PARAMETERIZATION OF MESOSCALE PROCESSES

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Summary: The aim of parameterization is to represent the effects of processes of a scale too small to be adequately resolved by a given numerical model. It is clear therefore that parameterization schemes should depend on the equivalent grid lengths of the model and further that it would be preferable if motion on the gridlength scale be artificially damped to avoid aliasing. Current gridlengths of numerical weather prediction models imply that processes of horizontal scale smaller than about 200 km should be so parameterized. Most schemes available at present concentrate on scales which have a large separation from this gridsize. However there is growing evidence of the existence in the atmosphere of processes on the mesoscale which, with a typical range of scales of 20-100 km, have neither a large scale separation from the grid scale nor have traditionally been accounted for in parameterization schemes. At least three such mesoscale phenomena are of importance: frontal zones, moist slantwise convection, and orographic drag. In this paper we shall describe some aspects of the dynamics of moist slantwise convection and suggest a simple parameterization scheme based on an adjustment method. Results from the use of a version of this scheme in a semi-geostrophic frontogenesis model will be described. Aspects of orographic drag will be mentioned in other papers in this volume, whilst problems of the representation of frontal zones will be briefly discussed in this paper.

1. INTRODUCTION

It is fundamental to dynamical meteorology that the equations of motion have certain scalings which represent atmospheric motion on various horizontal and vertical scales. Consequently it is possible to construct approximate equation sets which neglect terms small for the particular scales of motion so described. Thus the quasi-geostrophic approximation allows for the description of the synoptic scale with motion of a horizontal scale of order 1000 km. The semi-geostrophic approximation provides an accurate representation of the dynamics of systems with horizontal scales of order 100 km; such weather systems can be classified as mesoscale. For small scale motion of horizontal scale of order 10 km or less the Coriolis forces can be neglected providing yet another approximate equation set. Meteorological models can be divided up into those which use one of these approximate equation sets to understand basic dynamics and those using the primitive, or unapproximated, equations used for numerical weather prediction. Even models in the second category usually make the hydrostatic approximation.

The importance of such scaling considerations is that they provide a way of classifying weather systems according to their essential dynamical framework rather than in an *ad hoc* way from observations. As an example we observe tropical cyclones to have a horizontal scale of order 1000 km which might suggest they are synoptic scale. In fact as we shall show later the Rossby number of such systems is larger than that associated with typical synoptic scale motion and this places tropical cyclones, from a *dynamical viewpoint*, in the mesoscale category. Another example suffices to make this point. Satellite pictures often show mid-latitude and tropical squall lines to have upper cloud shields of a scale of order 100 km. Does this make squall lines a mesoscale phenomena? The answer is no because the dynamics can be described using equations which neglect the Coriolis forces.

These arguments are sometimes viewed with an amount of bemusement by those not directly working on mesoscale problems. It appears unnecessary to define the mesoscale, it is simply a convenient label to apply to a range of observed meteorological phenomena. This is true only until we have to consider the problem of parameterization in large-scale models when it becomes very important to identify the dynamics of scales not resolved in the model. In fact in the sense I am to define the term mesoscale it has, until relatively recently, been an open question as to whether the atmosphere releases energy on this scale. It has sometimes been referred to as the mesoscale gap in the energy spectrum. However it is now apparent that on the scale of 100 km a form of convective instability occurs which may account for a significant amount of mid-latitude and tropical precipitation and which, currently, is not being directly included in parameterization schemes.

Another way to view the parameterization problem is to examine the energy spectrum of atmospheric motion. The most comprehensive such spectrum has been compiled by Lilly, 1983; see Figure 1. It is evident that, far from there being a mesoscale gap, there is a nearly continuous slope to the spectra of $k^{-5/3}$ for scales less than about 1000 km. Lilly attributes this feature to either two-dimensional stratified turbulence, which can transfer energy to larger length scales in contrast to three-dimensional turbulence which has to transfer to smaller scales, or inertial gravity waves. Clearly other phenomena such as moist slantwise convection will also contribute. The extent to which these inherently mesoscale processes are currently accounted for in NWP models is an open question.

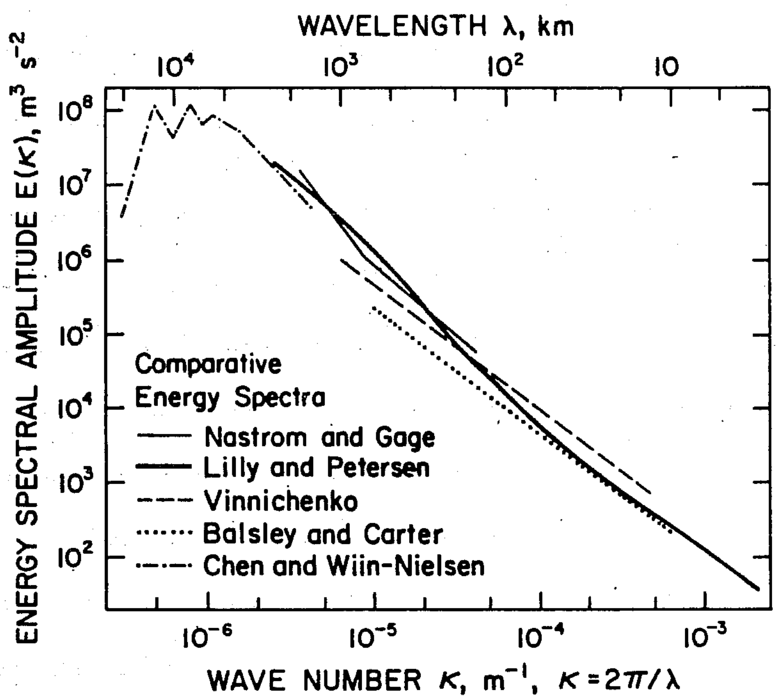

Fig.1. *Horizontal energy spectra over the wavelength range from about 5 km to the earth's circumference. The Balsley-Carter and Vinnichenko spectra were originally produced from time records, while the Nastrom-Gage and Lilly Petersen data were obtained from jet aircraft aircraft records. Other details are given by Lilly and Peterson (1983) and Nastrom and Gage (1983).*

Apart from a brief description of their main properties inertial-gravity waves will not be covered here; other papers by Mason and Shutts in this volume consider their generation by orography, role in modifying the synoptic scale flow, and the parameterization of the consequent orographic drag.

2. SCALES OF MOTION

In order to make a proper dynamical definition of mesoscale we will summarize the scalings involved in various approximate equation sets. These arguments are in the spirit of Emanuel (1983) but develop the definition in a different way. The working definition of mesoscale is that it is that scale which is too small to be described by the quasi-geostrophic equations but large enough that the hydrostatic approximation can be retained.

2.1 Quasi-geostrophic approximation

The quasi-geostrophic approximation assumes that the acceleration terms in the equations of motion can be approximated by using the geostrophic wind i.e.

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} \simeq \frac{\partial \underline{v}_g}{\partial t} + (\underline{v}_g\cdot\underline{\nabla})\underline{v}_g = \frac{D_g\underline{v}_g}{Dt}$$

Thus the quasi-geostrophic Boussineasa equation set on an f-plane can be written as

$$\frac{D_g}{Dt}u_g - fv_a = o \qquad (1)$$

$$\frac{D_g}{Dt}v_g + fu_a = o \qquad (2)$$

$$\frac{\partial \phi'}{\partial z} = g\frac{\theta'}{\theta_o} \qquad (3)$$

$$\frac{D_g}{Dt}\left(g\frac{\theta'}{\theta}\right) + wN^2 = o \qquad (4)$$

$$\underline{\nabla}\cdot\underline{v}_a = o \qquad (5)$$

where suffix a means ageostrophic, a prime indicates deviation from a stratified basic state , and $N^2 = g\frac{d}{dz}(\ln\bar{\theta})$. The scalings implied in these equations can be obtained by using the typical horizontal and vertical length scales and horizontal velocity scale L, H and $\tilde{U}$ respectively.

