

Objective analysis for numerical weather

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ABSTRACT

The field of objective analysis for numerical weather prediction is enjoying rapid development because of theoretical advances in analysis methods coupled with progress in observing and computing technology. This review considers current and projected work on objective analysis under three main headings: theory, verification and practice.

Recent developments in the theory suggest that there are close links between all analysis methods based on optimality principles. Moreover, simpler analysis methods such as successive correction can be modified to converge to an optimal method in simple circumstances. The basic job of any analysis algorithm is to filter random observation error from the data and then interpolate the filtered data to a regular grid. A new viewpoint on O/I shows the algorithm in a simple light with an intimate connection between the filtering and interpolation capabilities.

Verifications of observations, forecasts, and analyses are reviewed, using statistics on one-point and two-point height and velocity correlations. The two-point correlations are inter-related through the kinematic equations of two-dimensional turbulence. In the limit of vanishing spatial lag they degenerate to the one-point correlations. These diagnostics offer a means to compare the error characteristics of different observing systems. When used to verify short-range forecasts, they provide a complete description of the structure of the forecast errors. Phillips' simple theory of forecast errors can reproduce many of the features of the empirically determined structures. New results indicate that there may be a need for anisotropic and non-separable correlation functions for analysis.

The correlation diagnostics offer new and rigorous ways to estimate analysis error at observation points. They provide a quick and easy measure of the efficiency of an analysis system. They will prove useful in identifying the reasons for system dependence in the results of observing system experiments.

In reviewing current and future research we consider the special problems of analysis in data rich areas over land and near mountains; the problems of analysis over the mid-latitude oceans; and then the problems of tropical

analysis. Extensions of O/I offer new approaches to these problems which will be explored in the near future. In the slightly longer term, new developments in the theory of 4-dimensional analysis may well supersede current methods. Finally we consider developments in data quality control, an area of great importance for all other developments. New developments here, using Bayesian methods, offer promise.

1 INTRODUCTION

In recent years we have seen the advent of successful global operational forecasting (Bengtsson 1985); we have also seen the successful production of the FGGE III-B data, providing global analyses for 1979 from a vast and heterogeneous observational database. The development of objective analysis and data assimilation methods up to the completion of the main FGGE analyses is discussed in the review by Bourke, Seaman and Puri (1985). Their discussion pays particular attention to the many simulation studies that were undertaken to prepare the observational plan for FGGE. Objective analysis of the FGGE data has been the subject of a number of extensive reviews (National Research Council, 1985, ECMWF 1984).

There has been notable progress in understanding the kinematics, statistics, and mathematics of analysis methods in recent years. A theoretical framework is available which connects the many apparently unrelated methods which have been developed for meteorological analysis. The basic starting point in all methods is that the observations are insufficient to determine the continuous (or discrete) fields of interest. One must use additional a-priori information to resolve the under-determinacy. The a-priori information may be a forecast and the statistics of its errors, or constraints of continuity and/or differentiability on the solutions. The a-priori information and the observational information are combined in a Bayesian (maximum likelihood) or minimum-variance calculation which minimises a functional of the unknown solution; the advantage of the Bayesian methods is that they can handle non-gaussian statistics.

The method of optimal interpolation (O/I, Gandin 1963, Eliassen 1954) has an important role in establishing links between the many different analysis methods in existence. (The method is also called statistical interpolation since the optimum statistics can never be known exactly). The multivariate theory of O/I has been developed (Hollingsworth and Lonnberg 1986a, Lonnberg and Hollingsworth 1986, hereafter called HL/LH) using the kinematics of two-dimensional turbulence. These kinematic methods extend the capabilities of the O/I formalism in large scale, synoptic scale and meso-scale analysis and in tropical analysis. In addition they provide powerful diagnostic tools.

Assimilation systems have improved to the point where the 6-hour forecast is as accurate as the observations in data-rich areas of the Northern Hemisphere (Hollingsworth et al 1986). This offers new possibilities for improving the WWW data through long term monitoring. It also places new demands on that data since the noise to signal ratio is now approaching 1, if both noise and signal are measured relative to the first-guess. Improved communication between data producers and data users would be useful as the users know a lot about errors in the data.

The utilisation of available data over the oceans is still far from satisfactory. In the O/I context there is a need for more extensive documentation of forecast error statistics over the ocean. Further efforts are also needed to simulate the oceanic data distributions over land so that one may use independent data to define the three dimensional structure of the oceanic forecast errors. The extensive operational experiment planned by WMO for 1987/88 in the North Atlantic will be a particular stimulus to such efforts.

Analysis methods in the Tropics have improved through the imposition of physical balance requirements (Krishnamurti et al 1983, 1984) and the recognition of the importance of treating the very largest scales in the analysis (Cats and Wergen 1982, Daley Cats and Wergen 1986); further gains may come from improvements in the synoptic-scale wind analysis to take account of divergence (Daley 1985). In conjunction with improvements in model resolution, parameterisation and orography, the analysis improvements that have been implemented so far have led to marked improvements in the quality of operational forecasts in the Tropics. Despite these improvements, the over-riding problem remains the lack of observations in the Tropics.

To maximise the utilisation of the available observations, more effective methods will be needed to incorporate the time dimension in the analysis algorithm. The adjoint method proposed by Le Dimet and Talagrand (1986) appears to offer a sound mathematical foundation for tackling this problem. As pointed out by Lorenc (1986), the more explicit use of an accurate model in the assimilation also offers the possibility of including in the analysis non-linear aspects of balance, such as strong fronts in balance with a large-scale fronto-genetic field.

The plan of the paper is as follows. We begin in section 2 by reviewing the structure of current assimilation systems, emphasising the need to use the most accurate model possible in the data assimilation; this has only recently been achieved at some centres. We also discuss the need for accuracy in the multivariate interpolations of the background field from the model grid to the observation position.

Theoretical work establishing connections between many different analysis methods is discussed in section 3, together with experimental work on comparisons of the accuracy of analysis methods.

The basic task of any analysis system is to filter as much error as possible from the observations and then to make an interpolation of the resulting information to a regular grid. The intimate connections between the filtering capabilities and the interpolation capabilities of O/I are discussed in section 4.

Section 5 reviews recent developments using the kinematics of two-point velocity and geopotential correlations, and discusses how the capabilities of observing systems may be evaluated by comparison of atmospheric spectra observed by the different systems. Section 6 discusses progress in the determination of the correlation structure of forecast errors. Theoretical explanations of the empirically determined structures are also considered. One-point velocity correlations are used to survey the anisotropies in the forecast errors. Section 7 reviews the many different methods which have been used to estimate analysis error. Particular attention is given to new methods which permit a proof that all useful information has been extracted from the observations.

Section 8 considers current and future research in operational objective analysis including the problems of high resolution analysis in data-rich land areas and in the presence of mountains, the problems of analysis over the mid-latitude oceans, the problems of analysis in the Tropics, and new developments in quality control. The main themes of the paper are summarised in the concluding section.

Inevitably, there are modest overlaps with the companion reviews by Machenhauer (1986), and Puri and Gauntlett (1986), but we have tried as far as possible to avoid duplication. Inevitably also, most attention is given to the work of present and former colleagues. Our viewpoint has been described as EC-centric, but it is hoped that it is nonetheless useful.

The basic structure of most current assimilation systems is similar. A forecast model generates a 6-hour or 12-hour forecast to provide a background field for the next analysis; the background field is interpolated to the observation position; the differences between observation and first-guess are filtered, interpolated spatially, and added to the background field to form the analysis, and the process is repeated.

Current operational analysis systems at the larger centres use one of two forms of 4-dimensional assimilation, either a continuous insertion or an intermittent insertion (Bengtsson 1975, Miyakoda 1985, Lorenc 1981, Bergman 1979, McPherson et al 1979, Kistler and Parrish 1982, Dey and Morone 1985, Baker 1982). In both of these approaches the methods of Non-Linear Normal Mode Initialisation (NNMI, Machenhauer 1977, Baer 1977, Temperton and Williamson 1980, Williamson and Temperton 1980) can play an important role. In the intermittent insertion method used at most of the major Centres (Gustafsson 1981) NNMI plays a vital role in assuring the balance of the background field which is essential in quality control (Van Maanen pers. comm. 1982) and for the validity of the basic multivariate assumptions (Lorenc 1981, 1986). One goal of research on intermittent assimilation is the unification of the analysis problem and the assimilation problem (Williamson and Daley, 1983). Variational formulations of the problem are amenable to solution by direct methods and by adjoint methods. In continuous assimilation schemes (Bengtsson 1985, Lyne 1982, Miyakoda 1985, Miyakoda et al. 1986) the differences between observation and background may be inserted in the model in a repetition of the last forecast, and in the forecast which generates the background for the next analysis.

Success in data assimilation requires accuracy in all four phases of the calculation: the forecast (para 2.2), the forward interpolation, (para 2.3), the analysis (section 3 et seq.) and the initialisation (para 2.4). There have been substantial improvements in most of these areas in recent years. To set the scene for these discussions, it is useful to review recent progress at one institute chosen as an example.

2.1 Impact of Model and Analysis Changes on Forecast Skill at ECMWF

To get an overall impression of the impact of analysis improvements and model improvements we consider the ECMWF forecast performance for a four year period, June 1982 to April 1986. Fig 1 (from Shaw et al 1987) shows the rms 500mb geopotential forecast errors for the eleven months from June through April in the four successive years 1982/83 - 1985/86, for each hemisphere between latitudes 20 and 80. The May data were excluded because some of the more important model and analysis changes discussed below occurred in May.

