THE EXISTENCE OF REGIMES OF FLOW IN THE ATMOSPHERE

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Summary: The notion of "statistical equilibrium" in a turbulent system, that is basic to the classical theory of general atmospheric circulation and is also invoked in some theoretical interpretations of persistent anomalies, is re-examined in its physical foundations by means of a simple example of turbulent baroclinic flow.

1. INTRODUCTION

Since the discovery of baroclinic instability, hundreds of studies have been devoted to analyzing the growth process of unstable disturbances at the expense of potential energy made "available" by the differential absorbtion of solar radiation in the earth's atmosphere.

Most of the papers on the subject discuss,under various simplifying assumptions,linear stability analysis of known stationary solutions of the equations for atmospheric flow. Such papers essentially propose an elaborated version of the original formulation, by Charney (1947) and Eady (1949), of baroclinic instability theory.

Much less attention has been devoted to nonlinear stability analysis of baroclinic flows. The best known contributions on this matter are due to Pedlosky, starting from his classical paper of 1970. In the studies performed "in Pedlosky's spirit" some properties of the unstable system are usually

analyzed by weakly nonlinear perturbation expansions. In most cases, however, the authors fail to seek the link between the global properties of phase-space and the statistical properties of baroclinic turbulence.
On the other hand the classical theory of general circulation dictates that "average" (statistical equilibrium) states are essentially maintained by turbulent fluxes. Statistics must somehow be recovered. This is done in a number of papers, devoted to studying baroclinic conversion under conditions sufficiently realistic to permit comparison with observations, by considering a basic state near the observed time-average. Interestingly enough such comparisons systematically reveal discrepancies of horizontal scale and vertical structure between observed baroclinic waves and their theoretical counterparts (see, for example, the classical paper by Gall (1976)).

All in all very little conclusive work appears the literature concerning the maintenance of statistical equilibrium in the general circulation. Apart from the pioneering studies by Lorenz (1962, 1963), more concerned, though, with the annulus dynamics, I believe the relevant contributions are those of Charney (1959), Green (1970), Wiin-Nielsen and Fuenzalida (1975) and White (1977). The mechanism highlighted in these works is essentially that of baroclinic instability - barotropic stability emerged at the end of the fourties from the studies of Charney (1947) and Kuo (1949). Comparison with observed statistics is confined to fluxes of heat and momentum and seem to confirm the plausibility of the hypothesis that the jet structure is maintained in the atmosphere by the combined effect of baroclinic instability and barotropic stability.

Other ideas appear here and there in the literature. An interesting example is the phenomenological analysis by Stone (1978,1982) based on the assumption that the mechanism of baroclinic conversion must eventually stabilize near marginal stability. Although never clearly specified, the implied mechanism of stabilization seems to be that of depletion of available potential energy instead of barotropic stability.

More recently the issue of statistical equilibria has been the object of renewed interest in the context of the controversy regarding "multiple equilibria" . This matter is well documented elsewhere , e.g. Benzi and Speranza (1986) and in the literature quoted therein. The physical problem is in essence the same as in the case of ordinary baroclinic instability: we want to understand what type of scale interaction maintains the statistical equilibrium (equilibria!) and how to describe correctly the fluctuations with respect to the equilibrium (the transitions between different equilibria). The only difference is that, this time, we are dealing with the dynamics of low frequency variability.

In this presentation I prefer not to address the question of multiple equilibria , although of major concern to me, but rather concentrate on discussing the basic problem of the maintenance of statistical equilibria. This I will do by means of a simple, but statistically realistic , example of baroclinic flow recently set up by Malguzzi and myself in order to explore a number of properties of baroclinic waves, specially concerning their interaction with the mean flow. After a description of the model and its statistical properties as derived from a 10 year integration (Section 2), I will examine the system in the framework of dynamical system theory (Section 3), from the point of view of classical meteorologists (Section 4) and, at last, in my own way (Section 5). Some very tentative conclusions will eventually be drawn.

2. GENERAL PROPERTIES

2.1 The model

Recent experience with high resolution numerical models, e.g. that of Benzi et al. (1986), shows that localized features, like isolated vortices, can play an important role in the global balances of turbulent flows. However the atmospheric circulation is dominated by mean flows that certainly play a remarkable role in determining the dynamics of fluctuations. In fact the classical theory of general circulation deals almost exclusively with global interactions in the wave-mean flow form. Although recognizing the potential role of space-time intermittence, I will concentrate here on global interactions, in line with the meteorological tradition.

The simplest model representing the dynamics of interaction of baroclinic waves with a zonal flow and displaying earthlike statistics, turns out to be a minimal vertical truncation (two layers or modes) of the equations of motion with enough latitudinal resolution to guarantee adequate description of barotropic interaction between baroclinic waves and the zonal flow.

I will describe here the quasi-geostrophic version in two layers, following the notation of Pedlosky (1979). Starting from the potential vorticity equations with Laplacian dissipation, written in terms of the barotropic $\Phi=(\Psi_1+\Psi_3)/2$ and baroclinic $\tau=(\Psi_1-\Psi_3)/2$ components of the streamfunction, and introducing a baroclinic forcing $\tau*$, we obtain:

$$\partial_t\nabla^2\Phi + J(\Phi\ ,\nabla^2\Phi+\beta y) + J(\tau\ ,\nabla^2\tau) = = -\nu_E/2\,\nabla^2(\Phi-\tau) \qquad (1)$$

$$\partial_t(\nabla^2\tau - 2F\tau) + J(\Phi\ ,\nabla^2\Phi + \beta y) + J(\tau\ ,\nabla^2\tau) = = \nu_E/2\ ^2(\Phi-\tau) - \nu_S\nabla^2\tau + 2F(\tau-\tau*). \qquad (2)$$

We separate now the symmetric component:

$$\Phi(x,y,t) = -\int U(y,t)dy + \Phi'(x,y,t) \qquad (3)$$

$$\tau(x,y,t) = -\int m(y,t)dy + \tau'(x,y,t) \qquad (4)$$

and introduce the separated form:

$$\Phi'(x,y,t) = \sum_{1}^{+\infty}{}_n(A_n(y,t)\ g_n(x) + (*)) \qquad (5)$$

$$\tau'(x,y,t) = \sum_{1}^{+\infty}{}_n(B_n(y,t)\ g_n(x) + (*)). \qquad (6)$$

The main purpose of the assumptions (5-6) is to allow simple representation of nonlinear wave-wave interactions. In fact, by inserting (5-6) into (1-2) and projecting onto the functions $g_n(x)$, equations in which nonlinearity is lumped into scalar products of g-functions can be obtained. Here, however, we shall concentrate on the wave-mean flow interactions.