Equations (5) and (4) imply that $w \simeq u_a H/L$ and $w \sim \frac{\tilde{U}}{LN^2}\left(g\frac{\theta'}{\theta_o}\right)$ respectively. Further equation (3) gives $g\frac{\theta'}{\theta_o} \sim \phi'/H$ and the geostrophic assumption demands that $\phi' \sim f\tilde{U}L$. Substituting these scalings we can obtain the relation that the horizontal ageostrophic velocity scales as:

$$u_a \sim \tilde{U}^2 fL/N^2H^2 \qquad (6)$$

Now the momentum equations (1) and (2) indicate that:

$$\tilde{U}\frac{\tilde{U}}{L} \sim fu_a \text{ or } u_a \sim \tilde{U}^2/fL \qquad (7)$$

Taking the scalings of equations (6) and (7) together gives the implied horizontal length scale to be:

$$L \sim NH/f \qquad \text{Rossby radius of deformation}$$

For typical values of $N \sim 10^{-2}\ s^{-1}$, $H \sim 10$ km, and $f \sim 10^{-4}\ s^{-1}$ we get $L \sim 1000$ km which defines the synoptic scale of motion. Further for this scale the Rossby number $R_o = u_a/\tilde{U} = \tilde{U}/fL \sim 0.1$, and the Richardson number $R_i = N^2/(\partial u/\partial z)^2 \sim \frac{N^2H^2}{\tilde{U}^2} = \frac{1}{R_o^2} \sim 100$. The typical time scale $T \sim L/\tilde{U} = NH/f\tilde{U} \sim 1$ day.

2.2 Semi-geostrophic approximation

In this case the advection velocity in the acceleration terms is taken, more accurately, to be the total velocity giving:

$$\frac{D}{Dt}\underline{v} = \frac{\partial \underline{v}_g}{\partial t} + (\underline{v}.\nabla)\underline{v}_g = \frac{D\underline{v}_g}{Dt}$$

Thus the equation set is as in equations (1) → (5) except that the suffixes g on the substantial derivatives are removed. The scalings involved are the same except that in the momentum equation we admit the possibility that

$$u_a\frac{\tilde{U}}{L} \sim fu_a \quad \text{or} \quad L \sim \tilde{U}/f \qquad (8)$$

For typical values this gives a horizontal scale $L \sim 100$ km which becomes the dynamical definition of mesoscale. Notice that now $R_o \sim 1$, and the typical time scale $T \sim {}^1/f \sim 3$ hours.

2.3 Primitive equations

In the full equations these and other smaller scalings are included. For example it can be shown that if we imagine motion in which the transit time through a system, such as a cumulonimbus cloud, is substantially less than $^1/f$ then rotational effects can be neglected. In this case the hydrostatic assumption cannot be made and the scalings imply $L \sim H$, $R_o = \tilde{U}/fH \sim 10$, and $T \sim H/\tilde{U} = 20$ min.

In summary it is possible to provide a dynamical definition of certain scales of motion which indicate which terms in the equations of motion, and hence which processes, can be neglected for their description. These scaling arguments do not prove the existence of atmospheric phenomenon at these scales. However it is known that fronts can be described using the semi-geostrophic equations, see for example, Hoskins, 1982. The existence of mesoscale phenomena of horizontal scale 100 km clearly poses problems for large scale models. For example if a large scale model grid square indicates the presence of convective instability then it can be assumed, because individual cumulus clouds are very much smaller than the gridlength, that the effects of an ensemble of such clouds needs to be parameterized. The scale separation for mesoscale phenomena is much less implying the existence of only one or two entities within the grid-square. The mesoscale structures we shall discuss here are inertial-gravity waves, frontal zones and moist slantwise convection.

3. INERTIAL-GRAVITY WAVES

Most disturbances in the atmosphere generate inertial-gravity waves which can radiate energy to places remote from the source region. The scale of these waves is related to the geometry of the source region but it is difficult to give a precise specification of this relationship. Inevitably a significant part of the inertial-gravity wave spectrum so produced falls in the mesoscale range. As most large-scale models either filter out or distort inertial-gravity waves it is an important question as to whether the effects of this energy propagation should be parameterized in these models. It is, however, still not known whether these effects are of critical importance to the larger scale flow. In the case of waves generated by flow over topography it is thought to be a major contributor to the strength of upper jets, see Shutts this volume.

3.1 Linear equations

As inertial-gravity waves are such a ubiquitous feature of the atmosphere it is worth summarizing their main properties. Consider a basic flow in the y-direction in hydrostatic, geostrophic and thermal wind balance. Further we will consider infinesimal perturbations in this flow and linearize the Boussinesq equations in height coordinates:

Basic state $\underline{\bar{v}} = (o, \bar{v}(z), o)$; $\bar{\phi}_x = f\bar{v}$; $\bar{\phi}_z = \frac{g\bar{\theta}}{\theta o}$; $\frac{g\bar{\theta}_x}{\theta o} = f\bar{v}_z$

Perturbations $(\underline{v}', \theta', \phi') = (\underline{v}(z), \theta(z), \phi(z)) \exp(i(kx + ly - \sigma t))$

Linearizing the equations of motion gives:

$$\left.\begin{aligned} - i\Omega u + ik\phi - fv &= o \\ - i\Omega v + \bar{v}_z w + fu + il\phi &= o \\ - i\Omega n w + \phi_z - \frac{g\theta}{\theta o} &= o \\ iku + ilv + w_z &= o \\ - i\Omega g\frac{\theta}{\theta o} + f\bar{v}_z u + N^2 w &= o \end{aligned}\right\} \qquad (9)$$

where $\Omega = \sigma - l\bar{v}$ and n is a tracer parameter which if zero gives the hydrostatic approximation.

The inertial-gravity wave equation can be obtained from these equations by elimination:

$$(\Omega^2 - f^2)\ddot{w}_{zz} + w_z 2if\bar{v}_z\left(k + il\frac{1}{\Omega}\right) \qquad (10)$$

$$+ w\left((N^2 - \Omega^2 n)p^2 + 2if\frac{1}{\Omega}k\bar{v}_z + \bar{v}_{zz}\frac{(i\Omega fp^2 + lk(\Omega^2 - f^2))}{(\Omega k + ifl)}\right) = 0$$

where $p = (k^2 + l^2)^{\frac{1}{2}}$ is the total horizontal wavenumber. This is, in general, a non-constant coefficient equation which requires numerical solution. Some special cases suffice to show the wave characteristics.

3.2 Wave properties in special cases

For a barotropic atmosphere $\bar{v}_z = \bar{v}_{zz} = 0$ and $\bar{v} = v_o$ a constant so equation (10) becomes:

$$w_{zz} + wp^2\frac{(N^2 - \Omega^2 n)}{(\Omega^2 - f^2)} = 0 \qquad (11)$$

The solution of this equation in an unbounded domain is $w \alpha e^{lmz}$ and the dispersion relation is given by:

$$\sigma = lv_o \pm \left(\frac{p^2N^2 + m^2 f}{np^2 + m^2}\right)^{\frac{1}{2}} \qquad (12)$$

If $v_o = 0$ it can be seen, for the non-hydrostatic set with $n = 1$, that $f < \sigma < N$. As a mesoscale example consider the following values $m = 2\pi/3\text{km}$, $k = l = 2\pi/100$ km with $N^2 = 10^{-4}\ s^{-2}$ giving $\sigma \simeq \left(f^2 + \frac{p^2N^2}{m^2}\right)^{\frac{1}{2}}$ and $1/\sigma \sim 40$ mins.