The 1982/83 forecasts, for the first of these periods, were made with the N48 grid point model (Burridge and Haseler 1977, Tiedtke et al 1979, Hollingsworth et al 1980, Bengtsson and Simmons 1983). Between the first and second period there was an important model change. The forecasts for the second period, 1983/84, were made with a T63 spectral model using the same physical parameterisation package. This spectral model had been shown to have superior forecast performance (Girard and Jarraud 1983) and replaced the grid-point model as the operational model in late April 1983. At the same time the orographic representation was modified to an envelope orography (Wallace et al 1983).

Between the second and third periods there was a major change in analysis and some changes in the model. The changes in the analysis system in May 1984 (Shaw et al 1984) affected many aspects of the system including quality control, data selection algorithms, and structure functions. In addition an important change was made to the vertical interpolation scheme in Nov 1984 (Unden 1985). The forecasts for the third period, 1984/85, period were made with the same model as in the 83/84 period except for two changes in the radiation scheme. The first change, in May 1984, was the introduction of a diurnal cycle (Tiedtke and Slingo 1984). The second change, in December 1984, modified the representation of the transmission functions and led to a substantial reduction of the forecast error biases in the tropical temperatures in the medium range (Ritter 1985).

Between the third and fourth periods there were major changes in the model. In May 1985 the resolution of the spectral model was upgraded to T106 (Jarraud, Simmons and Kanamitsu 1985). At the same time there were major changes to the parameterisation package. A new representation of shallow

convection was introduced, the partitioning of moisture convergence between heating and moistening in the convection scheme was changed substantially, and the cloud parameterisation was radically revised (Tiedtke et al 1987).

There were clear improvements at all forecast ranges in the Southern hemisphere (Fig 1a) between the first and second periods, after the introduction of the T63 spectral model in May 1983, and again in 1984/85 after the introduction of the analysis modifications in May 1984. There were further gains resulting from the introduction of the T106 model with improved physics in 1985; these improvements were somewhat larger than expected from increases in resolution alone (Jarraud Simmons and Kanamitsu 1985, Tiedtke et al 1987) and may be partly attributable to the physics changes and partly to the gradual build-up in the number of drifting buoys for TOGA from late 1985 onward. Over the four year period the largest improvement in the southern hemisphere forecasts clearly came from the analysis improvements.

In the Northern hemisphere, Fig 1b, the results are quite different. The introduction of the T63 spectral model in 1983 led to a modest gain in forecast skill, as expected (Girard and Jarraud 1983). The analysis changes in 1984 also gave a modest improvement in medium range forecast skill; we shall see below that the 6-hour forecasts, and therefore the analyses, were clearly better. The introduction, in 1985, of the T106 model with improved physics led to a marked improvement in Northern hemisphere forecast skill.

The Northern Hemisphere and Southern Hemisphere results can be interpreted in different ways. Improved analysis methods led to a clear improvement in forecast skill in the data-sparse Southern Hemisphere in 1984/85, before the build up in the number of TOGA drifting buoys from late 1985 onward. The modest impact of the high resolution model on Southern hemisphere forecast skill may be attributable to the still large analysis uncertainties (Jarraud Simmons and Kanamitsu, 1985). If this is true then analysis error rather than model error is still the dominant contributor to forecast error in the southern hemisphere.

In contrast, on the basis of experiments with FGGE data and the 1982 model, Arpe et al (1985) suggested that model error rather than analysis error was the main source of forecast error in the Northern Hemisphere in the range from

two days out to six days and perhaps even further. The relative effects of improving the analysis in 1984 and improving the forecast model in 1983 and 1985 suggest that this conclusion was well founded for the 1982 model. Similar experiments with the 1985/6 model would be of interest. This interpretation should be tempered by the fact that the more accurate 1985/6 model probably improved the analyses through the data assimilation cycle (Leith 1984), as is clearly implied by the marked improvement in the one day forecasts. It would be interesting, but costly, to estimate how much of the improvement in the medium range is due directly to the improved model and how much is due to its indirect effect on analysis quality.

2.2 The Forecast Model

The importance of an accurate forecast model for data assimilation cannot be over-emphasised. Accuracy of the forecast model is the key to success, as it justifies the simplifying assumptions used in the other three phases: forward interpolation, analysis and initialisation. The smaller the forecast errors, the more linear will be the multivariate relations between the mass field and the wind components, the smaller will be the changes made by the analysis, and the more reliable will be the linearisations used in the initialisation.

From the point of view of Numerical Weather Prediction it is desirable to use the same model for data assimilation and for the main production forecast so as to minimise shocks to the model. This is easier to arrange for global models than for limited area models, because of the boundary conditions. The advantages are substantial: the background field for the analysis is as accurate as possible, the effects of the forward interpolations are minimised, and the adjustment between physics and dynamics at the start of the forecast, the 'spin-up', is minimised or eliminated entirely.

A most important test of an analysis, for practitioners of Numerical Weather Prediction, is the quality of the ensuing short-range forecast. Apart from operational considerations, this test has the important advantage that the errors of forecast and observations are statistically independent, unless the observations are biased. This test also provides a good evaluation of the internal balance of the analysis because a large gravity wave component in the first guess error is easily detected by the methods discussed in section 6.

There is an appreciable interaction between the quality of the analysis and the quality of the forecast model. Fig 2a shows the perceived, or measured, forecast errors in the wind field over N America in the three 3-month periods Jan-Mar 1983, Jan-Mar 1984, and Dec84-Feb85 as measured by rawinsondes (Shaw et al. 1987). There is a clear reduction in the magnitude of the perceived forecast errors as we progress from one year to the next. Between the first and second winter there was an important model change. The forecasts for the '83 winter were made with the previously described N48 operational grid-point model. The forecasts for the 83/84 winter were made with the T63 spectral model using the same physical parameterisation package and envelope orography. This spectral model had been shown to have superior forecast performance (Girard and Jarraud 1983) and this is evident already in the six hour forecasts, from Fig. 2a.

Between the second and third winter there was the major change in the analysis, and some changes in the model already described. The forecasts for the 84/85 period were made with the same model as in the 83/84 period except for the introduction of a diurnal cycle a modified representation of the radiative transmission functions. It is unlikely that either change had much effect on 6-hour wind forecast errors over North America, and so the gains in forecast skill between the second and third winter were mainly due to the analysis changes.

There is a clear implication from the results in Fig 2a that both the introduction of the spectral model in 1983, and the analysis revision in 1984, had important benefits for the accuracy of the short term-forecasts. To document this conclusion more clearly, Fig 2b shows the estimates of wind prediction error for each of the three periods, calculated as in HL/LH. The improvement in the quality of the 6-hour forecast errors is quite evident, and demonstrates clearly that both model improvements and analysis improvements can contribute to short range forecast improvements. As a check on the results, Fig 2c shows the observation error for wind for each of the three periods, estimated in same way. There is substantial agreement between the estimates of observation error for all three periods.

2.3 The Forward Calculations

In the language of geophysical inverse theory (Menke 1984), the interpolation of the background field to the observation point may be referred to as the 'forward' problem, and the interpolation of the observations back to the model may be referred to as the 'inverse' problem. Historically, most attention in meteorological assimilation has been given to the inverse problem i.e. to the analysis. The perception of the forward problem as a trivial interpolation to the observation position needs to change, because it is critical for a successful analysis, and it is difficult.

For a univariate problem Franke (1985) demonstrates the importance of accuracy in the interpolation of the first-guess to the observation position by showing that the advantage of O/I over competing methods can be completely negated if the O/I algorithm uses an inferior interpolation scheme for the forward calculation.

Bourke, Seaman and Puri (1985) note that two features of prediction models which influence assimilation systems are the use of terrain-following sigma-coordinates, and the use of temperature as the thermodynamic variable. They comment that 'in multivariate analysis a sigma coordinate system does not readily allow application of the geostrophic wind law, and temperature is less easily used than is geopotential. Accordingly the analysis component of the assimilation is frequently implemented in pressure coordinates. The model prediction is thus interpolated from model to pressure coordinates, corrected by available data, and the resultant increments or changes interpolated back to the model sigma domain'.

The importance of accuracy in the forward calculation is much greater in such a multivariate problem, since considerations of balance are of central importance in multivariate analysis. The theory of O/I assumes that the first-guess is in balance; it then calculates (linearly-) balanced increments from the differences between observations and first-guess. It is quite difficult to make univariate interpolations of the mass and wind components, and still preserve subtle properties such as approximate geostrophy or non-divergence. Wergen and Simmons (1986) have shown that serious consequences can arise when error in the forward calculation generates an unbalanced first guess. The imbalance in their study arose because mass and

wind were interpolated univariately from the model's coordinates to provide a first guess on standard pressure levels. Even though the model was balanced on sigma surfaces, the first-guess on pressure surfaces was seriously unbalanced due to a substantial mis-match between the model's vertical resolution and the set of standard levels on which the first guess was needed for the analysis. In their case the background field was needed at 10, 20, 30, 50, 70 and 100 hPa in the stratosphere, and the original model had wind and temperature levels at 25, 75 and 130 hPa. They compared the first guess generated at 50 hPa from this assimilation, with the first guess from an assimilation where the revised model had levels at approximately 10, 30, 50, 70, 100 and 130hPa. The balance of the first guess was seriously defective in the one case, and quite good in the other. If the first-guess is unbalanced and the observations are balanced, then the observation minus first-guess deviations will be unbalanced, and will be heavily filtered (section 4). As a result, much useful information will be rejected by the assimilation, because of errors in the forward interpolation. When they ran an extended assimilation with enhanced vertical resolution in the assimilating model, Wergen and Simmons found that there were important improvements in the medium range forecasts as well as in the short range forecasts. They showed that the medium range improvements were largely attributable to the improvement in the initial data.