Consequently we assume:

$$g_1(x) = \exp(ikx) \quad , \quad g_n(x) = 0 \quad \text{for } n\neq 1 \qquad (7)$$

This reduces (dropping superfluous indexes) the equations of motion to:

$$\partial_t U + \nu_E/2(U - m) + 2k\ \mathrm{Im}(AA^* + BB^*) = 0 \qquad (8)$$

$$\partial_t(m_{yy} - 2Fm) + \nu_S m_{yy} - \nu_E/2(U - m)_{yy} - 2F\nu_H(m - m^*) + \\ + 4kF\ \mathrm{Im}(A^*B)_{yy} + 2k\ \mathrm{Im}(AB^* + BA^*)_{yyy} = 0 \qquad (9)$$

$$\partial_t(A_{yy} - k^2 A) + (\nu_E/2 + ikU)A_{yy} + \\ - [ik^3 U + ikU_{yy} - ik\beta + \nu_E k^2/2]A + \\ -(ikm_{yy} + ik^3 m - \nu_E k^2/2)B + (ikm - \nu_E/2)B_{yy} = 0 \qquad (10)$$

$$\partial_t[B_{yy} - (k^2 - 2F)B] + (\nu_E/2 + \nu_S + ikU)B_{yy} + \\ - [ik^3 U + ikU_{yy} - ik\beta + \nu_E k^2/2 + k^2\nu_S + 2F\nu_H + 2ikFU]B + \\ - (ikm_{yy} + ik^3 m - \nu_E k^2/2 - 2ikFm)A + (ikm - \nu_E/2)A_{yy} = 0 \qquad (11)$$

where any trace of wave-wave interaction has disappeared, except for the momentum and heat fluxes in the zonal flow equations. Since A and B are complex, (8-11) constitute a set of six field equations in latitude y and time t. Notice that when U and m are fixed, as in some cases of marginally unstable flow that we shall consider later, the wave amplitude vector (A,B) can be written explicitly in terms of the zonal flow (U,m) as exp(Tt), where T is the matrix of the evolution operator in (10,11). In general, though, the evolution operators of different instants T(t), T(t') do not commute and the generalized exponential solution cannot be written.

2.2 THE STATISTICAL PROPERTIES

Experience shows that the system (8-11) displays realistic statistical properties and these are conveniently modelled by means of a spectral representation in terms of a few tens of modes. The results we show here are relative to an integration with a leap-frog scheme in time (time-step ≈ 1/10 day) and a pseudospectral representation of fields in terms of 32 latitudinal components. The integration is carried on for 10 years (≈ 350.000 steps corresponding to about 1 hour on a CRAY1).

Values of the dissipation coefficients are respectively $\nu_E = 0.45$ (decay-time ≈ 2.5 days in dimensional form) and $\nu_S = 0.1157$ (decay-time ≈ 10 days in dimensional form). The external forcing, operating only on the gravest latitudinal mode, is m* = 1.41. This corresponds to a thermal forcing of 9. The zonal wavenumber is k = 1.3 (≈ 4800 km). The energy cycle is correctly closed and very realistic.

Fig.1 displays the scatter of different components of states sequentially occupied by the system in time and Fig.2 the relative probability densities. Fig.3 shows the average in time of the zonal flow; although characterized by a high degree of variability from year to year, the average is quite stable after ten years. The average of wave amplitude is obviously zero. Power spectra are shown in Fig.4. It should be kept in mind that our dissipation is not scale-selective and, therefore, we are not dealing with an inertial range: the -3 spectrum is not that of two-dimensional turbulence theory! The time spectra, shown in Fig.5, confirm the "turbulent" nature of the system. Combined wavenumber-frequency spectra reveal the energetic dominance of the lowest harmonics (Fig.6). These statistical properties are in reasonable agreement with those of the real atmosphere. For the sake of comparison I show in Fig.7 a typical histogram of short baroclinic waves (see again Fig.2).

The problem I want now to address is that of setting up a consistent dynamical theory of the statistical equilibria observed in our model atmosphere.

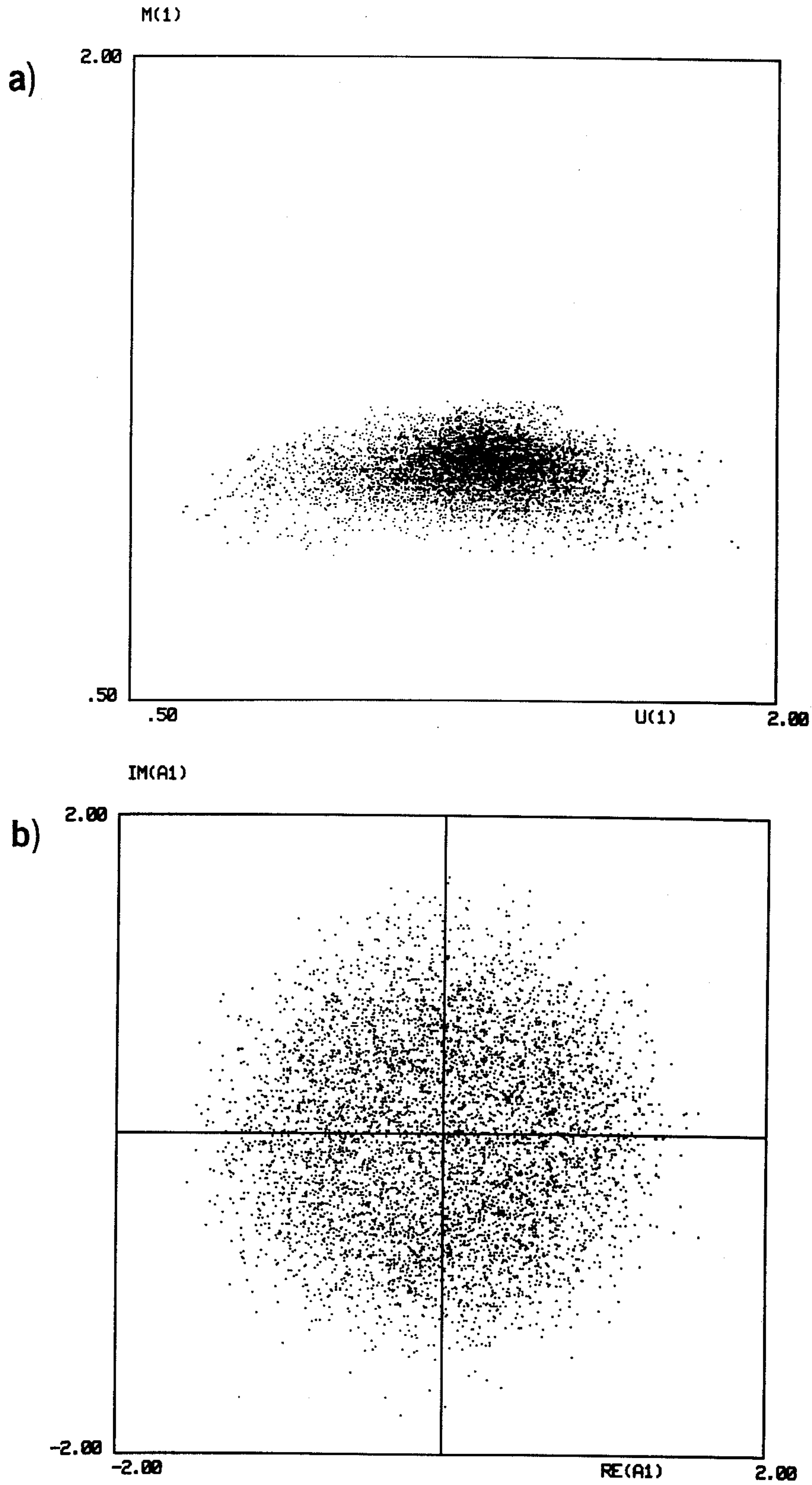


Fig.1 a) Scatter diagram of the first meridional component of zonal wind shear (m_1) versus the first component of the zonal wind (U_1). Units are dimensionless (one unit corresponds to 10 m/sec). The total time of integration is about 10 years.
b) Same as a) but for the real and imaginary parts of the first barotropic meridional component of the wave-field (A_1).