The group velocity vector, $\underline{c}_g$, defines the direction of energy propagation:

$$\underline{c}_g = \left(\frac{\partial\sigma}{\partial k}, \frac{\partial\sigma}{\partial l}, \frac{\partial\sigma}{\partial m}\right)$$

It is easy to show that, in the case of $v_o = 0$, $\underline{c}_g.\underline{k} = 0$ where $\underline{k} = (k,l,m)$ in the wave vector. Therefore the group velocity vector is perpendicular to the wave vector and consequently parallel to the direction of the wave fronts, see Fig.2.

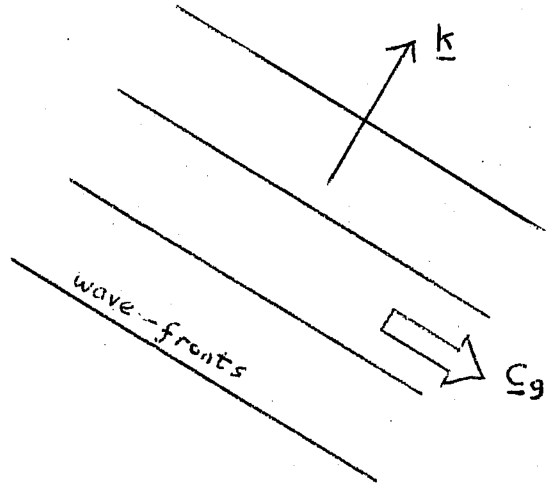

Fig.2 *Orientation of wave fronts and the group velocity vector for an inertial-gravity wave.*

For the smaller scale waves we can neglect the effects of rotation by setting f = o in equation (10) giving:

$$w_{zz} + m(z)^2 w = o \qquad (13)$$

where $m^2 = \frac{l\bar{v}_{zz}}{\Omega} + \left(\frac{N^2}{\Omega^2} - 1\right)p^2$

In many atmospheric cases m varies significantly with height making the solution to equation (13) more difficult. If m^2 changes sign from positive to negative at some height then the flow reflects the gravity waves from this level. Alternatively, if $\Omega = o$ at some level there exists a critical level at which the wave energy is probably absorbed. It is a matter of current concern as to whether these energy propagation and absorption characteristics should be accounted for in large-scale models.

4. FRONTS

The process of frontogenesis is one which is likely to be a severe test for a large-scale model to represent in a dynamically accurate way. The dynamics of baroclinic instability are now relatively well understood and it is an inevitable consequence of the growth of the mid-latitude cyclone that localized regions of frontogenesis occur. This implies a cascade of energy to increasingly small horizontal scales as the front forms. Although the along-front scale is of order 1000 km, the across-front scale of the dynamics decreases until, in theory, a discontinuity forms at the surface. Particularly as most mid-latitude precipitation arises in frontal zones it is important to determine the appropriate parameterisation of these processes. We shall concentrate here on two aspects of these problems; the formation of the discontinuity, and the moist processes which occur on the mesoscale.

4.1 Theory of frontogenesis

It is necessary to appreciate the dynamical processes on the mesoscale at fronts to summarise aspects of the theory of frontogenesis as developed by Hoskins, see for example Hoskins and Bretherton, 1972 and Hoskins, 1982. It is simplest to discuss two-dimensional frontogenesis but most of the ideas have been extended to a full three-dimensional description.

From the scale analysis described in section 2.2 it can be shown that if $R_o \sim 1$ then the flow along the front can be approximated by its geostrophic value but that the along-front acceleration term is not small. This leads to the semi-geostrophic equations:

$$\left.\begin{aligned} &\frac{Dv_g}{Dt} + fu_a = o \\ &\frac{D}{Dt}\left(\frac{g\theta'}{\theta_o}\right) + N^2 w = S \end{aligned}\right\} \qquad (14)$$

where we have allowed for a diabatic forcing term S to represent the effects of latent heat release. Given thermal wind balance, equation (14) can be appropriately differentiated to give the so-called Sawyer-Eliassen equation for the ageostrophic circulation about the front needed to maintain that thermal wind balance in the presence of a geostrophic frontogenetic forcing increasing the horizontal thermal gradients:

$$N^2\Psi_{xx} - 2fv_z\Psi_{xz} + f\zeta\Psi_{zz} = -2Q - \frac{g}{\theta_o} S_x \qquad (15)$$

where $(u_a,w) = (-\Psi_z, \Psi_x)$, $\zeta = f + \bar{v}_x$, the geostrophic forcing $Q = f\left(\frac{\partial u_g}{\partial z}\frac{\partial u_g}{\partial x} - \frac{\partial u_g}{\partial x}\frac{\partial v_g}{\partial z}\right)$, and a more detailed derivation can be found in Thorpe, 1984.

There are several factors about equation (15) of concern to us here but it is first useful to make a mathematical transformation to geostrophic momentum coordinates (X,Z) where

$$X = x + v/f \qquad Z = z$$

As described by Eliassen, 1962 equation (15) then becomes:

$$\frac{g}{\theta_o f^3}(P\Psi_X)_X + \Psi_{ZZ} = -\frac{2Q}{f\zeta} - \frac{g}{f^3\theta_o}S_X \qquad (16)$$

where $P = \underline{\zeta}\cdot\underline{\nabla}\theta$ is the Ertel potential vorticity (PV), which in this two-dimensional case is given by $P = \frac{\theta_o}{g}(\zeta N^2 - fv_z^2)$. If $P > o$ equation (16) is elliptic representing the forced ageostrophic circulation. For a frontogenetic region with $Q > o$ then the circulations in transformed and physical space are shown in Fig.3.

It is clear from equation (16) and the Jacobian of the transformation that the horizontal scale implied is given by:

$$\underline{L} \sim \frac{H}{\zeta}\sqrt{\frac{gP}{\theta_o f}}$$

Thus when the front is not well marked $\zeta \sim f$ and $P \sim f\theta_z$ giving

L ~ NH/f ~ 1000 km or synoptic scale. However as the front intensifies the vertical vorticity becomes large and the horizontal scale decreases, assuming that the PV is constant. This latter assumption is an important one and appears to be justified in the troposphere as long as moist processes are not dominant.

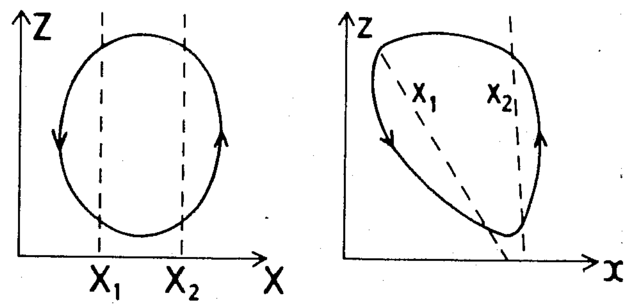

Fig.3 *Cross-frontal ageostrophic circulation in geostrophic momentum coordinates and physical coordinates. Frontogenesis in geostrophically forced by Q > o in the centre of the domain. (from Hoskins, 1982).*

Another important feature of the circulation equation is its relationship to the inertial gravity wave equation for this baroclinic zone with Q = o, which is similar to that obtained by making the two-dimensional assumption l = o in equation (10):

$$\frac{\partial^2}{\partial t^2}(\Psi_{xx} + \Psi_{zz}) + N^2\Psi_{xx} - 2fv_z\Psi_{xz} + f\zeta\Psi_{zz} = \frac{-gS_x}{\theta_o} \qquad (17)$$

For constant coefficients and no diabatic forcing this gives the dispersion relation:

$$\sigma = \left(\frac{N^2k^2 - 2fv_zmk + f\zeta m^2}{m^2 + nk^2}\right)^{\frac{1}{2}} \qquad (18)$$

Thus equations (15) and (16) give the response to frontogenesis forcing whilst equation (18) is the dispersion relation for waves generated in this type of front. In the hydrostatic limit n = o, the dispersion relation is shown in Fig.4. Note that there is a minimum frequency $\sigma_{min} = (fgP/\theta_o N^2)^{\frac{1}{2}}$

at which the group velocity is zero.