The work by Wergen and Simmons is the latest of a series of developments at ECMWF aimed at improving the vertical interpolation algorithms. The first implementation of the ECMWF analysis system (Lorenc 1981), interpolated the first guess from model coordinates to pressure coordinates, evaluated the new analysis fields in pressure coordinates and then interpolated these univariately to model coordinates. A revision to this scheme was introduced by Talagrand in 1980, whereby the analysis increments were interpolated multivariately from the data points to the horizontal projection of the model grid in pressure coordinates. The analysis increments were then interpolated univariately from pressure coordinates to model coordinates in the vertical. This procedure has since been adopted at NMC Washington (Dey and Morone 1983) and at NASA's Goddard Laboratory for Atmospheres (Baker et al, 1984) and elsewhere.

Uden (1984) implemented an algorithm where the analysis increments can be interpolated three-dimensionally and multivariately from the observation points to model coordinates, taking account of the usual constraints of near-geostrophy and near non-divergence of the increments. This requires the use of continuous positive-definite representations of the vertical correlation functions, ensuring that all correlation matrices generated with these functions are positive-definite. Implementation of these ideas has improved the multivariate properties of the analysis.

Improvements in the multivariate aspects of the forward interpolation can come both from improving the resolution of the prediction model, so that the interpolation to observation position is over a shorter distance, and by improving the order and accuracy of the interpolation. The importance of errors in the forward problem will be highlighted again when we discuss developments in the retrieval of information from the satellite sounders.

2.4 Convergent Evolution in Assimilation and Initialisation Techniques

It has long been recognised that some concept of balance is essential for meteorological analysis. If the concept is ignored then the introduction of new data may generate noise in the assimilation, resulting in the rejection of very expensive information. The techniques of intermittent assimilation and continuous assimilation both address this problem, but the methodologies of the assimilation methods were initially rather different in several important respects (Bengtsson 1975).

On the whole, the intermittent methods use more elaborate spatial interpolation methods for the observational data. The methods are often multivariate, they use more observational data in the calculation for a given point, and they use fairly simple concepts of balance, a linear concept in the analysis and a linearisable concept in the initialisation. The continuous insertion methods avoid the explicit imposition of balance concepts, and typically use fewer observational data in the calculation of updates for a given point. The continuous method relies on the model's dynamics and physics to spread out the observational information in such a way as to achieve an accurate and balanced analysis.

As a result of the availability of the FGGE datasets, extensive and detailed comparative studies of the response of analysis systems to particular types of data have been undertaken (Hollingsworth et al 1985, Barwell and Lorenc 1985, Baede et al 1985, Bromley et al 1985, Kashiwagi 1985, Uppala et al 1985, Puri et al 1985, Atlas et al 1984). These studies served to underline the relative strengths and weaknesses of the different methods. As the studies have been extensively reviewed already (ECMWF 1984, National Research Council 1985), there is no need for great detail here, and a few general comments will suffice.

There is evidence of convergence in the evolution of the assimilation systems (Williamson and Daley 1983). As discussed in a companion review, Machenhauer (1986), the intermittent assimilation systems are trying to incorporate more of the model's physics in the initialisation. Many studies have shown that the adiabatic normal mode initialisation had a serious damping effect on the tropical divergence field. This can be overcome by eliminating some of the damping through selective filters (Puri 1983, Kitade 1983), or through the incorporation of the large-scale and slowly varying components of the model's diabatic tendencies in the initialisation technique (Wergen 1982).

At the same time, it has been found desirable, in some continuous assimilation schemes, to be more explicit in the way in which wind data is spread out in the vertical and balanced with height in the horizontal (Miyakoda et al. 1986, Barwell and Lorenc 1985). The availability of advanced assimilation systems with different design philosophies has been a valuable stimulus to deeper understanding of the problem as a whole.

3 THEORETICAL AND EXPERIMENTAL COMPARISON OF ANALYSIS METHODS

Not only has there been convergence in practical aspects of assimilation, there has also been a growing understanding of the close theoretical relations between many of the methods used for the analysis step of the assimilation. Theoretical work has shown that analysis methods based on optimality principles are closely related to each other, and a synthesis of the theory of analysis methods is now emerging (para 3.1). The theory of O/I has an important role in this synthesis.

The adjoint technique discussed in para 3.2 is also based on optimality principles. It offers the prospect of a more effective exploitation of the time history of the atmosphere. The value of a better use of the time history will occur at several points in our later, more practical, discussions. Experimental work confirming that the theoretical advantages of O/I can also be realised in practice is discussed in para 3.3.

3.1 The Theory of 3-dimensional Multivariate Analysis

Phillips (1982) argued that a multivariate O/I analysis using all the available observational data is 'complete' in the sense that no further information can be extracted from the data through a variational analysis with balance constraints. The underlying continuous multivariate analysis (see section 4) defines the slow mode components (Leith 1980) of the mass and wind field increments completely, subject to the requirements that 1) the covariances used are appropriate for slow modes, and 2) the forecast is sufficiently accurate that it represents the fast modes to a reasonable degree. Phillips (1982) argues that one should use as much data as possible in a single calculation, and that one should analyse as large an area as possible with that data, ideally the whole globe. The box technique developed by Lorenc (1981) therefore finds solid support from theoretical arguments. Lorenc (1986) comments that a limited or restricted data selection is appropriate when the background field has little useful prior information. When the background field is accurate, then there is much to be gained from an extensive data selection, provided the statistics of forecast error are reasonably well known.

Ikawa (1984a,b,c) has made a detailed investigation of the relationship between variational analysis as proposed by Sasaki (1959,1970) and the multivariate O/I algorithm. He confirms Phillips' result on the essential equivalence of the slow-mode O/I analysis and the variational analysis. Ikawa first shows how weak and strong constraints can be formulated in essentially similar ways (Ikawa 1984a). He then shows that the slow-mode covariance matrix as used in O/I has the same sort of filtering properties as the variational analysis (Ikawa 1984b,c).

Lorenc (1986) has taken the synthesis of analysis methods a stage further by demonstrating that O/I can be derived by variational arguments, as also shown by Ikawa, and that O/I can be derived by Bayesian methods, provided the statistics are gaussian. We therefore have three independent demonstrations of the equivalence of O/I and constrained variational analysis. These results are extensions of earlier work by Kimeldorf and Wahba (1970).

Lorenc (1986) formulates the analysis problem in a very general way. The observations, of themselves, are incapable of defining the state of the atmosphere. Prior information in the form of model equations, or a model forecast, or climatology, or requirements of smoothness, or some combination of these, is needed to resolve the under-determinacy. The analysis problem is posed as the minimisation of a penalty function consisting of terms depending on the distance of the analysis from the data, and on the distance of the analysis from the prior information, each distance being weighted by the accuracy of the corresponding information source. A particular feature of Lorenc's derivation is the way in which any number of weak constraints can be incorporated into a single penalty function.

Lorenc goes on to demonstrate the relationship between O/I and the smoothing spline, or thin-plate spline, approach of Wahba and Wendelberger (1980). In the latter approach no information on the mean field (or its first few derivatives) is taken from the first guess. Both the curvature of the interpolating function and the noise level of the observations are adjusted by Bayesian calculations which maximise the likelihood of the final solution. This method is most applicable when no first guess is available, or when the first-guess has little useful information on the larger scales. As shown by Hollingsworth et al (1986) the first guess produced by a good forecast model has much valuable information.

The main difference between the results using continuous correlation functions on the one hand, Phillips (1982) and section 4 below, and those presented by Ikawa (1984) and Lorenc (1986) on the other, is that the latter authors have used a discrete formulation for the spatial dependence of the correlation functions. The use of a discrete formulation has theoretical and practical advantages, since finally the continuous representations used here and in Phillips' discussion must be discretised for the forecast model. As noted by Hollingsworth (1984) the difference between the differential gradient and the finite-difference gradient can be large. However it is not clear from the work of either Ikawa or Lorenc how to determine the discrete correlation representation from observational data. This is likely to be a tricky problem in the multivariate case.

3.2 More Effective Incorporation of the Time Dimension

There are a number of outstanding problems in analysis which will probably yield only to methods which make more effective use of the time history of the atmosphere. Examples include situations where the state of atmospheric balance is strongly non-linear, as it is near sharp fronts, and the determination of the planetary scales in the Tropics from scattered sparse observations.

The general analysis framework proposed by Lorenc (1986) can accommodate 4-dimensional as well as three dimensional analysis. When the forward problem is linear, Lorenc shows that the optimal 4-dimensional analysis reduces to a sequence of 3-dimensional analyses, using the forecast from each to provide the background for the next analysis. This is the Kalman-Bucy filter (Ghil et al 1981). There is a close analogy between approximations to this process and the implementation of many operational intermittent-insertion assimilation systems. The process of initialisation can be incorporated explicitly in the Kalman-Bucy formalism (Ghil et al 1981). Alternatively, it can be imposed implicitly through a linear weak constraint that the analysis should mainly consist of slow modes.

Implementation of the full Kalman-Bucy formalism is prohibitively expensive because one needs to deal with matrices whose order is given by the model's degrees of freedom. The minimisation of a four-dimensional penalty function which includes the model equations as a constraint poses formidable problems.

Le Dimet and Talagrand (1986) propose integrating the model forward in time for N time-steps, remembering the model's trajectory at each time level, and then integrating the adjoint model backward in time, using the remembered trajectory in the adjoint calculation. One can then determine the direction of descent for minimisation of the penalty function. Iteration of this procedure gives an optimal trajectory over the period of assimilation. If one wishes to start a forecast at day 0, with an assimilation period of 1 day beforehand, then the starting analysis at day -1 is repeatedly adjusted so as to bring the model evolution as close as possible to the observed evolution during the assimilation period. In this way not only will the atmospheric state at day 0 be well analysed; its tendency, or time derivative, will also be well represented. In a sense the method is a rigorous way to achieve the goal of the simpler forward-backward schemes of the late 1960s, (Miyakoda and Moyer, 1968).