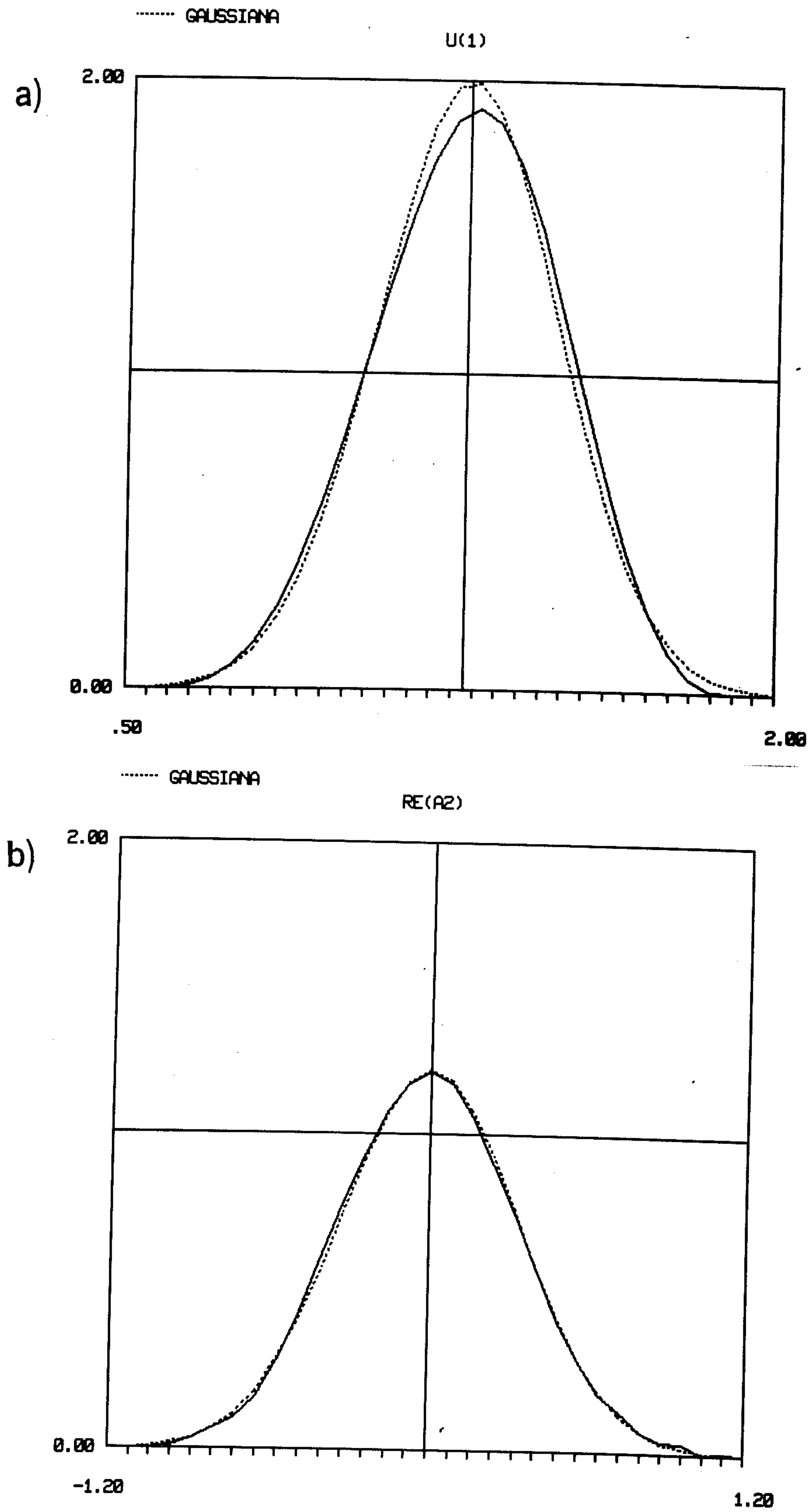


Fig.2 a) Histograms of U_1 (solid line). The dashed line represents a gaussian having the same mean and variance as the distribution of U_1. b) Same as a) but for the real part of A_2.

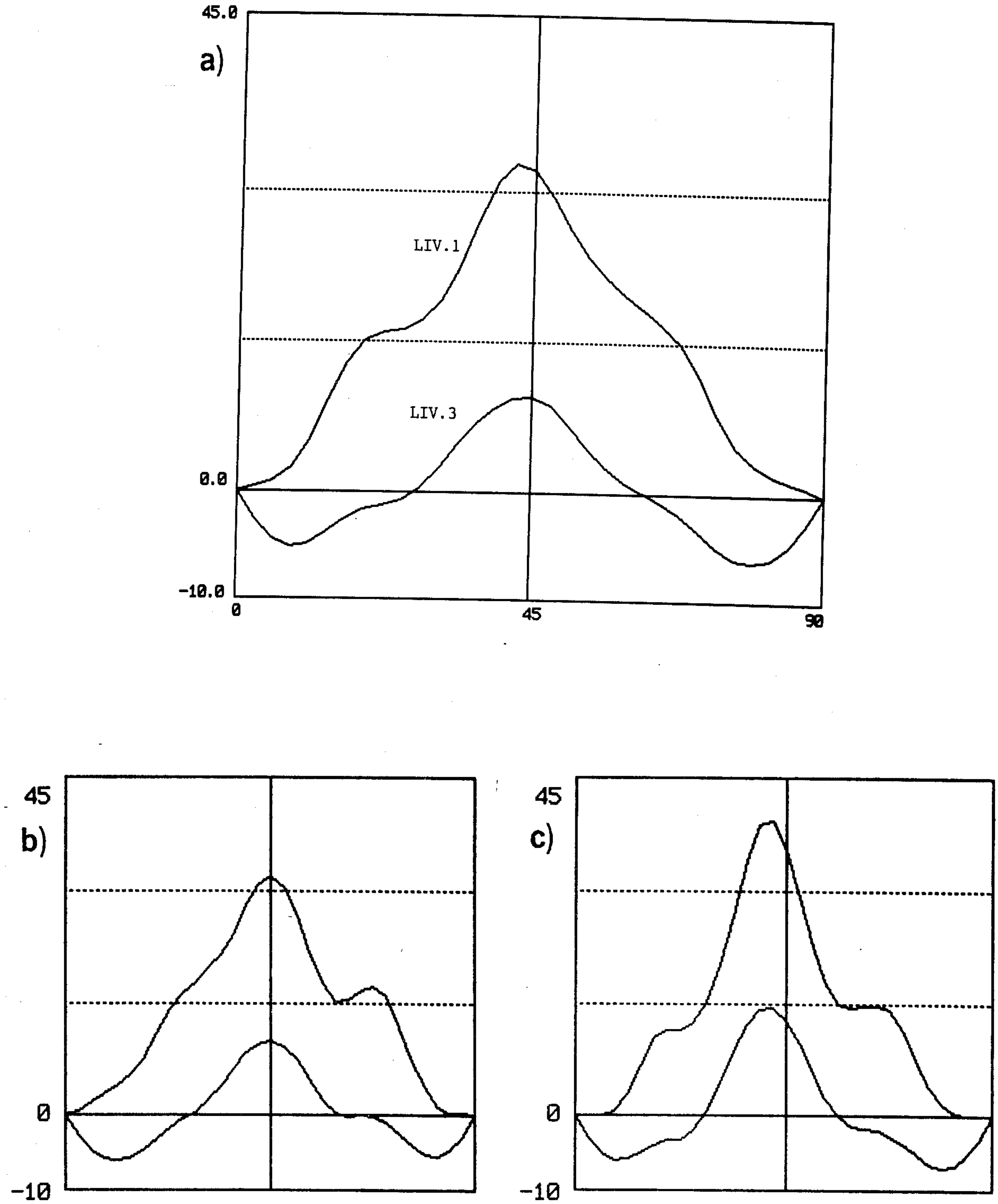


Fig.3 a) ten years average of the zonal wind at the upper level (liv.1) and lower level (liv.3). Units are m/sec. Examples of averages over individual years are shown in b) and c).