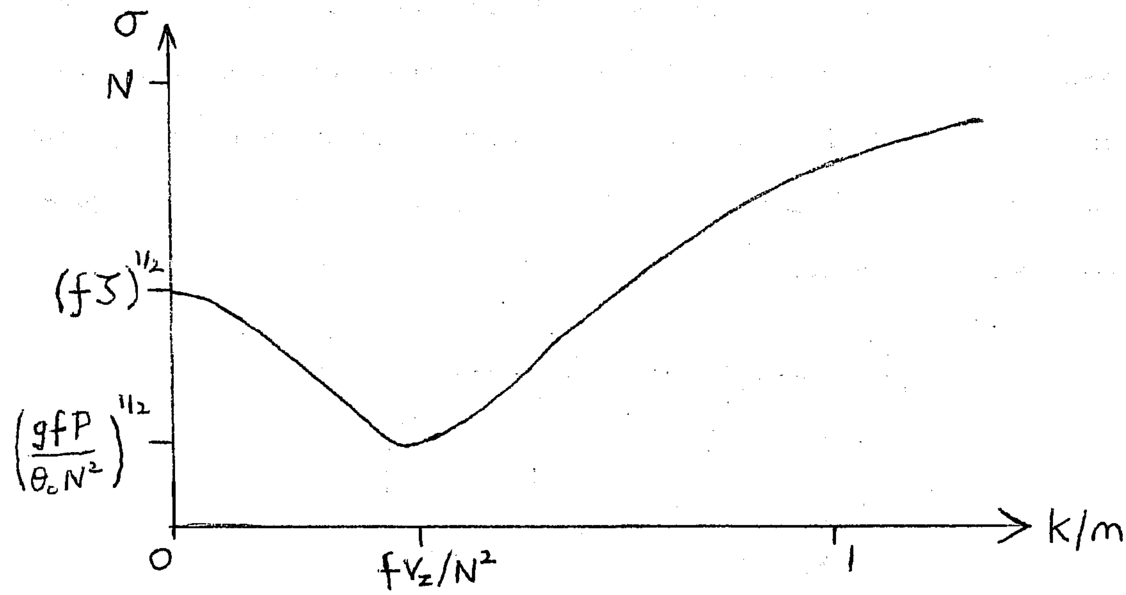

Fig.4 *The dispersion relation for inertial-gravity waves in a baroclinic flow showing the existence of a minimum frequency.*

4.2 Moist processes

The effects of latent heat release can be included by writing a diabatic term S consistent with the conservation of θ_e:

$$S = -\frac{Lv}{cp}\frac{Dq}{Dt} \simeq -\frac{Lv}{cp} w \frac{\partial q}{\partial z}$$

where q is the specific humidity or mixing ratio. If we define $N_e^2 = N^2 + \frac{Lv}{cp}\frac{\partial q}{\partial z} = \frac{g}{\theta_o}\frac{\partial \theta_e}{\partial z}$ then the previous equations for the forced circulation (15) and (16) and for inertial gravity waves (18) are as before but with N_e^2 replacing N^2 and $P_e = \underline{\zeta}\cdot\nabla\theta_e$ replacing P. A more rigorous definition of N_e^2 can be found in Durran and Klemp, 1982. This is a simple physical parameterization of the moist processes. Notice that the horizontal scale in the circulation equation and the minimum wave frequency become:

$$L \sim \frac{H}{\zeta}\left(\frac{gP_e}{\theta_o f}\right)^{1/2} \text{ and } \sigma_{min} = \left(\frac{fgP_e}{\theta_o N_e^2}\right)^{1/2}$$

Now there is an intriguing possibility that if $P_e < o$ when $P > o$ and $N_e^2 > o$, then the minimum frequency is imaginery and an instability develops. This is moist symmetric instability and it leads to the occurence of moist slantwise convection. The condition that $P_e < o$ is:

$$R_e = \frac{N_e^2}{v_z^2} < \frac{f}{\zeta}$$

This criterion on the moist Richardson number can be satisfied even in conditions when the atmosphere is stable to vertical moist adiabatic displacements. At present this moist slantwise convection is not being directly accounted for in convective parameterization schemes. It may however make up part of the so-called large-scale or stable condensation, see Sundquist this volume.

The other important aspect of moist slantwise convection is that the circulation equation (15) and (16) becomes hyperbolic for instability ($P_e < o$) or parabolic for neutrality ($P_e = o$). This suggests that there will be a strong response to such latent heating in a frontogenetically forced atmosphere. The semi-geostrophic equations which lead to the circulation equation cannot describe slantwise convection directly as unbalanced motion is filtered out. However, as we shall see later, it is possible to parameterize slantwise convection in the semi-geostrophic formulation. This aspect as well as the structure of slantwise convection will be discussed in sections 6 and 7.

Upright convection also occurs at fronts when $N_e^2 < o$. One interesting aspect is that the upright convection can change the structure of the sloping frontal ascent of Fig.3. An attempt to parameterize upright convection in a front is described in Thorpe and Nash, 1984, and Thorpe, 1984 and 1984a. Here the latent heating term S is made dependent on the low-level convergence which is large at low-levels where the front is forming. The result is that a 'dipole' circulation is superimposed on the frontal circulation giving structures as shown in Fig.5. This type of parameterization is related to that known as the Kuo scheme and may be most appropriate for frontal (upright) convection.

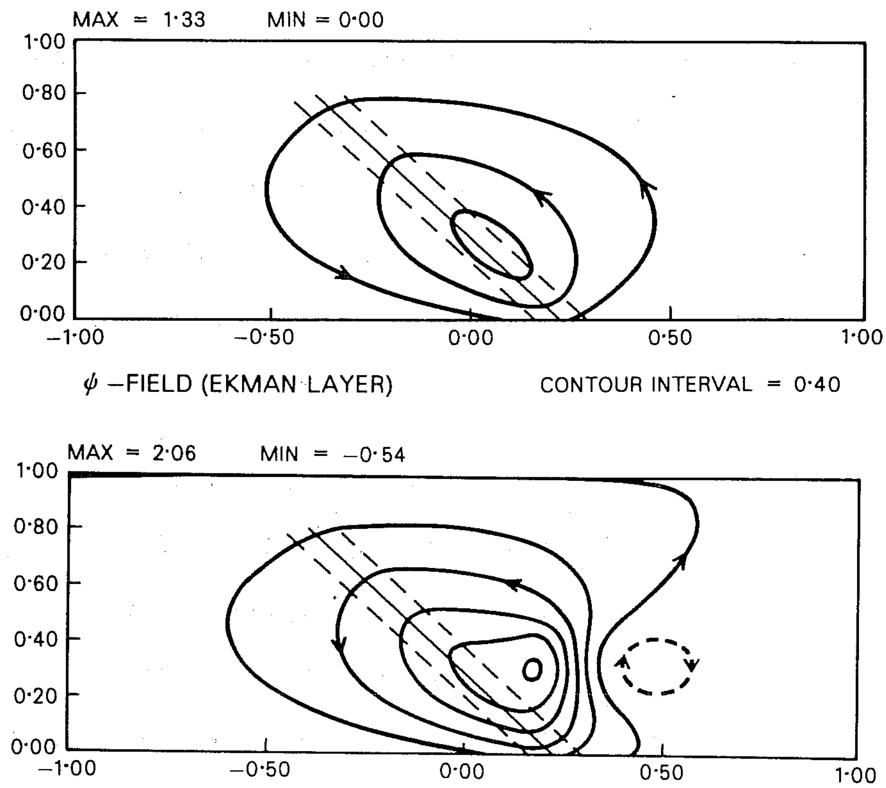

Fig.5 *Cross-frontal circulations with and without an upright convective parameterization in which the latent heating is made proportional to the low-level convergence. The dashed lines indicates the position of the frontal zone. (from Thorpe and Nash, 1984).*

4.3 Generation of turbulence

As well as the occurence of convection the collapse of a front produces local regions of turbulence and gravity wave generation. This occurs because the vertical vorticity becomes large which is consistent with the Richardson number falling below a quarter. An important question is to determine whether the effects of these processes should be included in the large-scale models.