Talagrand (1985) and Courtier (1985) have applied the method to a one-level primitive equation model with encouraging results; Lewis and Derber (1985) have made similar calculations with filtered models. Extension of the work to multi-level models will undoubtedly follow. A major difficulty will be the treatment of non-differentiable processes, such as convection, for which a suitable adjoint cannot be defined. Nevertheless, the preliminary results are so encouraging, and so many interesting possibilities exist for approximate treatment, that this will be an area of much interest and activity.

On a much simpler level, synopticians make considerable use of the surface pressure tendency field for analysis and forecasting. Extension of the O/I formalism to include tilted structure functions for mass and stream-function, coupled with the divergent wind field, may permit an effective use of pressure tendency data within the O/I formalism. The potential value of such data has been discussed by Bengtsson (1978) in the context of analysing developing systems.

3.3 Experimental Comparison of Methods for Univariate Analysis

Seaman (1983) and Seaman and Hutchinson (1985) have compared the accuracy of different univariate analysis algorithms using a 20-year record of surface pressure reports in S.E Australia, with climatology as a background field. They investigated Statistical Interpolation, The Successive Correction Methods

of Cressman (1959), and Barnes (1964), Laplacian Spline Fitting with Generalised Cross-validation (Wahba and Wendelberger 1980), and Statistical Interpolation with Generalised Cross Validation to determine the noise to signal ratio of the observations being analysed. They found that the best results were obtained by statistical interpolation using appropriate forms of correlation function. However the Cressman successive correction method was only slightly less accurate than statistical interpolation with the best functions, and was better than statistical interpolation with a Gaussian correlation function. They also found some suggestion that when the covariance structure is not well known, a modest improvement in statistical interpolation accuracy may be achievable by changing the 'noise to signal' parameter from case to case, using generalised cross validation.

Franke (1985) has examined the errors in objective analysis arising from two sources, the interpolation of the gridded first guess to the observation position, and the interpolation of the observation minus first guess information from the observation points back to the grid. His results on the second question, the observation-point to grid interpolation, are in general agreement with those of Seaman (1983) and Seaman and Hutchinson (1985) in showing that the O/I approach has an edge over most other methods in univariate problems with uniform observation errors. Franke's results on the importance of the forward calculation have been noted already. The final point made by Franke is that O/I is more successful at making an analysis than at estimating the analysis accuracy. The reason is that the calculation of the second moment of the analysis error makes no allowance for error in the assumed covariances of forecast error.

These extensive tests show that O/I is better than competing methods on univariate problems, with observations of uniform quality. In operational analysis one deals with more complex problems of multivariate analysis with data of variable quality. The theory of statistical interpolation suggests that a properly tuned O/I analysis would do even better than the competing methods in such circumstances (Seaman 1977). The methods for analysis verification discussed in section 7 provide a framework in which to compare the performance of different analysis methods in a multivariate context; comparative studies of this kind have yet to be made.

The basic task of any analysis method is to filter as much noise as possible from the observations, and to interpolate the information to a model grid as accurately as possible. Since the O/I algorithm plays a central role in theoretical discussions of analysis method, it is worthwhile to present a simple interpretation of the filtering and interpolation properties of the algorithm. It is always helpful in scientific problems to have an intuition about how the solution will look, and it is not easy to develop intuition about the inverses of large matrices; the results presented below may help.

The O/I algorithm is a filter of the observations, relative to the eigenvectors of the weight matrix $\underline{\underline{P}}(\underline{\underline{P}}+\underline{\underline{D}})^{-1}$, followed by an interpolation with a set of continuous functions which are interpolators of the eigenvectors of the prediction error correlation matrix $\underline{\underline{P}}$, where $\underline{\underline{P}}$ and the scaled observation error covariance matrix $\underline{\underline{D}}$ are defined below. In many cases of practical importance the eigenvectors are identical, and the algorithm then has a particularly simple representation (Eqs A.3-A.5).

The results of para 4.1 provide a complete description of the response characteristics of the O/I system when all available data has been used by the algorithm. This is normally impractical. An approximation to the ideal data selection was introduced by Lorenc (1981), based on a box method. The response characteristics of this algorithm are discussed in para 4.2 following the work of Daley (1985). The effect of correlated observational error on the O/I filter is discussed in para 4.3.

4.1 The Connection Between Filtering and Interpolation in O/I

As demonstrated in many papers (Gandin 1963, Rutherford 1972, Schlatter 1975, Bergman 1979, Lorenc 1981) the O/I analysis equation may be written in the form

$$a(r_k) = [p(r_k, r_i)]^* (\underline{\underline{P}} + \underline{\underline{D}})^{-1} \cdot [d(r_i)] \quad 4.1$$

where r_i indexes the position and the observed variable at the observation points;

r_k indicates the position and variable at the analysis point (it is usually convenient to treat the spatial dependence as continuous);

$[d(r_i)]$ is the vector of differences between the observed and background values normalised by the standard deviation of the background error; we shall speak of these quantities as the observation minus first-guess (Ob-Fg) differences; the notation $[]$, or a single underline, will be used for a vector;

$a(r_i)$ is the difference between the analysed and background values normalised by the standard deviation of the background error;

$[p(r_k, r_i)]$ is the vector of background error correlations between the observed variables at points r_i and the analysed variable at point r_k , and the asterisk indicates transpose;

$\underline{\underline{P}}$ is the correlation matrix of the background field error for the observed variables at points r_i ; a double underline will denote a matrix;

$\underline{\underline{D}}$ is the corresponding observational error covariance matrix, normalised by the variance of the background field error; and all variables are assumed to be unbiased.

We may write the vector of analysed values at the observation points as

$$[a(r_i)] = \underline{\underline{P}} \cdot (\underline{\underline{P}} + \underline{\underline{D}})^{-1} \cdot [d(r_i)] \quad 4.2$$

so that Eq 4.1 may be rewritten as

$$a(r_k) = [p(r_k, r_i)]^* (\underline{\underline{P}})^{-1} \cdot [a(r_i)] \quad 4.3$$

Thus the analysis calculation may be thought of as a linear filter (Eq 4.2) of the observed values to produce the analysed values at the observation points, followed by an interpolation (Eq 4.3) of the filtered observations to the analysis points.

The nature of the filtering and interpolation are discussed in the appendix. Here we discuss the special case where $\underline{\underline{P}}$ and $\underline{\underline{D}}$ commute. This is a good approximation to many cases of practical importance, such as $\underline{\underline{D}} = \sigma \underline{\underline{I}}$, when the observation errors are random, uniform, and independent.

When the matrices \underline{P} and \underline{D} commute, the relation between the filtering and the interpolation is very simple. As shown in the appendix, if the original vector $[d(r_i)]$ is expanded in eigenvectors of \underline{P} as

$$[d(r_i)] = \sum_1^N c_j \underline{e}_j \quad \text{A.3}$$

then the analysed values at the observation points are given by

$$[a(r_i)] = \sum_1^N c_j \frac{1}{1+v_j} \underline{e}_j \quad \text{A.4}$$

where v_j is the ratio of observational error to prediction error in the j^{th} component. The analysed value at the point r_k is given by

$$a(r_k) = \sum_1^N c_j \frac{1}{1+v_j} \mathcal{E}_j(r_k) \quad \text{A.5}$$

where the function \mathcal{E}_j interpolates the vector \underline{e}_j according to Eq. 4.3. The simple relation between the data representation (Eq A.3), the representation of the filtered data (Eq A.4) and the representation of the continuous multivariate analysis (Eq A.5) is aesthetically pleasing, and provides a simple insight into the way the O/I analysis algorithm works. For a scalar field with a gaussian correlation, the filter has simple scale dependent properties (Daley 1985). If one introduces constraints of non-divergence or geostrophy, the multivariate filter will selectively damp divergent or anti-geostrophic components in the data, as well as retaining its general scale-dependent character (Daley, 1985). The existence of observational error correlations can substantially modify the scale selectivity properties. These topics will be discussed presently. The continuous multivariate fields of analysis increments produced by the analysis A.5 have all the properties of differentiability, scale dependent geostrophy and scale dependent non-divergence that one wishes to impose. When the analysis increment is evaluated on a finite difference mesh, or in a truncated spectral representation, these desirable properties may be retained only in an approximate fashion.

4.2 Scale Dependent Response of the O/I Algorithm

An implementation of the O/I system using all the data available at a given time would provide a multivariate and three-dimensional interpolation of the observational data satisfying all the requirements of differentiability, linear balance and near non-divergence one wishes to impose. This ideal will

remain a dream for some years, and it is necessary to make intelligent approximations to the ideal. One such approximation was devised by Lorenc(1981), using a box method for the analysis.

The results of Phillips (1982) and Lorenc (1986) provide theoretical support for the box approach to O/I analysis introduced by Lorenc (1981). Seaman (1977), Lonnerberg (1982), Andersen (1982) and Daley (1983, 1985) addressed theoretical and practical questions concerning the response of the box-based analysis system to data for different variables, and to data on different scales. Early experience with the ECMWF system indicated that the system had difficulty in analysing small scale rapidly developing disturbances. It was not clear which features of the system were responsible for these defects. It has since emerged that several aspects of the system needed refinement, including both the horizontal and vertical resolution. A good theoretical basis and a statistical database were needed to address these questions.