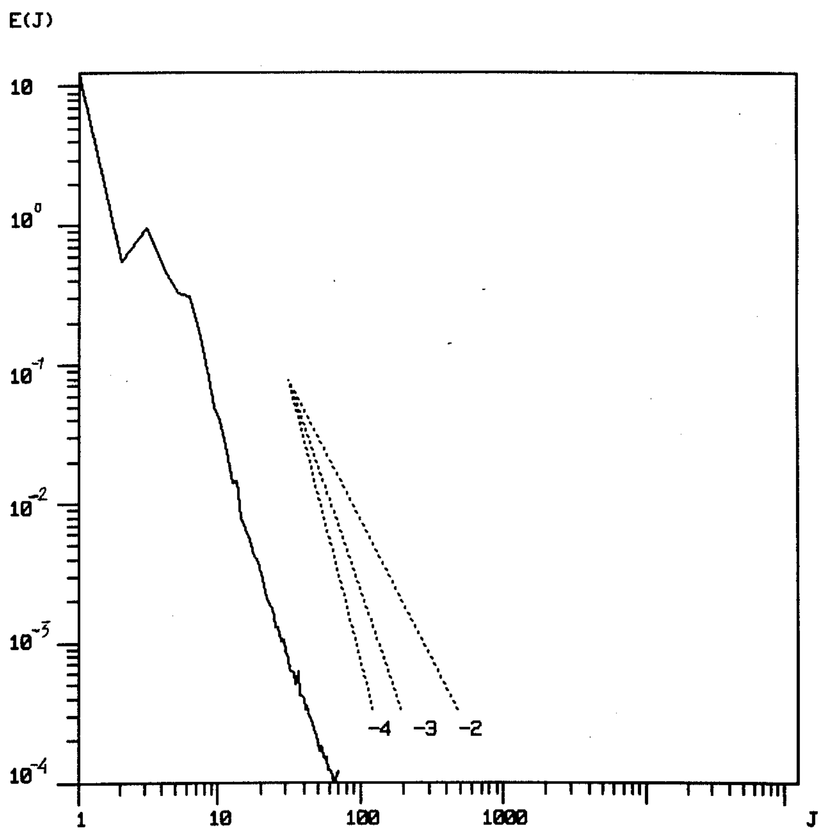


Fig.4 Fourier spectrum of the total energy (kinetic + available) as a function of the meridional wavenumber $w_j = \pi j/L_y$, j = 1,2,3,... The slope of a w_j^{-n} law, n = 2,3,4, is also plotted for comparison.

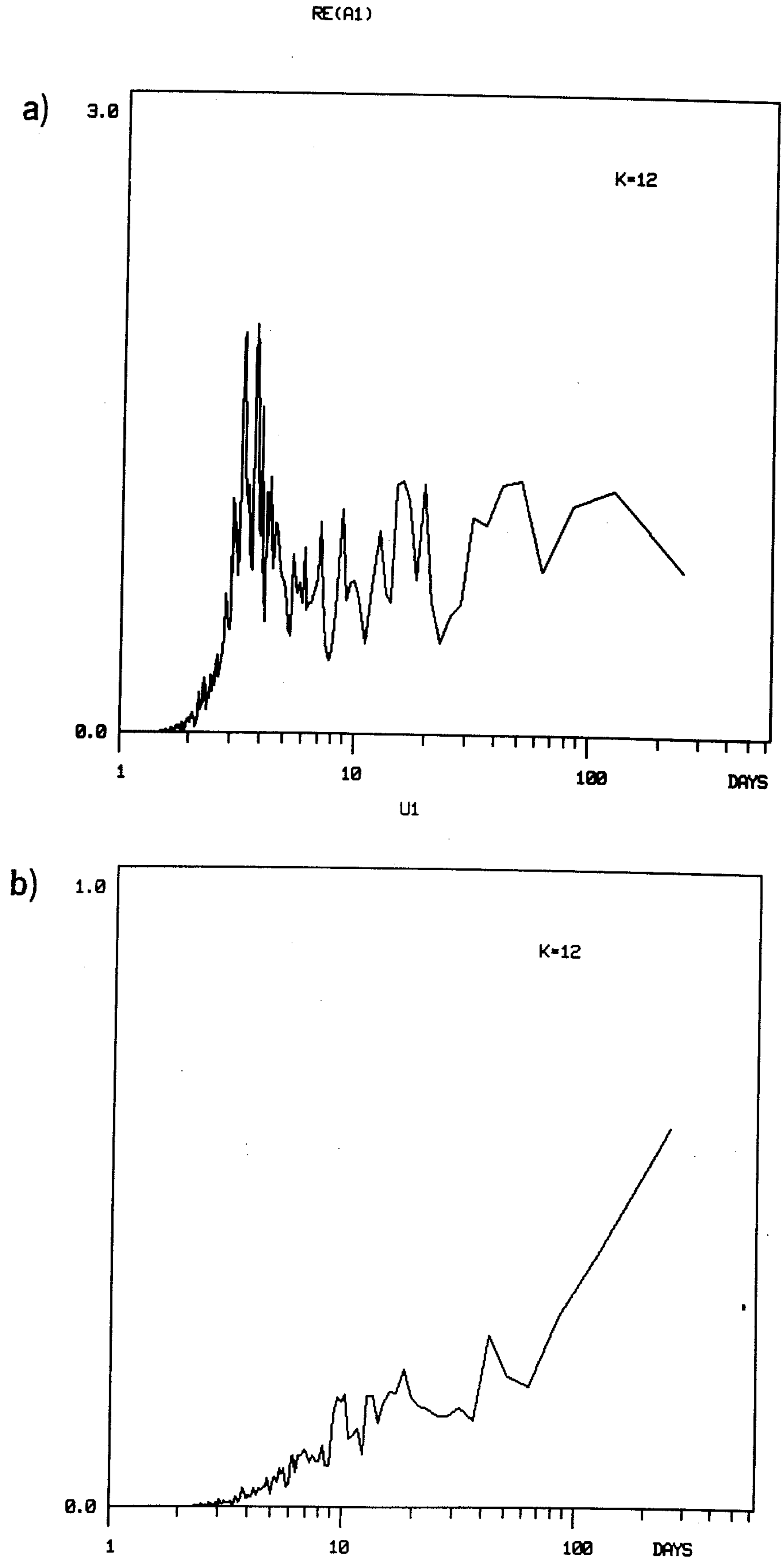
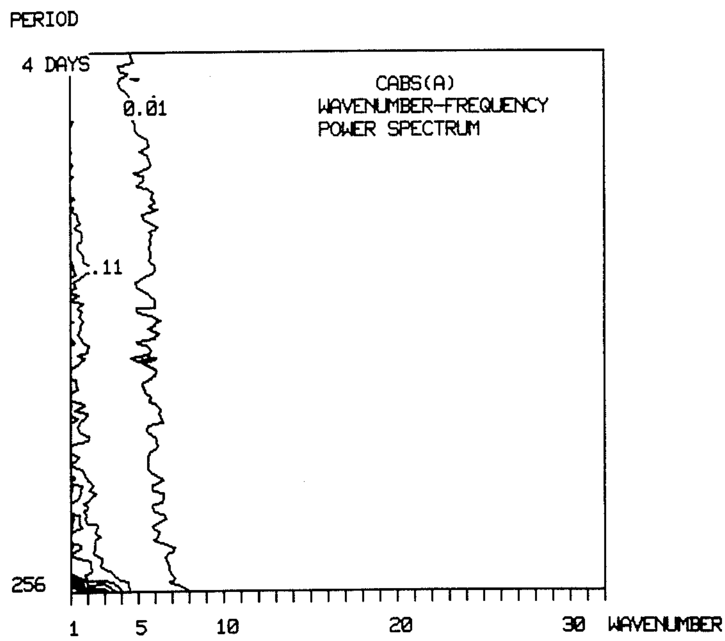