The frontal discontinuity occurs at the ground which is where one would expect this turbulence to be first generated. Recent work by Cullen and Purser, 1984, suggests however that this discontinuity can propagate away from the surface. This is an interesting possibility as it implies flow separation at the surface: a feature which large-scale models certainly cannot represent at present. It is clear that a proper parameterization for the boundary layer in the disturbed frontal environment has yet to be devised.

The other region of strong atmospheric frontogenesis is near the tropopause. It can be shown that a discontinuity cannot form there but large gradients are produced and consequent regions of turbulence are generated. Shapiro, 1981, has incorporated a parameterization of clear-air turbulence in a simulation of upper frontogenesis.

5. TROPICAL CYCLONES

We include here a brief discussion of tropical cyclones merely to indicate that mesoscale processes are not exclusively a mid-latitude problem. As mentioned in section 2 we take the typical horizontal scale for the mesoscale to be $L \sim \tilde{u}/f$. For tropical cylones $\tilde{u} \sim 20\text{ms}^{-1}$ and $f \sim 2 \times 10^{-5}\ \text{s}^{-1}$ giving $L \sim 1000$ km which is in fact the observed scale of such systems. They can therefore be thought of as mesoscale as they cannot be well desscribed by quasi-geostrophic theory.

The main point to be made here is that the convection in tropical cyclones is probably characterized by moist slantwise convection rather than upright convection over much of the vortex. Again if this is true then at present this convection is not being accounted for.

in parameterization schemes. Observations and models of tropical cyclones suggest that they are in a state of neutrality to slantwise convection, that is $P_e = o$. As we shall describe more fully in the next section another way of stating this criterion is that θ_e surfaces are parallel to the angular momentum (m) surfaces. For an axisymmetric tropical cyclone it can be shown that:

$$m = \tfrac{1}{2} f r^2 + v r$$

where r is the radius and v is the gradient wind speed. The m and θ_e surfaces are nearly vertical in the lower troposphere but flair out to become nearly horizontal in the upper troposphere, see Fig.6.

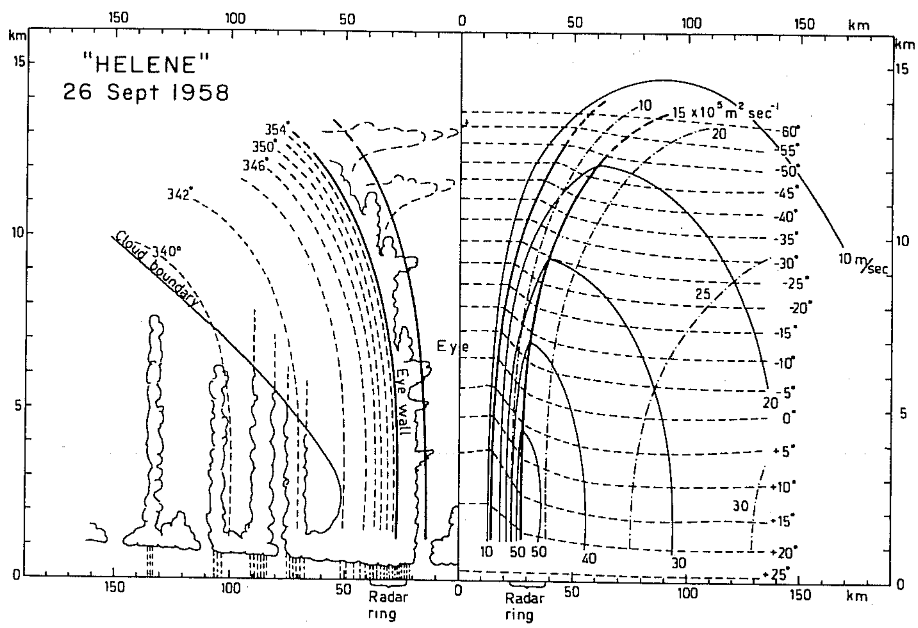

Fig.6 *Surfaces of θ_e, m, v, and T for tropical cyclone Helene indicating the close correspondence between θ_e and m. (from Palmen and Newton).*

It may therefore be most appropriate to parameterize this slantwise convection by returning the atmosphere to a condition of neutrality along angular momentum surfaces. The critical point is that these surfaces are often vertical but occasionally, in frontal zones, and tropical cyclones, can be significantly tilted relative to the vertical. Mesoscale aspects of tropical cyclone structure are considered in Ooyama, 1982, Thorpe, 1985 and 1985a, and Emanuel, 1985.

6. PARAMETERIZATION OF MOIST SLANTWISE CONVECTION

The important conclusion about moist slantwise convection, as described previously, is that the criterion for its development is that $P_e < o$. An example of the structure of the convection is shown in Fig.7 which is from a non-linear slantwise convection cloud model.

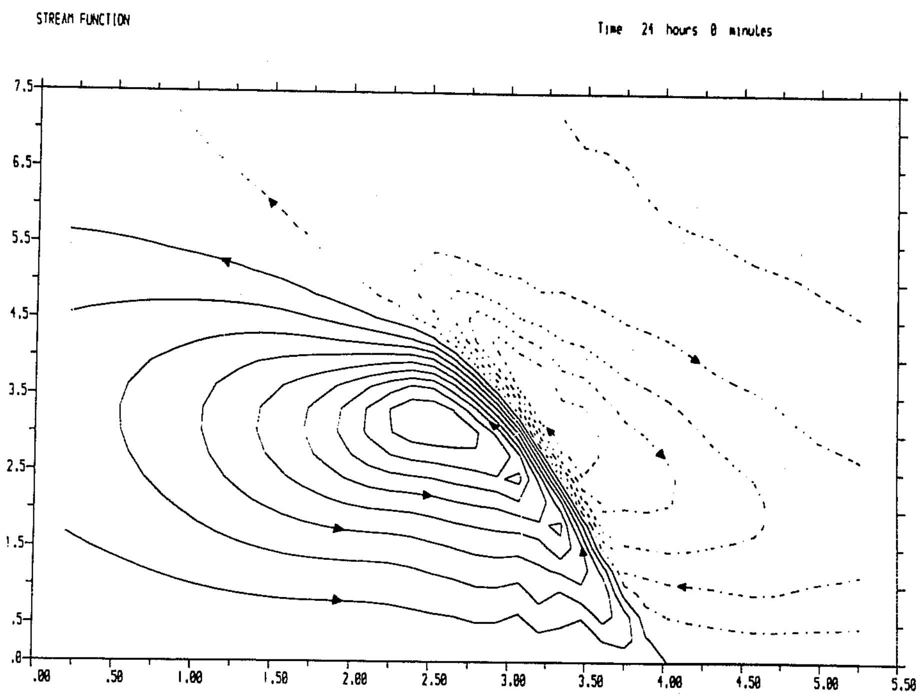

Fig.7 *Circulation in a numerical simulation of a moist slantwise cloud. Horizontal scale is in units of 100 km. The updraught slope is close to that of the* θ_e *surfaces while the downdraught return flow conserves* θ.