As formulated by Lorenc (1981) the analysis algorithm selects data on mass and wind for up to 9 levels within and around each analysis box of about 660km on a side. A multivariate analysis is then performed for mass and wind at all the grid-points and levels within the box. A refinement to the algorithm known as the 'box overlap' was introduced by Andersen in 1981. In this refinement the analysis for each box is evaluated for a region extending outside the box to the centre of the neighbouring boxes. This provides several additional estimates of the analysis for each box. These are combined in a simple weighted mean. It was this version of the analysis that was examined by Daley (1983, 1985).

To study the scale dependence of the response, Daley sampled simple sinusoidal functions with a regular array of observing points, and examined the magnitude of the response as a function of the observation spacing and the scale of the disturbance. For these experiments the geopotential auto-correlation was a gaussian with a length scale of 600km, the geostrophic coupling coefficient was 0.95, and the wind structure function was strictly non-divergent.

Fig 3 summarises his results. Fig 3a shows the response (ratio of output amplitude to input amplitude) in the geopotential field to geopotential observations. There is a marked sensitivity of response to both the scale of the input and the observation spacing. For observation spacings larger than

200km the response is less than 10% for planetary wave-numbers higher than 35 (quarter wave-length=285km); for observation spacings larger than 300km the response is less than 30% for planetary wave-numbers higher than 25 (quarter wave-length=400km).

The non-divergent wind response to non-divergent wind observations in Fig 3b is rather better than the height response, with improved response at all wavenumbers between 10(quarter wave-length 1000km) and 35. However there are clear indications of problems in the analysis of the very largest scales when the observation spacing is equal to or larger than the length scale of the gaussian. This has been remedied by recognising that the correlation function must include a representation of forecast errors which are of such large scale that they are essentially constant over the region for which analysis calculations are made (HL/LH). The square of the normalised observation error was 0.25 in both Fig 3a and Fig 3b so the extra response on synoptic scales in the wind field must come from the non-divergence constraint and the fact that two wind-components are analysed simultaneously.

The effect of analysing height or wind using both height and wind observations is shown in Fig 3d. Comparing Fig 3d with 3a shows that the presence of the wind data improves the height analysis on short scales, while the presence of the height data improves the wind analysis on large scales.

In general terms one may think of Fig 3d as the mid-latitude response to height or wind data, Fig 3a as the tropical response to height data, and Fig 3b as the tropical response to non-divergent wind data. The response to divergent wind data at any latitude is shown in Fig 3c. Although the structure functions are non-divergent, they are only applied in local calculations with non-global data selection. As discussed by Lorenc (1981, 1986), it is then possible for the system to respond to divergent wind fields which are representable on a 6-degree grid, and so have a minimum wave-length of 12 degrees. These arguments are borne out by Fig 3c, which shows that the response is negligible beyond wavenumber 25. For wave-number 5 or lower there is a response in excess of 30% provided the observation spacing is closer than 600km.

These results are dependent on the width of the gaussian correlation function used for the calculations. The spectral representation of forecast error discussed in section 6 offers a general approach to the problem of improving

resolution at small scales while maintaining good response characteristics at large scales. Simple calculations provide a valuable check on the properties of any proposed revisions of the correlation function.

4.3 The Effect of Correlated Observational Error

In many applications the observations have correlated errors (Schlatter 1981). A correct treatment of these errors can change the filtering effect of the O/I algorithm. Once the filtering step is complete, the observation error correlations have no further effect on the interpolation step. The role of correlated observational error has been discussed by Bergman and Bonner (1976) and by Seaman (1977) who argue that the presence of correlated error in mass-field observations can effectively improve the resolution of the analysis on small scales. Hollingsworth, Lonnberg and Andersen, 1987, show that one may gain useful insight on the problem by considering the filtering step of the O/I algorithm. As noted previously

$$[a] = \underline{\underline{P}} (\underline{\underline{P}} + \underline{\underline{D}})^{-1} \cdot [d] \quad 4.2$$

where $[a]$, $[d]$, $\underline{\underline{P}}$ and $\underline{\underline{D}}$ are as defined earlier. Suppose first that the observation errors are uncorrelated and of uniform magnitude

$$\underline{\underline{D}} = \sigma \underline{\underline{I}}$$

where $\underline{\underline{I}}$ is the identity matrix. If the original data and the filtered data are expanded in terms of the eigenvalues of $\underline{\underline{P}}$, then the response R_i to the i^{th} component is

$$R_i = \frac{\lambda_i}{\lambda_i + \sigma} = \frac{1}{1 + \frac{\sigma}{\lambda_i}} \quad 4.4$$

where λ_i is the i^{th} eigenvalue of $\underline{\underline{P}}$. This gives a simple filtering of the observations depending on the ratio of σ to λ_i . For a scalar field like the mass field this translates readily into a scale-dependent filter.

Suppose now that the observations have a correlated error, and that the error can be represented as the first two terms in a power series expansion in $\underline{\underline{P}}$:

$$\underline{\underline{D}} = \sigma \underline{\underline{I}} + \epsilon \underline{\underline{P}}$$

where we shall speak of the terms on the right as the un-correlated and correlated parts of the observation error respectively. Empirical support for such a representation of the vertical error correlation of TOVS data can be had by comparing the results of Schlatter (1981) with those of LH for

thickness forecast errors. It follows that the response in the i^{th} component in this case becomes

$$R_i = \frac{\lambda_i}{(1+\epsilon)\lambda_i + \sigma}$$

$$= \frac{1}{1+\epsilon} \cdot \frac{1}{1 + \frac{\sigma}{(1+\epsilon)\lambda_i}} \quad 4.5$$

The presence of correlated observational error has two effects. Firstly it damps all components uniformly by an amount $\frac{1}{1+\epsilon}$. Secondly it reduces the scale-selective damping of the algorithm by reducing the term $\frac{\sigma}{\lambda_i}$, which controls the scale selectivity in Eq. 4.4, to $\frac{\sigma}{(1+\epsilon)\lambda_i}$. If there is no uncorrelated error as defined above, then the response is simply $\frac{1}{1+\epsilon}$. The filter is no longer scale-selective, and all components of the data are treated equally.

In this limiting case the data is treated as if it were exact, except for the uniform factor $\frac{1}{1+\epsilon}$; this occurs because of the relative magnitudes of first-guess error (normalised to 1) and observation error, given that both have the same spatial correlation. This result indicates that care is needed in the specification of the observation errors. Polar orbiting satellites are well known to have both vertically and horizontally correlated observation errors, arising mainly from the retrieval techniques. Current estimates of forecast error structure and observation error structure for satellites indicate that the associated length scales are not too dissimilar, so the simple example is relevant for practical work.

The particular strengths of the O/I algorithm are the ease with which it can handle heterogeneous observations, and the simplicity of the multivariate formulation. If constraints such as near-geostrophy or non-divergence are imposed on the covariance functions, they will be satisfied by the continuous analysis. The ability of the O/I system to cope with correlated observational error is important in practice. The central role played by the error characteristics of the observations and of the first-guess in the O/I filtering and interpolation indicates that one must determine these statistics as accurately as possible.

In the next few sections we discuss a general approach to the determination of the statistical structure of atmospheric fluctuations, of forecast errors and of analysis errors. The kinematics of two-dimensional turbulence provide the connecting link between all three discussions. Having outlined the basis of the techniques we discuss

- i) the statistical properties of atmospheric fluctuations as seen by different observing systems (section 5.2)
- ii) the statistical structure of short-range forecast errors (section 6),
and
- iii) the verification of analyses (section 7).

Most of the results are concerned with the isotropic component of the statistics; the anisotropic components of the statistics are briefly discussed in section 6.3

5.1 Two-point Correlations and the Kinematics of Two-Dimensional Turbulence

The kinematic theory of 2-point velocity correlations in homogeneous conditions was worked out in the 1930s and 1940s by workers on turbulence theory. The results are summarised in standard texts (Batchelor 1954, Panchev 1972). These results were used by Hutchings 1955, Buell 1972, Brown and Robinson, 1979, in discussions of the statistical structure of fluctuations about the climatological norm.

Daley (1985) used the kinematic equations of 2-dimensional turbulence in a Cartesian framework to formulate the O/I analysis problem for mass and wind in a plane, with no constraint of non-divergence. HL/LH 1986 used the kinematic equations in the coordinate system commonly used in turbulence theory to estimate the three auto-correlations $\langle \phi, \phi \rangle$, $\langle \psi, \psi \rangle$, $\langle \chi, \chi \rangle$ and the three cross-correlations $\langle \phi, \psi \rangle$, $\langle \phi, \chi \rangle$, $\langle \psi, \chi \rangle$, where ϕ , ψ , χ represent the fluctuations of geopotential, streamfunction and velocity potential.

Assuming homogeneous conditions, and defining

$$F(r, \theta) = \gamma^2 \langle \psi, \psi \rangle$$

$$G(r, \theta) = \delta^2 \langle \chi, \chi \rangle$$

$$H(r, \theta) = \gamma \delta \langle \psi, \chi \rangle$$

where $\gamma = E_\psi / E_v$;

$$\delta = E_\chi / E_v$$

and E_ψ , E_χ and E_v are the standard deviations of the ψ , χ and velocity component fluctuations, the kinematic equations for the winds may be written

$$-\nabla^2(F+G) = \langle v, v \rangle + \langle t, t \rangle$$

$$L_1(F-G) - 4L_2(H) = \langle v, v \rangle - \langle t, t \rangle$$

5.1

$$L_2(H) + L_1(F-G) = -\langle v, t \rangle$$

where v, t denote the velocity components along (longitudinal) and perpendicular to (transverse component) the line joining two observation points. Using polar coordinates (r, θ) for the displacement between two stations, the operators in Eq 5.1 have the form

$$\nabla^2 = r^2 R^2 + 2R + T^2, \quad R = \frac{1}{r} \frac{d}{dr}, \quad T = \frac{1}{r} \frac{d}{d\theta}$$

$$L_1 = r^2 R^2 - T^2,$$

$$L_2 = T \left(\frac{1}{r} - rR \right).$$

Solutions of these equations may be found from empirical data on the velocity correlations by least squares fitting procedures using expansions in Fourier-Bessel series.

