source. A substantial fraction of this may however be attributable to inadequately modelled mountain barrier effects where cold air damming gives a sizeable pressure difference across an orographic ridge.

The Canadian Climate Centre have independently developed a gravity wave drag parametrization scheme for use in their T21 spectral climate model. (Boer et al, 1984). Although some differences in approach exist (they use Lindzens convective overturning criterion for wavebreaking) their scheme behaves very similarly to the one described here. A substantial beneficial impact is also found. Another related approach to alleviating systematic errors was introduced by Wallace et al (1983) whereby the height of the model orography is raised in proportion to the standard deviation of the real orography about the grid box average. Tibaldi (1985) has shown that its principal effect is to increase the zonal-mean mountain torque and reduce the overall strength of the extratropical westerlies.

In summary, there can be little doubt that the representation of orography in numerical models leaves much to be desired. It has been argued that the inclusion of a parametrization scheme for gravity wave drag can have a substantial beneficial impact and that the need for such a scheme is indicated by both observations and theory. Much work remains to be done on such topics as upstream blocking, lee cyclogenesis and the influence of steep orography before the true impact of mountains can be represented in models.

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