The ascent is nearly parallel to the (sloping) θ_e surfaces whereas, in this example, the descent is dry and conserves θ. A horizontal scale for the circulation can be based on the slope of the θ_e surfaces, giving:

$$L \sim H\frac{\partial\theta_e/\partial z}{\partial\theta_e/\partial x} = H\frac{Ne^2}{fv_z\gamma}$$

where $\gamma = (\partial\theta_e/\partial x)/(\partial\theta/\partial x)$ which, as will be discussed later, is of order unity. This expression can be rewritten in terms of the Richardson number as:

$$L \sim \frac{v_z H}{f} \frac{R_e}{\gamma} \sim \frac{v}{f} \frac{R_e}{\gamma}$$

Now the criterion for moist slantwise convective instability is that $R_e < f/\zeta$, which is of order unity so that L ~ 100 km for strong baroclinic zones.

An alternative way of viewing this type of convection in two-dimensions is highlighted by considering the relationship of the gradient of θ_e in the vertical compared to that along X-surfaces:

$$\left(\frac{\partial\theta_e}{\partial Z}\right)_X = \frac{\partial\theta_e}{\partial z} - \frac{v_z^2 \gamma f \theta_o}{\xi\, g} = \frac{P_e}{\zeta} \qquad (19)$$

So the criterion $P_e < o$ is equivalent to $(\partial\theta_e/\partial Z)_X < o$, that is that the gradient of θ_e along X-surfaces is negative.

6.1 Basis of proposed parameterization scheme

It is apparent that in tropical cyclones, frontal zones, and other mid-latitude baroclinic zones, that after a period of slantwise convection the atmosphere is returned to a state of neutrality i.e. $P_e \to o$, see Fig.6 and Fig.10 in Emanuel, 1983a. This is a form of moist adiabatic adjustment but along sloping absolute momentum surfaces rather than in the vertical. The condition $P_e = o$ is, in the two-dimensional case, consistent with $(\partial\theta_e/\partial Z)_X = o$ and consequently equation (19) predicts the (stable) gradient of θ_e in the vertical for which the atmosphere is neutral to moist slantwise convection. This stable gradient is the one which the atmosphere will be adjusted to in the parameterization scheme.

The extension of the formulation developed in the last sub-section to three-dimensions is necessary for the scheme to be useful in a realistic atmospheric model. In three-dimensions we can write:

$$\frac{P_e}{\zeta} = \left(\frac{\partial\theta_e}{\partial Z}\right)_{X,Y} = \frac{\partial\theta_e}{\partial z} - \frac{v_z}{\zeta}\frac{\partial\theta_e}{\partial x} + \frac{u_z}{\zeta}\frac{\partial\theta_e}{\partial y}$$

where $\zeta = f + v_x - u_y$ is the vertical component of absolute vorticity, and certain terms involving Jacobian of u and v have been neglected on scale grounds. Consequently for $\partial\theta_e/\partial Z = o$ we require:

$$\left(\frac{\partial\theta_e}{\partial z}\right)_a = \frac{f\theta_o}{\zeta g}\left(\gamma_X v_z^2 + \gamma_Y u_z^2\right) \qquad (20)$$

where $\gamma_X = (\partial\theta_e/\partial x)/(\partial\theta/\partial x)$, $\gamma_Y = (\partial\theta_e/\partial y)/(\partial\theta/\partial y)$, and suffix a indicates adjusted. Before the adjustment the gradient of θ_e was:

$$\left(\frac{\partial\theta_e}{\partial z}\right)_i = \left(\frac{\partial\theta_e}{\partial Z}\right)_{X,Y} + \frac{f\theta_o}{\zeta g}\left(\gamma_X v_z^2 + \gamma_Y u_z^2\right) > o$$

$$\text{or} \quad o < \left(\frac{\partial\theta_e}{\partial z}\right)_i < \frac{f\theta_o}{\zeta g}\left(\gamma_X v_z^2 + \gamma_Y u_z^2\right) \qquad (21)$$

Thus equation (21) is the criterion that an atmospheric layer is unstable to moist slantwise convection and equation (20) gives the value of the gradient of θ_e which a period of such convection will produce. A key point is that this adjustment can be made in each grid column separately given that currently the horizontal grid length of the large-scale model at 200 km, is greater than the horizontal scale of the convection

6.2 Implementation of proposed scheme

A possible implementation of this scheme, which is the subject of current work at ECMWF, has been developed in conjunction with Betts as an extension to the upright convective adjustment scheme described by Miller in this volume. The details are beyond the scope of this lecture but some important aspects will be summarized.

From equation (20) it is possible to compute a correction to the moist adiabat at each level to determine the relative buoyancy of a parcel including the slantwise component. If the variables $\gamma_x, \gamma_y, \zeta, u_z$, and v_z are constants then this correction can be obtained by integration of equation (20):

$$(\Delta\theta_e)_a = \frac{f\theta_o \Delta z}{\zeta g}(\gamma_x v_z^2 + \gamma_y u_z^2) \qquad (22)$$

where $(\Delta\theta_e)$ is that difference in θ_e over the layer Δz which makes the layer neutral to slantwise convection. The adjustment scheme will then find a temperature and humidity correction consistent with this adjusted θ_e structure, in the manner described in Miller, this volume. This adjustment is only done if $(\Delta\theta_e)_a > (\Delta\theta_e)_i$ for the layer, where $(\Delta\theta_e)_i$ is the initial difference in θ_e over the layer in question. An alternative way to describe this process is that for every possible cloud-base a new profile of θ_e is constructed with $\Delta\theta_e = (\Delta\theta_e)_i - (\Delta\theta_e)_a$ over each layer. If $\Delta\theta_e < o$ then a parcel ascending through the depth Δz will be positively buoyant and slantwise convection is assumed to occur on the sub-grid scale leading to the establishment of a neutral profile. If $\Delta\theta_e > o$ then there is negative buoyancy predicted and no convection or adjustment is required. Cloud top is therefore defined as being the first level at which $\Delta\theta_e > o$.

For shallow layers an approximation to equation (22) is acceptable:

$$(\Delta\theta_e)_a = \frac{f\theta_o}{\zeta g}\frac{(\gamma_x(\Delta u)^2 + \gamma_y(\Delta u)^2)}{\Delta z} \qquad (23)$$

where Δu and Δv are the changes in wind velocity components over the layer. The ratio quantities γ_x and γ_y depend on the local moisture gradients in the x and y directions. If we make the assumption of constant relative humidity then $\gamma_x = \gamma_y = \gamma$ and we can estimate γ from the definition of θ_e:

$$\delta\theta_e = \frac{\theta_e}{\theta}\left(\delta\theta + \frac{L_V\theta\delta q}{C_pT}\right) \simeq \gamma\delta\theta \qquad (24)$$

where $\gamma = 1+(L_V^2 q/c_p R_V T^2)$. A typical range of values of γ are for $q \sim 4gkg^{-1}$ and $T = 0^{\circ}c$, $\gamma \sim 1.8$ whereas for $q \sim 16gkg^{-1}$ and $T = 0^{\circ}c$ then $\gamma \sim 4$. In equation (24) we have made the approximation that $\theta_e/\theta \simeq 1$. Thus equation (23), using these estimates, becomes:

$$(\Delta\theta_e)_a = \frac{f\theta_o\gamma}{\zeta g\Delta z}|\Delta \underline{v}|^2$$

For a layer of depth 4 km and velocity difference 15 ms^{-1} then a typical value from this expression is $(\Delta\theta_e)a \sim 3.5$ K, which is a significant thermodynamic adjustment. The timescale of this adjustment is of order $1/\zeta$.