Fig.5 Power spectrum of a) real part of A_1 and b) first component of the mean zonal wind U_1. In abscissa is the period (in days) on logarithmic scale.

a)



b)

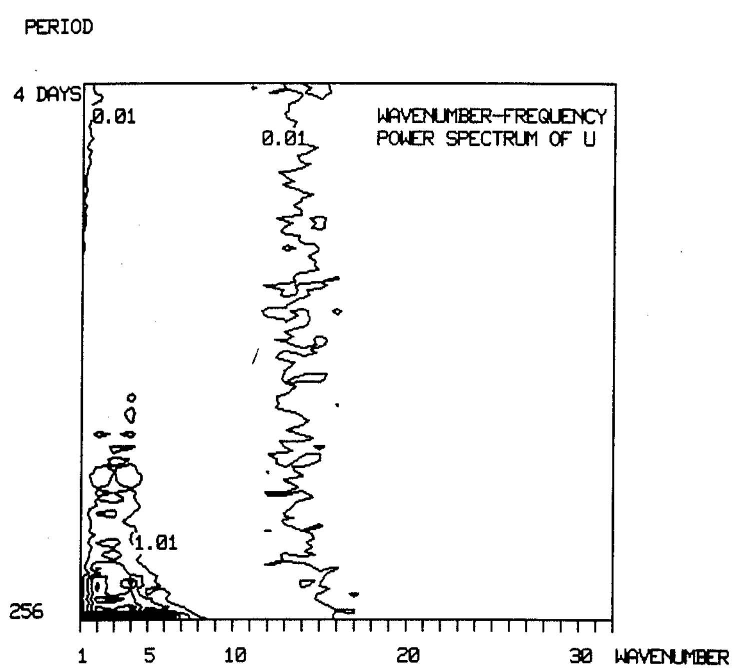


Fig.6 Wavenumber-frequency power spectrum for a) barotropic wave-component (contour interval .1) b) mean zonal wind. On the y-axis is the period on logarithmic scale.

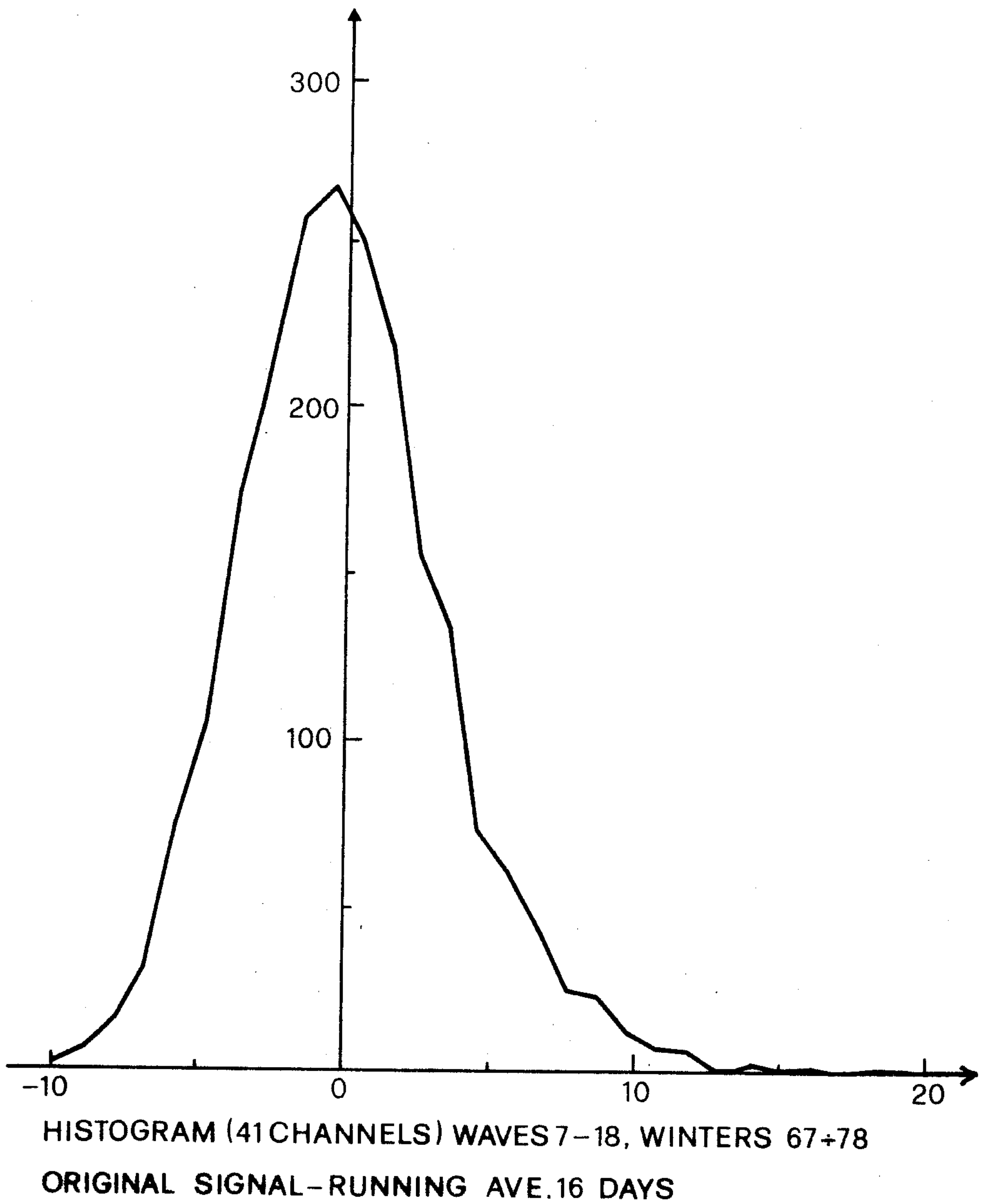


Fig.7 Histogram of spectral power of short (zonal wavenumbers 7-18) baroclinic waves in the northern hemisphere computed from 500 mb heights of winters 1966-1978. Heights are integrated in latitude between 30 and 75 degrees. The signal is detrended by operating a running average at 16 days. (From Benzi and Speranza, 1986).

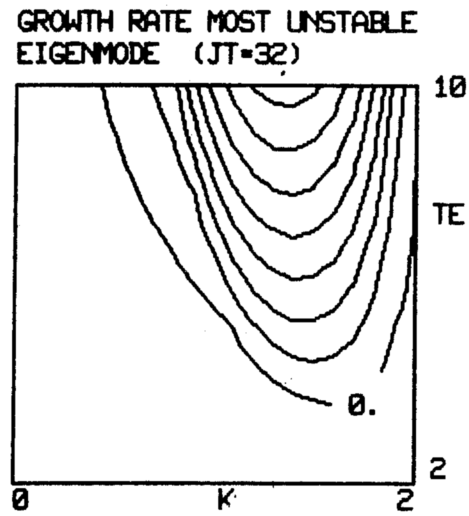
3. THE DYNAMICAL SYSTEM

The first approach I propose is the mathematical one: starting from stationary solutions (if any) the onset of turbulent chaos is followed by varying an order parameter, in our case the external thermal forcing.