Only the isotropic terms have been studied by HL/LH, and by most earlier investigators. In isotropic conditions the equations 5.1 simplify to

$$-(r^2 R^2 + 2R)(F+G) = \langle 1, 1 \rangle + \langle t, t \rangle$$

$$r^2 R^2 (F-G) = \langle 1, 1 \rangle - \langle t, t \rangle \quad 5.2$$

$$-r^2 R^2 H = \langle 1, t \rangle$$

If the turbulence is non-divergent then $H=G=0$, and the equations simplify to

$$-RF = \langle 1, 1 \rangle \quad 5.3$$

$$-(r^2 R^2 + R)F = \langle t, t \rangle$$

which implies a well known relation between the longitudinal and transverse velocity correlations in homogeneous isotropic incompressible flow (von Karman and Howarth 1938). The equivalent relation between the longitudinal and transverse velocity correlations in purely divergent flow were derived by Obukhov, 1954, in his thesis. Obukhov commented that he could not imagine an application which would need to consider flows which were partly divergent!

HL/LH found empirical expressions for $\langle \psi, \psi \rangle$, $\langle \chi, \chi \rangle$ and $\langle \psi, \chi \rangle$ by substituting spectral expansions for these quantities in the left hand sides of Eq. 5.2, and making a least squares fit of the resulting expressions to the empirical data on the right.

The determination of the cross-correlations of mass and wind is rather simpler than the wind-wind correlations, as it involves the solution of a pair of coupled first-order linear partial differential equations rather than a set of three coupled second-order PDEs. If $I(r, \theta)$, and $J(r, \theta)$ are defined by

$$I(r, \theta) = \gamma \langle \phi, \psi \rangle$$

$$J(r, \theta) = \delta \langle \phi, \chi \rangle$$

then for homogeneous conditions

$$I_r + \frac{1}{r} J_\theta = -\langle \phi, t \rangle \quad 5.4$$

$$J_r - \frac{1}{r} I_\theta = -\langle \phi, i \rangle$$

If conditions are isotropic then the equations are especially simple:

$$I_r = -\langle \phi, t \rangle \quad 5.5$$

$$J_r = -\langle \phi, i \rangle$$

These last equations have a simple physical interpretation if we consider the correlation of the geopotential at one radiosonde location with the radial and transverse wind components at surrounding stations. The height/streamfunction correlation is defined by the correlation between the geopotential and the swirl velocity component about the geopotential observing point, while the height/velocity-potential correlation is defined by the correlation between the geopotential and the radial velocity component. The $\langle \phi, \phi \rangle$ correlation is determined directly from observations using the methods of Rutherford (1972).

5.2 Intercomparison of Observing Systems

There have been extensive studies of the analysis of atmospheric fluctuations, using climatology as a background (Gandin 1963, Rutherford 1972, Buell 1971, Buell 1972a,b, Panchev 1971, Julian and Thiebaut 1975, Brown and Robinson 1979, Buell and Seaman 1984, Rinne and Jarvenoja 1985). Several of these studies have used the simplified non-divergent form of the kinematic equations to investigate atmospheric fluctuations.

There is however a further application of the methods which is suggested by Wylie et al (1985), who raise the important question: Do different observing systems see the same atmosphere? They studied the spatial correlation structure of the fluctuations of the tropical wind field about the long term mean, using the vector wind correlation $\langle i, i \rangle + \langle t, t \rangle$, the spatial correlation for wind speed, and Gandin structure functions for wind speed. They compared the statistics derived from rawinsondes, aircraft, and cloud track winds. Fig. 4 shows an example of their results for the vector wind speed in the tropical upper troposphere. It shows that there are some striking differences between the statistics of atmospheric fluctuations as derived from different

observing systems, indicating that the observing systems sample quite different scales of motion (cf. the correlations for automatic aereps and radiosondes). The very broad correlations for the cloud track wind data suggest that the data sample only large scales. Relative to radiosonde winds, the cloud track wind data therefore have correlated sampling errors.

Further work along these lines would be valuable. Comparison of spectra from observation systems with uncorrelated errors (e.g. rawinsondes and aircraft) would determine a spectral band where they both have equivalent response. It should then be possible by cross comparison with satellite data to quantify the structure of spatially correlated observation errors, giving a different approach from Schlatter (1981). One could, for example, also address the question of 'Do different wind observing systems have the same response to divergent flow?' by examination of the statistics on divergent flow resulting from solutions of Eqs 5.1 or Eqs 5.2, where the empirical data is derived from different observing systems. Such information would be quite important for tropical analysis.

5.3 Data Monitoring

Besides the elaborate methods just discussed for evaluation of observing systems, there are much simpler techniques which are also valuable. As discussed in section 8, every operational system needs quality control algorithms to reject data which may be corrupt for one reason or another. Studies of the interaction of data with the assimilation system can be valuable in identifying systematic defects in data.

d) Real Time Data Monitoring

An operational data assimilation system needs defences against data which has been corrupted for one reason or another. These defences serve a useful purpose by indicating the data which have problems, and need examination. Several tools besides rejection lists are available to identify data which need examination; maps of analysis increments and regional plots of the rms fit of the analysis and first guess to the data are both heavily used (Delsol 1985).

In cases of real time problems there is a typical sequence of investigations. The first concern is to make sure that the data has been correctly treated in the pre-analysis phase (correct decoding, proper climatological tests, etc.) Next there are checks that the differences between the data and the analysed fields are reasonable from the synoptic point of view. An analysis scheme designed for models with mesh sizes of 150-200km cannot resolve the core of a hurricane; data indicating the central pressure of an intense hurricane will then be rejected on the reasonable scientific grounds that it represents something the analysis and the forecast model cannot resolve. Similarly, data may receive little weight in the analysis if it is contradicted by adjacent good quality data and by the background field.

In many cases it can take just a short time to establish that either the assimilation system or the data has real problems. In other cases it can take several days to establish whether or not the treatment of a particular piece of data or type of data was correct. Depending on the outcome of the investigation, action will need to be taken, either by liaison with the builders of the assimilation system, or by liaison with the data producers in case of a new bug in an observational system.

e) Long Term Data Monitoring

The system of near real-time data monitoring at ECMWF has developed over the years and is very effective in detecting errors in the data, and weaknesses in the assimilation system. However it has been possible to refine the system even further by considering monthly or longer term statistics on the differences between the observations on the one hand and the back-ground and analysed fields on the other.

Hollingsworth et al (1986) demonstrated that monthly or longer term statistics on the treatment of data in the assimilation can reveal weaknesses both in the data and in the assimilation system itself. Calculations to partition 6-hour forecast errors, as perceived by rawinsondes, into spatially correlated prediction errors and spatially uncorrelated observation errors (which include sampling errors) indicated that the two components of perceived forecast error are of comparable magnitude when good quality data is regularly available, Fig 5.

There are three components of the assimilation system: the 6-hour forecast, the analysis, and the initialisation. If the forecast error and observation area are comparable in data rich areas, then most of the atmospheric evolution from one analysis time to the next must be accomplished by the forecast step. Hollingsworth et al (1986) demonstrated this by comparing the RMS changes made by each of the three steps in the assimilation from 0600GMT to 1200GMT in October 1983. For 500 hPa geopotential they found the northern hemispheric rms changes to be 21.6m for the forecast, 12.5m for the analysis and 6.5m for the initialisation. This state of affairs has come about because of many developments in the techniques of analysis assimilation and forecasting. Fig 6 compares the perceived 6-hour forecast errors over N America in the fall of 1974 (Hollett 1975) and in summer and winter 1983 (Hollingsworth et al 1986), which shows clearly the extent of the improvement in technique over the last decade.

The reliable quality of the analyses and 6-hour forecasts is exploited in a number of ways. Delsol (1985) has presented monthly means of the bias and standard deviations of the differences between the forecasts and retrieved thickness temperatures from the TOVS systems on the NOAA satellites. These have been helpful in identifying the areal distribution of biases, particularly at low levels, in the retrieved temperatures.

Delsol has also demonstrated that near mid-latitude jet-streams there are substantial biases in the differences between cloud track wind reports and the first-guess, while the equivalent biases for radiosondes are much smaller. Pierrard (1985) and Kallberg and Delsol (1986) have used rawinsonde, aircraft, and cloud-track winds in collocation studies which show clearly that there are indeed substantial speed-dependent biases in the operational Cloud Track wind data. Several possible solutions are under consideration. For the re-processing of the FGGE data it was decided to use the collocation studies to calibrate some of the cloud track wind data near the jet-streams of the Southern Hemisphere.

As emphasised in this and every other paper on O/I, the statistics of the first-guess error play a central role in the O/I analysis algorithm, both in determining the relative weights of first-guess and observation, and in determining the response characteristics. The assimilation uses the observations to correct forecast errors, and it does this most effectively by spreading out the information in the observations to the extent indicated by the forecast error correlations. Given that the O/I algorithm has been adopted for operational use at many different centres (Gustafsson 1981) it is surprising that so little has been published on the determination of the structure of forecast errors (Hollett 1975, Balgovind et al 1983, HL/LH, 1986, Thiebaut Shantz and Mitchell 1986), as distinct from the statistical structure of departures from climatology.