7. APPLICATION TO SEMI-GEOSTROPHIC FRONTOGENESIS

The only example currently available of the use of this parameterization scheme for moist slantwise convection is Thorpe and Emanuel, 1985. They considered a simple two-dimensional frontogenesis problem using a semi-geostrophic formulation. The frontogenesis is assumed to be produced by the action of a deformation forcing in which $U_g = -\alpha x$ and $V_g = \alpha y$, where $\alpha = 10^{-5}\ s^{-1}$, and occurs in an atmosphere which initially has a small horizontal temperature gradient on the lower and upper rigid horizontal boundaries of the domain. The atmosphere is further assumed to have uniform PV at the beginning of the simulation. The prognostic equation for PV is integrated forward in time allowing for the existence of diabatic forcing:

$$\frac{DP}{Dt} = \frac{1}{\rho}\underline{\nabla}\cdot(\underline{v} \times \nabla S + f\underline{k}S) = \frac{\zeta}{\rho}\frac{\partial S}{\partial Z} \qquad (25)$$

where the final equality is obtained by transformation to geostrophic momentum coordinates, and frictional stresses have been neglected.

The representation of moist slantwise convection arises from the conservation of θ_e and P_e in two-dimensional flow. This result can be obtained by consideration of the full equation for P_e, in the absence of friction.

$$\frac{D}{Dt} P_e = \frac{g}{\rho\theta_o} \underline{k}.(\nabla\theta_e \times \underline{\nabla}\theta) \qquad (26)$$

In three-dimensional flow it is possible to imagine a situation in which the source term on the right hand side is negative allowing generation of negative P_e and the development of moist slantwise convection; see Bennetts and Hoskins, 1979. However, in two-dimensions this source term is zero making P_e a conserved quantity. We imagine the following evolution of a front where during the three-dimensional development of a baroclinic wave local deformation promotes frontogenesis and further P_e becomes negative. The moist slantwise convection adjusts to a state of neutrality, with $P_e \simeq o$, after which a largely two-dimensional front develops. Notice that once in a two-dimensional mode there is no need to force P_e back to some value as it has to remain constant. In this sense we are not describing a parameterization but rather simulating the frontogenesis in an atmosphere nearly neutral to moist slantwise convection. Thus P_e, but not P, will be held constant for the integration. The source term S, as mentioned previously, is given by:

$$S = \frac{Lv}{Cp}\frac{Dq}{Dt} \simeq \frac{Lv}{Cp}\frac{\zeta}{\rho f} w^* \frac{\partial q}{\partial Z}$$

where $w^* = wf/\zeta$ is the transofrmed vertical velocity and we have assumed that the major contribution in changing q is via advection along X-surfaces consistent with the notion of slantwise convection. Hence from the definition of P_e and θ_e shown previously we can write:

$$S = (P-P_e) \frac{(w^* + 1w^*1)}{2}$$

see Thorpe and Emanuel, 1985, for details. We have taken a form for S which is non-zero only for ascent. This expression for S can be used in equations (25) and (16) to give an equation set suitable for the numerical simulation of moist frontogenesis. It should be noted that the circulation equation becomes parabolic for $P_e = 0$ so to avoid the consequent slow rate of convergence of the numerical method used to solve this equation we shall take $P_e = 0.07P_{initial}$ where $P_{initial}$ is that value of P typical of the undisturbed atmosphere remote from the region of frontogenesis. This small but non-zero value of the P_e allows the important effects of the moist slantwise convection to be readily apparent.

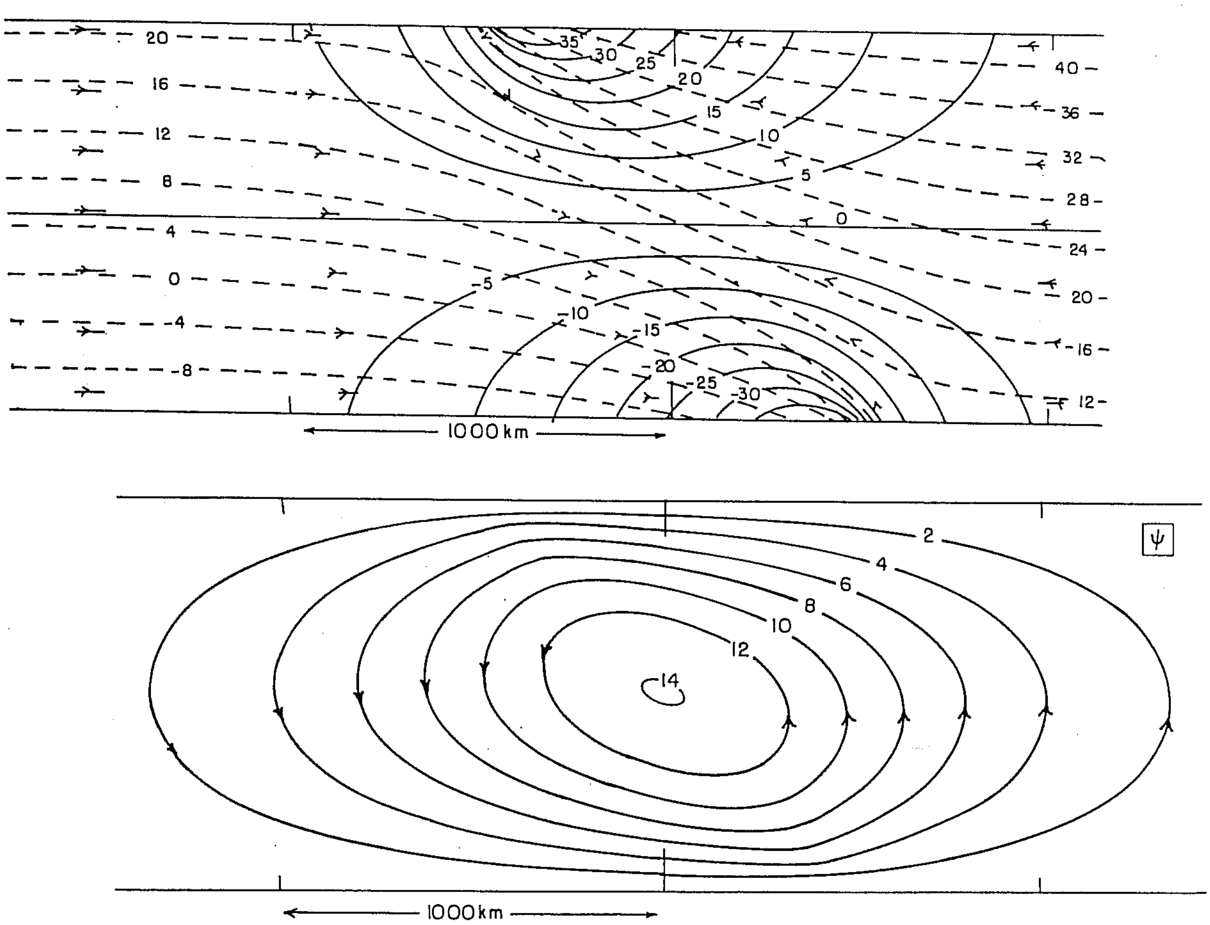

Fig.8. *Cross-sections showing the v, θ and Ψ fields from a two-dimensional semi-geostrophic model of frontogenesis in a dry atmosphere after 1 day of simulation (from Thorpe and Emanuel, 1985).*