The stationary solution is most easily determined as the "Hadley circulation":

$$\begin{aligned} A &= B = 0 \\ U &= m \\ \nu_S m_{yy} &- 2F\nu_H(m - m^*) = 0 \end{aligned} \tag{12}$$

that is characterized by zero wind in the lower layer. Linear stability analysis of this solution (a classical topic in meteorology!) gives the results shown in Fig.8 for the first and second mode. For k = 1.3 stability is lost around $\vartheta_E = 4$. Fig.9 illustrates the bifurcation sequence. At $\vartheta_E = 4$ the system is stable; the stable orbit, that at $\vartheta_E = 4.07$ winds several times before spiralling down to equilibrium, degenerates at $\vartheta_E = 4.2$ into a stable Hopf cycle. Between 4.2 and 4.3 another cycle is bifurcated: the motion is quasi-periodic, i.e. the ratio between the period of the new orbit and that of the first Hopf cycle is irrational and the ensuing vacillation is covering its toroidal phase space. At high values of the forcing the whole space near statistical equilibrium is filled. The sequence is of the type described by Feigenbaum, Kadanoff and Shenker (1982) who also provide an example of renormalization near the transition to chaos. The chaotic behaviour is generated near the second orbit in a fashion very similar to that described in KAM theory, although a)



b)

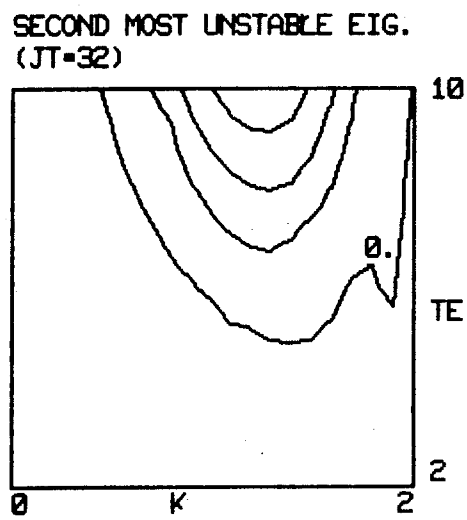


Fig.8 Growth-rates of the most unstable a) symmetric b) antisymmetric eigenmodes of the linear stability analysis of Hadley circulation versus the zonal wavenumber k and the external forcing. Contour interval is .1.

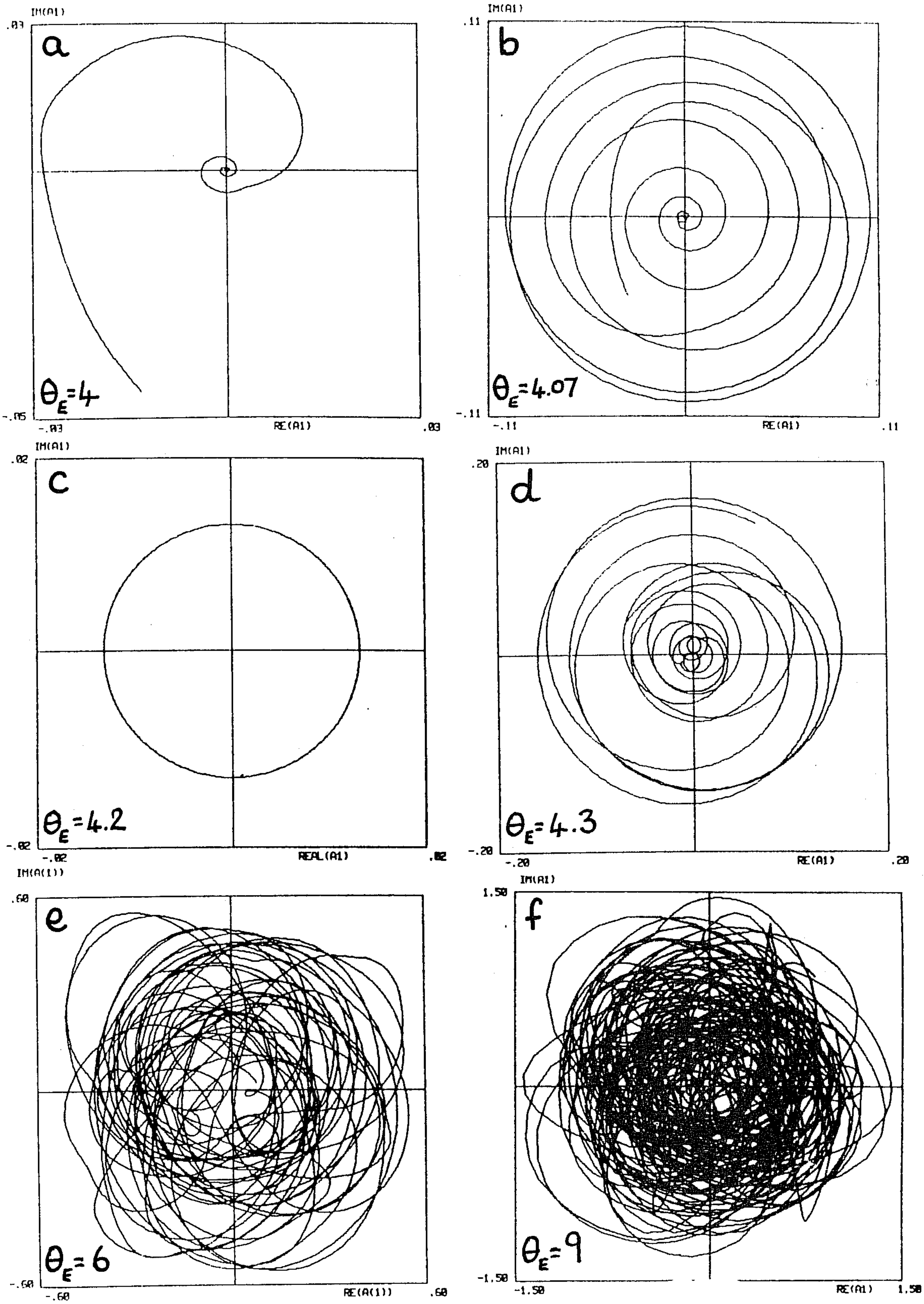


Fig.9 Projection of the phase-space trajectory on the $(\mathrm{Im}(A_1), \mathrm{Re}(A_1))$ plane for different values of the external forcing. All the trajectories describe a 1-year evolution of the system. a) $\vartheta_E = 4$. b) $\vartheta_E = 4.07$ c) $\vartheta_E = 4.2$ d) $\vartheta_E = 4.3$ e) $\vartheta_E = 6$. f) $\vartheta_E = 9$. The Hadley circulation loses its stability at $\vartheta_E \approx 4.158$.

here the system is dissipative.

Since the prototype of atmospheric behaviour we are interested in is very far from transition, there is not much quantitative knowledge we can gain from the study of the bifurcation sequence. However, as we shall see later, the qualitative understanding of the onset of baroclinic turbulence is crucial for setting up a dynamics of fluctuations near statistical equilibrium.

4. THE THEORY OF GENERAL CIRCULATION

The theory of general atmospheric circulation revolves, in many different fashions, around the basic idea that turbulent fluxes maintain an equilibrium circulation (the "stationary flow" determined by time-averaging) conspicuously shifted with respect to the true stationary solution (the Hadley circulation) and that any dynamical theory of fluctuations near statistical equilibrium, e.g. linear stability analysis, should be referred to such a state.