6.1 Horizontal Structure Functions for Mass and Wind

The papers by HL/LH used the formulation of the last section to study short range (6-hour) forecast errors in the ECMWF assimilation, using North American rawinsonde data for verification. The first step was to partition the perceived forecast error into horizontally correlated prediction error and horizontally uncorrelated observation error. The technique used, the extrapolation of horizontal lag correlations to zero lag, was introduced by Drozdov and Shepelevskii (1946), and has been widely used since then. The horizontally correlated prediction errors in height and wind are each partitioned into a large scale part, which is essentially constant over a large area, and a synoptic scale part. The synoptic scale wind components are partitioned into a rotational part and a divergent part. The Bessel expansions used for the extrapolation were truncated at a scale corresponding to planetary wave-number 68.

Forecast error and radiosonde observational error are of comparable magnitude for both height and wind over land areas of the Northern Hemisphere, and indeed in the Tropics in those areas where good quality data is available, Fig. 5 (HL/LH, Hollingsworth et al. 1986). The mid-latitude wind forecast errors on synoptic scales are mainly non-divergent and geostrophic. The Rossby number of the forecast errors is about 0.4, and the geostrophic coupling coefficient between geopotential and stream-function is about 0.9. The spectra of the correlations are well behaved, with spectra being flatter, or whiter, at lower levels than at higher. This indicates that

the length scales of the forecast error are shortest in the lower troposphere. The $\langle \psi, \chi \rangle$ correlation is practically zero. There is a marked correlation (0.25) between ϕ and χ , corresponding to excessive convergence in the lower troposphere when a low is forecast too deep. A similar feature is noted in the behaviour of the NMC regional model (N.A. Phillips, pers. comm.)

A substantial part of the forecast error in both height and wind is on horizontal scales larger than planetary wave no. 8; we shall call this the large scale error. The large scale wind error changes phase between the lower stratosphere and the lower troposphere. The large scale geopotential error is nearly barotropic, Fig. 7b. A study of the height and temperature errors of the Canadian assimilation (Thiebaut, Shantz and Mitchell, 1986) shows indications of a similar large scale error. These authors found that the horizontal correlation length was much shorter for temperature than for geopotential, and that part of the difference could be explained by a large scale error in the height field.

6.2 Vertical Structure Functions for Mass and Wind

Provided the assumptions of homogeneity and isotropy are valid for vertical differences of height or wind, the kinematic equations, Eq. 5.1, are as valid for the vertical differences as they are for the height and wind at a single level. HL/LH used this result to determine the vertical correlation matrices for forecast error and observation error in both height and wind.

Both Bergman (1979) and Lorenc (1981) formulated the 3-dimensional structure function for height and wind in separable form as a product of a function describing the horizontal dependence and a function (continuous or discrete) describing the vertical dependence. With this formulation, the maintenance of the geostrophic constraint required that the same vertical function be used for height and wind.

Figure 7, from HL/LH, shows plots of the vertical correlations of the synoptic scale and large scale height errors, and of the wind errors. The height and wind data have very different vertical dependences, which is clear evidence for non-separability. Moreover the vertical correlation structure of the individual terms in the expansions for the synoptic scale height field showed

marked variability from one term to the next, which was also inconsistent with the presumption of separability.

An explanation of these results was suggested by Phillips (1986) who modelled the forecast errors in a data rich area as an ensemble of slow modes with random phases. Phillips assumed that the ensemble had a white noise spectrum in the sum of potential and kinetic energy. Balgovind et al.(1982) had earlier proposed a model which assumed that forecast errors had a white noise spectrum in potential vorticity; neither spectrum quite fits the empirical results. Phillips has shown that if his formulation takes account of the vertical and horizontal resolution of the ECMWF model, he can reproduce many important features of the results of HL/LH, such as the overall variation in the forecast error amplitudes, and the sharper vertical correlation functions for wind as compared with height. Most operational systems use the separability assumption that a three dimensional correlation may be represented as a product of two functions - one describing the horizontal and the other the vertical dependence. Geostrophy then requires that the vertical correlations for height and wind be the same. Phillips' model is inherently non-separable, with a different vertical dependence for each horizontal component of the turbulent ensemble; Fig 7 is clear evidence for non-separability in the ECMWF assimilation. As a further result of the non-separability, Phillips' model of forecast error predicted that the horizontal length scale for temperature should be much shorter than the length scale for geopotential. This feature was found in the Canadian results and is confirmed in unpublished ECMWF results.

Altogether we now have substantial theoretical and empirical evidence for the existence of marked non-separability in the forecast error statistics of operational data assimilations. One must now consider seriously the need to use a non-separable correlation model to make an optimal use of data sources like rawinsondes and satellite thicknesses. This will be expensive in computer time, should it prove necessary, but the cost will be negligible compared to the cost of the observations.

There are important features of the HL/LH results which are not explained by the simple model of Phillips'. Notable amongst these is the maximum in forecast error near the tropopause, both in height, Fig 8, and wind (not shown). Advection of error from the oceans may be a contributing factor, but

is unlikely to be the only one. The shape of the spectra found in HL/LH's results also need explanation; they vary from about $k^{-1.5}$ at low levels to about k^{-3} at upper levels. The large scale errors also need explanation; there is some indication that they may be partly tidal in origin. Thiebaut et al found no evidence for tidal effects in their calculations. Accurate treatment of the tides in an assimilation requires a good diurnal cycle in the forecast model, Unden (1984), and a suitable initialisation condition, Wergen, (1986).

Balagovind et al (1983) found a marked latitudinal variation in the magnitude of mass-field forecast errors in their assimilation, which was larger than simple f-plane theory would indicate (Phillips 1986); they proposed a theoretical beta-plane model to interpret their results. Thiebaut et al (1986) found a similar empirical result. HL/LH did not pursue this line of investigation because of known problems in their system in large scale analysis in the tropics, in modelling of the tropics, and in assimilation of tides. Given recent progress in the assimilation of tidal data (Wergen (1986), and in the analysis and forecasting of large scale fields in the tropics (cf para 8.3), a re-examination of the latitudinal dependence of forecast errors would be interesting.

6.3 Significance of Forecast Error Structure in Data Rich Areas

It has been argued since Gandin (1963) that the structure functions have little effect on analyses in data rich areas, and are only important in data sparse areas; the comment was made in the context of analysis using climatology as a background field. The results of Seaman and Hutchinson (1985) however suggest that the correlation functions are indeed important in data rich areas. We have some results which support their conclusion, rather than Gandin's, in current operational work.

One of the major changes introduced in the ECMWF assimilation by Shaw et al (1984) was an increase in the vertical and horizontal resolution of the analysis, by using sharper structure functions; the changes were based on the results of HL/LH. There were other important changes as well, affecting data selection and quality control. The impact of these changes has been shown in the verifications of the short range wind forecasts over N America in the winter of 83/84 (after the introduction of the T63 spectral model) and in the winter of 84/85 after the introduction of the analysis changes (Fig 2). The

impact of the analysis changes has been very positive. One may conclude that the correlation functions do indeed matter in the relatively data-rich area of North America.

When using climatology as a background field (Gandin 1963) in an area such as North America or Europe, the observations are very close to each other relative to the width of the auto-correlation of background error. The larger scales find a much better response than the smaller scales; as a result the details of the correlation structure do not matter provided they represent the main broad scale features well.

When one is analysing forecast errors in an area such as North America, the scales of interest are much shorter than they are when analysing climatological fluctuations. The data density relative to the correlation length of the background field is correspondingly lower and as a result the importance of the statistics of the background field increases.

6.4 One-point Correlations, Anisotropy and Flow Dependence

A common criticism of operational O/I systems is that the correlations used are isotropic and make no attempt to use functions elongated along the flow direction. Some of this criticism is based on a mis-understanding of the fact that an operational system analyses forecast errors rather than the entire spectrum of atmospheric fluctuations. If the anisotropies have been captured by the first-guess, then the first-guess errors may not be anisotropic, and the analysis problem is therefore simplified.

Holopainen, (1978), Hoskins James and White, (1983), use the tensor of one-point velocity covariances, or Reynolds stress tensor, to summarise important aspects of the anisotropy of a two-dimensional velocity field. They write the velocity correlation tensor in the form

$$\begin{pmatrix} \overline{u'u'} & \overline{u'v'} \\ \overline{u'v'} & \overline{v'v'} \end{pmatrix} = \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} + \begin{pmatrix} M & N \\ N & -M \end{pmatrix}$$

$$\text{where } K = (\overline{u'u'} + \overline{v'v'})/2$$

$$M = (\overline{u'u'} - \overline{v'v'})/2$$

$$N = \overline{u'v'}$$

The prime denotes the departure from the ensemble mean and the bar denotes ensemble mean. The principal axes of the Reynold's stress tensor have magnitudes $K \pm (M^2+N^2)^{\frac{1}{2}}$, and the major axis has an orientation $\frac{1}{2} \tan^{-1}(\frac{N}{M})$ to the x-axis. In the northern hemisphere an orientation from South-West to North-East indicates a poleward momentum flux. The 'anisotropy' vector V , with magnitude $(M^2+N^2)^{\frac{1}{2}}$ and orientation along the major axis, gives a convenient summary of the main properties of the anisotropy for the one-point velocity correlations.