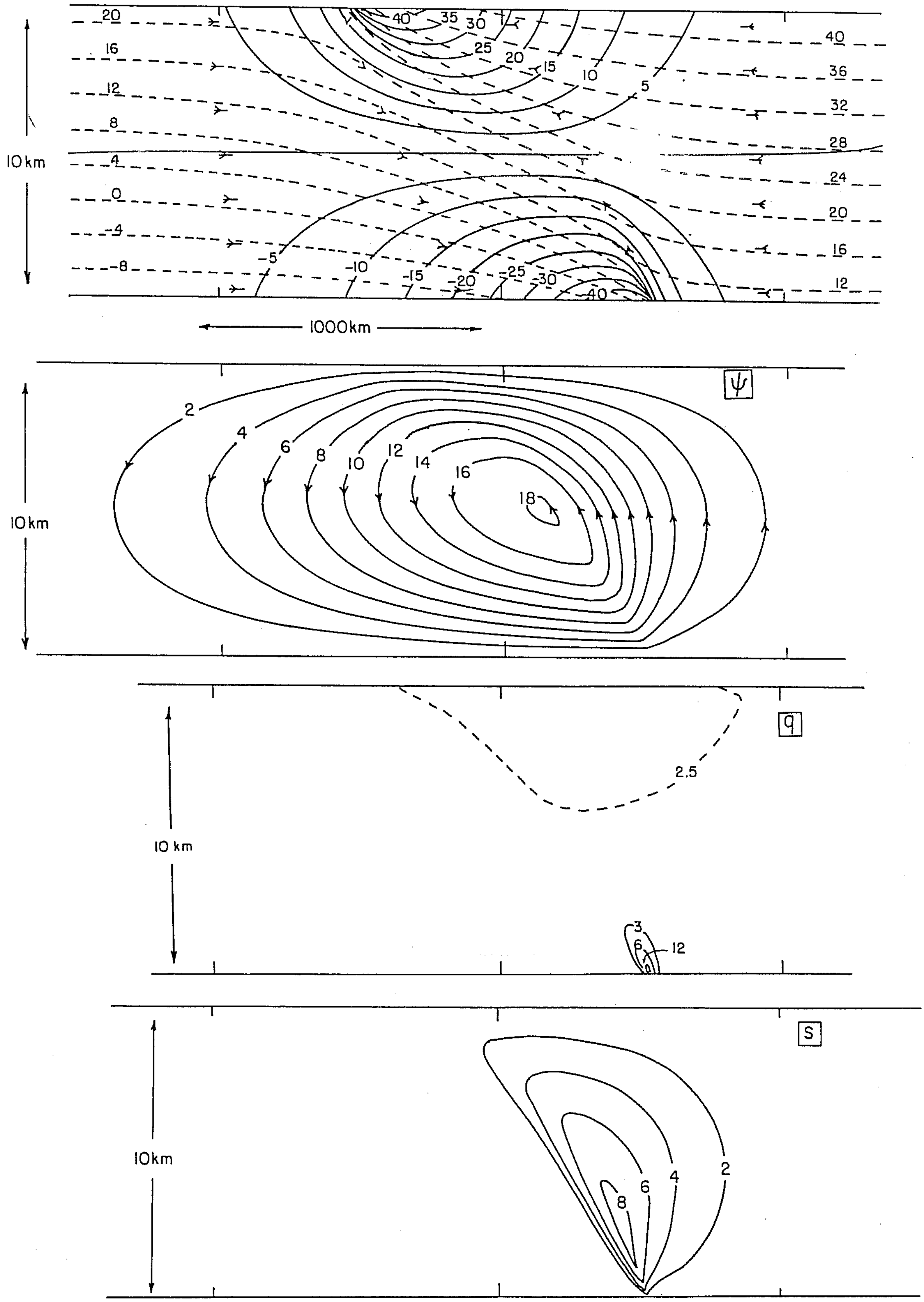

Fig.9. *Cross-section showing the v, θ; Ψ; P; and S fields from a moist frontogenesis simulation in which Pe was a small constant value, after 1 day. Note the increased frontogenesis, increased ascent, and changes in the PV structure due to the latent heating (from Thorpe & Emanuel 1985)*

In Fig.8 various fields are shown after 1 day of frontogenesis without moist processes and Fig.9 shows the structure including the moist processes. Notice that the rate of frontogenesis has been increased and the region of ascent has intensified and is reduced in horizontal extent. Of particular note is the PV structure in the moist case where a large local maximum is produced in the lower troposphere. The point is made in Thorpe and Emanuel, 1985, that this may create conditions suitable for a secondary baroclinic instability to develop giving an explanation of the periodicity of the convection along observed cold fronts. In summary it is evident that moist slantwise convection can change the structure of fronts significantly as well as being responsible for the generation and organisation of precipitation.

REFERENCES

Bennetts, D.A. and Hoskins, B.J., 1979. Conditional symmetric instability - a possible explanation for frontal rainbands. Quart. J. R. Met. Soc., 105, 945-962.

Durran, D.R. and Klemp, J.B., 1982. On the effects of moisture on the Brunt-Vaisaila frequency. J. Atmos. Sci., 39, 2152-2158.

Eliassen, A., 1962. On the vertical circulation in frontal zones. Geofys. Publikasjoner, 24, (No.4), 147-160.

Emanuel, K.A., 1983. On the dynamical definition(s) of "mesoscale" Mesoscale meteorology - theories, observations, and models. Reidel.

Emanuel, K.A., 1983a. On assessing local conditional symmetric instability from atmospheric soundings. Mon. Wea. Rev., 111, 2016-2033.

Emanuel, K.A., 1985. An air-sea interaction theory for tropical cyclones. Part I (accepted by J. Atmos. Sci.).

Hoskins, B.J., 1982. The mathematical theory of frontogenesis. Ann. Rev. Fluid Mech., 14, 131-151.

Hoskins, B.J. and Bretherton, F.P., 1972. Atmospheric frontogenesis models : mathematical formulation and solution. J. Atmos. Sci., 29, 11-37.

Lilly, D.K., 1983. Stratified turbulence and the mesoscale variability of the atmosphere. Ibid, 40, 749-761.

Lilly, D.K. and Petersen, E.L., 1983. Aircraft measurements of atmospheric kinetic energy spectra. Tellus, 35.

Nastrom, C.H. and Gage, K.S., 1983. A first look at wavenumber spectra from GASP data, Ibid, 35

Ooyama, K., 1982. On basic problems in theory and modelling of the tropical cyclone. Topics in Atmospheric and Oceanographic Sciences. (Intense atmospheric vortices). Springer-Verlag.

Palmen, F. and Newton, C.W., 1969. Atmospheric circulation systems, Academic Press.

Shapiro, M.A., 1981. Frontogenesis and geostrophically forced secondary circulations in the vicinity of jet stream-frontal zone systems. J. Atmos. Sci., 38, 954-973.

Thorpe, A.J. and Nash, C.A., 1984. Convective and boundary layer parameterizations in a diagnostic model of atmospheric fronts. Quart. J. R. Met. Soc., 110, 443-466.

Thorpe, A.J., 1984. Note on convective parameterization in a quasi-geostrophic model of fronts. J. Atmos. Sci., 41, 691-694.

Thorpe, A.J., 1984a. CISK as a basis for convective parameterization in baroclinic flows - application to frontal development. ECMWF proceedings of workshop on 'convection in large-scale models'. Dec. 1983, pp.139-168.

Thorpe, A.J., 1985. Diagnosis of balanced vortex structure using potential vorticity. J. Atmos. Sci., 42 (Feb. 15).

Thorpe, A.J., 1985a. Potential vorticity and the structure of tropical cyclones. 16th conference on Hurricanes and Tropical Meteorology (preprint volume).

Thorpe, A.J. and Emanuel, K.A., 1985. Frontogenesis in the presence of small stability to slantwise convection. J. Atmos. Sci., 42 (Aug. 15).

Cullen, M.J.P. and Purser, R.J., 1985. An extended Lagrangian theory of semi-geostrophic frontogenesis. Ibid, 41, 1477-1497.