I will try here to test the consistency of such a theoretical framework by analyzing our model atmosphere in its context. The statistical equilibrium circulation is well approximated by the average zonal flow of Fig.3. Linear stability analysis of this circulation gives the results shown in Fig.10. As expected the system is pushed by turbulent fluxes much nearer to marginal stability than with respect to the Hadley circulation (compare with Fig.8), although still remaining in the unstable range. This result at first sight confirms classical

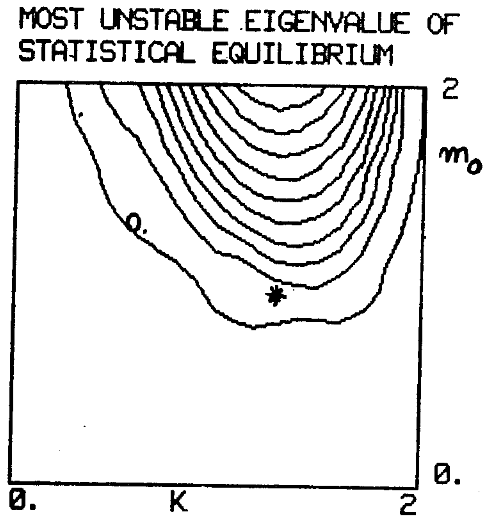


Fig.10 Growth-rate of the most unstable eigenmode of the linear stability analysis of time-averaged zonal flow. The vertical shear m is varied mantaining the profiles unchanged. Statistical equilibrium corresponds to m_c = 1. . Contour interval is .1 .

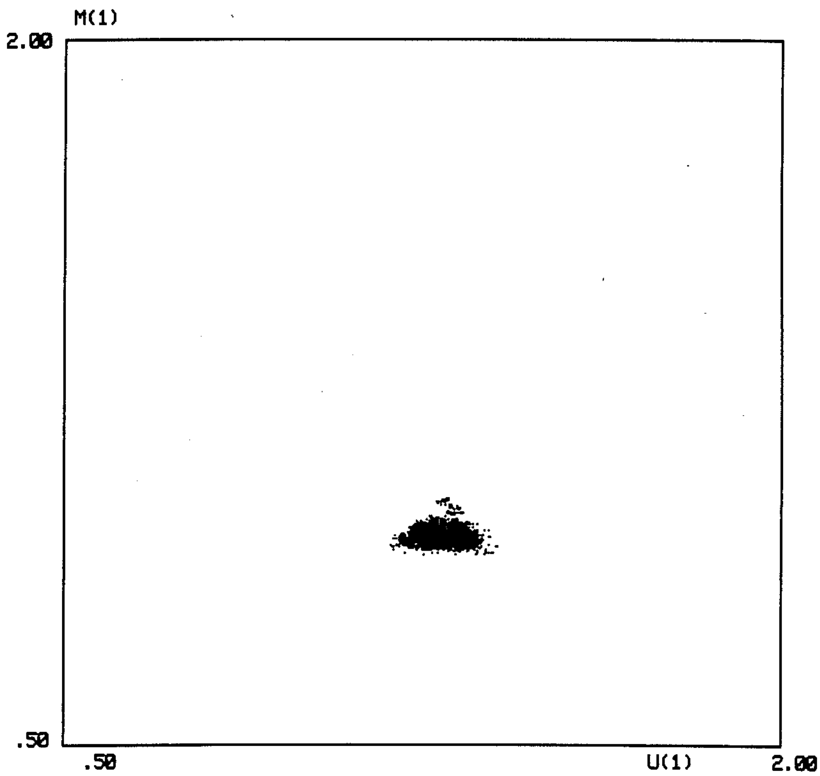


Fig.11 Scatter diagram of zonal wind and first wave-component of the system with external forcing producing a fixed point at the statistical equilibrium of the system with the scatter diagram shown in Fig.1.

conjectures about baroclinic turbulence in the atmosphere as, for example, those formulated by Stone (1978). However a thorough investigation of nonlinear theory reveals very dangerous inadequacies. In order to avoid the laborious (and boring!) mathematics typical of weakly nonlinear expansions I will try here to give an idea of the nature of the difficulties encountered in the formulation of a consistent theory, along the lines that are classical in meteorology, by means of a numerical experiment very similar to the basic one described in Section 2. The experiment consists of a ten year numerical integration of the system (8-11) for a choice of the external forcing that balances the time-mean state. As a consequence the new system has a fixed point exactly in the statistical equilibrium. The scatter diagrams of various components are shown in Fig.11. Comparison with the statistics of the original experiment (see again Fig.1) reveals two conspicuous problems: the variance is too small and the new statistical equilibrium is shifted with respect to the old one. The first problem is associated with the circumstance (previously considered favourable) that the system is very near to marginal stability of the time-mean state. In principle this problem can be handled by carefully reshaping the center manifold near marginal stability. At any rate the need for an accurate rewriting (renormalization) of the equations emerges already.

But where the physical nature of the limitations of classical theory most clearly appears is in the second problem: the shift of the equilibrium is caused by an essential symmetry of the system, namely that associated with the property of baroclinic instability to transport heat only in one direction so that nothing like an "antibaroclinic" instability exists. One can easily be convinced that this difficulty must be faced no matter how one modifies the stability problem; moreover it is not model-dependent and is, therefore, basic.

In conclusion the foundations of the theory of general circulation appear rather shaky. This perhaps explains why it is so often invoked, but never thoroughly formulated!

4. A TENTATIVE MINIMAL THEORY FOR STATISTICAL EQUILIBRIUM

A satisfactory solution to the problems outlined in the previous section will probably come only from a complete theory of renormalization of the baroclinic turbulence. I do not have even a hint of what the correct renormalization procedure may be. It will not come out to be an obvious one. What I can propose here is a line of approach emerging from the work that Malguzzi and I are performing on different truncations of the system (8-11). Truncation has, to a certain extent, the same qualitative effects as the decrease of the stress parameter: if the dimension of the truncated space is not sufficient for the embedding of the center manifold of the instability "generating" the turbulence, the system is stabilized. On the other hand the spectra of Fig.6 substantiate the hypothesis that, at least energetically, a limited number of components plays a dominant role. It is interesting, therefore, to consider the phenomenology of a sequence of trucations very much like the sequence of

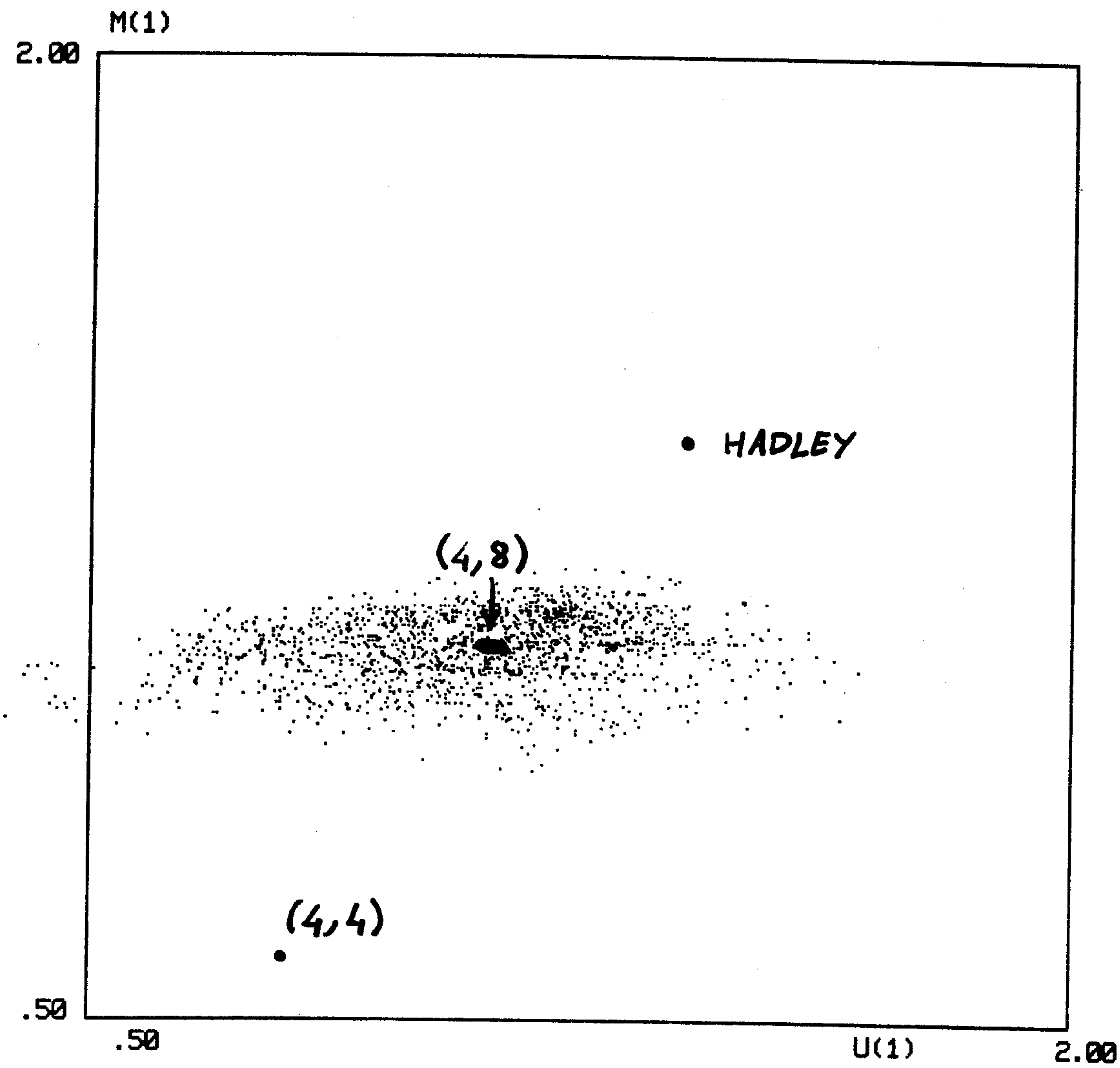


Fig.12 Scatter diagram in the (U_1, m_1) plane showing the position of the stable fixed point obteined in the (4,4) simulation, the stable orbit in the (4,8) case and the chaotic attractor in the (8,8) case. Also reported is the position of the Hadley circulation: $U_1 = 1.4$, $m_1 = 1.4$.

bifurcations discussed in Section 3.
Fig.12 shows the typical results obtained with truncation at 4 modes in the wave field and 4 modes in the zonal flow, 4 and 8, 8 and 8 respectively. Particularly interesting is the sequence of the zonal flow: the fixed point of the (4,4) system, strongly shifted with respect to the Hadley circulation because of the extreme efficiency of the waves in trasporting momentum and heat, becomes a stable,though less efficient, cycle for the (4,8) system and an unstable cycle for the (8,8) system.
Notice that we speak only improperly of a cycle because in the rest of the phase-space the orbits are ergodic and filling (remember that vacillations of waves are characterized by irrational period ratio). Comparison of the scatter diagram of the zonal flow in the unstable (8,8) system with the original one (in Fig.1) shows that the variance is very well distributed and, apparently, both the problems outlined in the previous section are solved: the instability of the cycle in the zonal flow is non-shifting and characterized by the right variance amplitude.

An adequate discussion of the physical mechanisms that are at the basis of such a striking result requires careful mathematical consideration of the properties of instabilities growing on periodic and quasi-periodic orbits versus those growing on fixed points centered on the average of the cyclic orbits themselves. Suffice here to remark that many properties are extremely different. Therefore, if the above results were confirmed for more complex and realistic systems, we might have a potential explanation of the discrepancies between the classical theory of baroclinic instability and available observations. Moreover, if the global properties of phase-space of the baroclinic system are associated with the instability of a basically cyclic process, the identification of such a process in the real atmospheric circulation becomes a primary task. An obvious hypothesis is at this point that the indicial circulation may be more "fundamental" then a priori thought. These problems are the object of active investigation.

5. CONCLUSIONS

Given the relevance of the subject debated, the mechanism of maintenance of statistical equilibrium of baroclinic turbulence in the atmosphere, I hesitate here to state any firm conclusion: much more work is needed on the statistical properties of various model atmospheres and, even more, on similar properties in the observed atmospheric circulation. I only hope I was able to show that there is no convincing prove concerning the internal consistency of the classical interpretation schemes of the general circulation and that some of their known limitations in explaining observed properties can perhaps be mitigated, if not eliminated, in the context of other theories.

I want to conclude with a general consideration. The atmospheric system is characterized by the simultaneous excitation of many spectral decades. The task of parameterizing small scales on large ones ("renormalizing" in the language of physicists) is not made simple by scale-separation, similarity, or any other obvious symmetry.

The problem of determining the collective dynamical effect of all the motions that will remain essentially unresolved, no matter what resolution will be achieved by future GCM's, is totally open. The time has come to take inspiration from other fields of science that have already faced similar situations (see, for example, Wilson (1975)) and try at least to outline some possible strategies of attack on the problems in question.

6. REFERENCES

Benzi, R., Paladin, G., Paternello, S., Santangelo, P., Vulpiani, A., 1986: Intermittency and coherent structures in two-dimensional turbulence. Submitted to Phys.Rev.Lett..

Benzi, R. and Speranza, A., 1986: The statistical properties of low frequency variability in the northern hemisphere. Submitted to Mon.Wea.Rev..

Charney, J.G., 1947: The dynamics of long waves in a baroclinic westerly current. J.Meteor., 4, 135-162.

Charney, J.G., 1959: On the general circulation of the atmosphere. Rossby Memorial Volume. New York, The Rockfeller Un. Press.

Eady, E.T., 1949: Long Waves and Cyclone Waves. Tellus, 1, 33-52.

Feigenbaum, M.J., Kadanoff, L.P., Shenker, S.J., 1982: Quasiperiodicity in dissipative systems: a renormalization group analysis. Physica D, 3, 370-386.

Gall, R., 1976: A comparison of linear baroclinic instability theory with the Eddy statistics of a general circulation model. J.Atmos.Sci., 33, 349-373.

Green, J.S.A., 1970: Transfer properties of the large scale eddies and the general circulation of the atmosphere. Quart.J.R.Met.Soc., 96, 157-185.

Kuo, H.L., 1949: Dynamic instability of two-dimensional non-divergent flow in a barotropic atmosphere. J.Meteor., 6, 105-122.

Lorenz, E.N., 1962: Simplified dynamic equations applied to the rotating-basin experiment. J.Atmos.Sci., 19, 39-51.

Lorenz, E.N., 1963: The mechanics of vacillation. J.Atmos. Sci., 20, 448-464.

Pedlosky, J., 1970: Finite amplitude baroclinic waves. J. Atmos.Sci., 27, 15-30.

Pedlosky, J., 1979: Geophysical Fluid Dynamics. New York, Springer.

Stone, P.H., 1978: Baroclinic adjustment. J.Atmos.Sci., 35, 561-571.

Stone, P.H., Ghan, S.J., Spiegel, D.,Rambaldi, S., 1982: Short-term fluctuations in the eddy heat flux and baroclinic stability of the atmosphere. J.Atmos.Sci., 39, 1734-1746.

White, A.A., 1977: The surface flow in a statistical climate model-a test of a parameterization of large-scale momentum fluxes. Quart.J.R.Met.Soc., 103, 93-119.

Wiin-Nielsen, A.C., Fuenzalida, H., 1975: On the simulation of axisymmetric circulation of the atmosphere. Tellus, 27, 199-214.

Wilson, K.G., 1975: The renormalization group: critical phenomena and the Kondo Problem. Rev.Mod.Phys., 47, 773-840.