We now apply these concepts to the anisotropies of forecast error fields. Fig 9a shows plots of the anisotropy vector for the 250 hPa wind forecast errors at the North American radiosondes for winter 1984/85. The vectors are oriented mainly North-South near the West Coast, and are oriented mainly East-West in the East. The magnitude of the anisotropy vectors for the forecast errors is about 1/25 of the atmospheric magnitude. Fig 9b shows the ratio $\alpha = (M^2+N^2)^{\frac{1}{2}}/K$, which gives a non-dimensional measure of the significance of the anisotropy in the wind forecast error. Typical values are 0.2-0.3. K , the denominator of α , contains contributions from observation error and forecast error since K is the energy of the measured forecast error. If the observational error is isotropic, and is comparable with the prediction error, the true anisotropy of the forecast errors may therefore be of order 0.5, which is comparable with values for atmospheric fluctuations. Thus anisotropy may be as significant a feature of the forecast errors as it is of the observed fields. Preliminary investigations indicate that the analysis copes with some but not all of this anisotropy.

The anisotropy calculations just discussed can only document the existence of components of the correlation structure with azimuthal wave number $m=2$, if the forecast errors are homogeneous. As discussed by HL/LH the components with odd parity ($m=1,3$, etc) occur in cross-correlations and may be important in describing the baroclinic phase tilts in the forecast errors.

Evidence for the existence of these components is forth-coming (A.M.Jorgensen 1986, pers comm).

A full investigation of the need for anisotropy in the forecast error correlation model will need to consider the balance between the mass and wind field. A convenient survey of the role of anisotropy and ageostrophy in the mass-wind coupling of the forecast errors is provided by the vector of one-point correlations ($\overline{\phi'u'}$, $\overline{\phi'v'}$). For atmospheric fluctuations this quantity shows the mainly non-divergent pattern expected from quasi-geostrophic flow (Holopainen 1978), with indications of the smaller terms arising from correlations of ϕ with the divergent wind (Hoskins and Sardeshmukh 1986). Calculations of this vector for the forecast errors shows that it is noisier than the anisotropy vectors. At some stations the velocity-height correlations in the forecast errors are as large as 0.5. The correlation model used at most operational centres assumes that this correlation is zero.

These preliminary results suggest that there may be a need to take account of anisotropy and divergence in the correlation models used for analysis, but the question needs further study. The one-point correlations shown in Fig 9 are the limits, in the case of vanishing spatial lag, of the two-point velocity correlations discussed earlier, provided the forecast errors are homogeneous and the observation errors are isotropic. Anisotropic components of the forecast errors can be incorporated in a correlation model using the same formalism as the isotropic component.

The view that the correlation functions ought to be flow-dependent has often been expressed. Operational centres have not given high priority to this effort. An explanation may be that it is profitable to ensure that simple things are done well, before tackling difficult problems. The use of flow-dependent correlation functions for wind analysis in highly curved flows is inherently self-referential, and therefore requires iterative solutions to non-linear problems. The adjoint method offers a more complete approach to the problem. Baker et al (1986) have approached the problem in a straight-forward fashion by stratifying the correlation structures of analysis increments according to the background vorticity, but using isotropic representations for each of the vorticity categories. The results seemed to indicate modest improvements.

The analysis error field is, by definition, the difference between the analysed field and the true field. Since the true field can never be known exactly, the analysis error field can never be known exactly. It can nevertheless be estimated, if only approximately. Many techniques have been used to assess the magnitude and significance of analysis errors. Forecast error was discussed earlier as a proxy for analysis error. Comparisons of independent analyses and a-priori estimation using the O/I technique are discussed in para 7.1 and 7.2. Analysis verification using two-point correlations of observation minus analysis differences is discussed in para 7.3; analysis verification using one-point correlations has been touched on already in para 6.4.

For those with access to a good forecast model, the best estimate of analysis accuracy comes from verification of short range-forecasts based on the analyses. The advantage is that the errors of the verifying observations are independent of the forecast errors, if the observations are unbiased. With a 2-day doubling time, analysis errors should amplify by less than 10% in 6 hours, and so the forecast errors are a very good estimate of the analysis errors. Moreover the forecast verification provides an excellent measure of the internal balance of the analysis.

7.1 Comparison of Operational and FGGE Analyses

With the availability in recent years of many different analyses of the FGGE data, and, of several different operational analysis sets, there have been numerous comparisons of the properties of different analyses. Many of these studies are reviewed in Miyakoda 1985, National Research Council (1985) and WMO (1985b). These investigations have used a variety of methods to compare the analyses. Some have compared the energetics of the analyses, some have examined the intensity of the diabatic processes implied by the analyses; others have studied the accuracy of forecasts made from different analyses, and yet others have considered the internal balance of the analyses. Implicit in all these studies has been an attempt to estimate the accuracy and consistency of the analyses by establishing the range of uncertainty in the analyses (WMO 1985b).

The simplest comparison method is a computation of the rms differences between independent analyses of the same observations, Arpe (1980), Rinne and Jarvenoja (1983), Lau(1984) and many other authors. If the analyses are

assumed to be of equivalent accuracy, and if their errors can be assumed to be uncorrelated, then the variance of the analysis differences may be attributed equally to errors in both analyses. This method has been used extensively by Rinne and Frisk (1979) and by Rinne and Jarvenoja (1983) to estimate analysis errors of long series of operational analyses, and of analyses of the FGGE data; their results demonstrate the high accuracy of the FGGE analyses.

In addition, Rinne and Jarvenoja (1984) developed an alternative technique for estimating analysis error. They projected sets of 500 hPa height analyses onto empirical orthogonal functions (EOF's). They then developed optimal linear predictors for the evolution of the time series of the EOF coefficients. These methods provide an estimate of the noise in the time series of coefficients, which they attribute to analysis error. The results they found with this method had many important similarities with the results they obtained from computations of the rms differences.

7.2 The O/I Estimate of Analysis Error

The O/I analysis method provides an estimate of the second moment of the analysis error (Gandin 1965). Seaman (1977, 1983) has shown that the analysis error can be calculated in a similar way for any analysis method which defines an analysis increment to a background field by a linear combination of the observations. The estimated analysis error is used in quality control of data (Lorenz 1981), in estimating the accuracy of short-range forecasts made from the analyses (Lorenz 1981, Ghil et al 1981), and therefore in deciding on the relative weights to be given to observations and the back-ground field. The estimates appear to be rather reasonable in data sparse regions (Rinne and Jarvenoja 1984).

However the estimated analysis error appears to be over-optimistic if a great deal of data is used in the estimation of analysis error in a data rich area. Experiments have shown that if height data from a single radiosonde in Central Europe is withheld from the analysis, then the estimated error in 500 hPa height is of order 2m, when the actual analysis error is between 5 and 8m, due allowance being made for observation error (Lonnberg and Shaw, 1984). As discussed by Seaman (1977) Franke (1983) and Lorenz (1986), the O/I estimate of analysis error is probably much less reliable than the O/I analysis in these circumstances. The main reason is that the estimate makes no allowance for inevitable errors in its own statistics, and the estimate

tends to zero as the number of data becomes large. Since the estimated analysis error is a monotonically decreasing function of the number of data used, it will tend to give unrealistically optimistic estimates of analysis error if large volumes of data are used.

7.3 Verification of Analyses using Two-point Correlations

Computations of rms differences between analyses and observations are a standard verification tool. At best such comparisons can tell if the data were used in the analyses, but there is little further information. Hollingsworth and Lonnerberg (1986b) have shown that a study of the spatial correlation structure of the observation minus analysis (Ob-An) differences can yield upper and lower bounds on the magnitudes of analysis errors, and on the spectra of analysis errors, in much the same way as study of the correlation structure of the observation minus forecast (Ob-Fg) differences can quantify the magnitude and spectrum of forecast errors.

a) The Theoretical Covariance of Observation Minus Analysis Differences

To provide a theoretical context for the discussion of the observation-minus-analysis (Ob-An) differences, Hollingsworth and Lonnerberg (1986b) show that in the case of a complete O/I analysis

$$\underline{A} = \underline{P} - \underline{P} \cdot (\underline{P} + \underline{D})^{-1} \underline{P} \quad 7.1$$

where \underline{A} represents the covariance matrix of the analysis errors at the observation points. From this result one may derive an expression for the covariance of the difference between the observed and analysed values

$$\text{Cov}\langle (d-a) \cdot (d-a)^* \rangle = \underline{D} - \underline{A} \quad 7.2$$

If the observations were not used in the analyses the covariance of the Ob-An differences would be

$$\underline{D} + \underline{A}'$$

where \underline{A}' is the analysis error covariance if the observations in question are not used. If the observations were used but the analyses were less than ideal one would expect to find that

$$\text{Cov}\langle (d-a) \cdot (d-a)^* \rangle = \underline{D} + \underline{A} - 2 \underline{E}_o^* \cdot \underline{R}_{o/a} \cdot \underline{E}_a \quad 7.3$$

where $\underline{R}_{o/a}$ is the correlation matrix of observation error and analysis error at the observation points, and $\underline{E}_o, \underline{E}_a$ represent the vectors of rms observation

error and analysis error at the observation points. Consistency of this result with Eq 7.2 requires that in the complete O/I analysis:

$$\underline{E}_o^* \cdot \underline{R}_{o/a} \cdot \underline{E}_a = \underline{E}_a^* \cdot \underline{R}_{a/a} \cdot \underline{E}_a \quad (= \underline{A}) \quad 7.4$$

where $\underline{R}_{a/a}$ is the auto-correlation matrix for analysis error. Since $\underline{R}_{a/a}$ tends to the identity matrix \underline{I} at zero separation, Eq. 7.3 suggests that the spatial correlation of Ob-An differences should be negative at short separations, provided observation errors are spatially uncorrelated and sampling errors are negligible. We shall now demonstrate this in some simple calculations.

b) Two-point Correlations of Observation-Analysis Differences

Given a set of analyses and observations, the spatial variation of the covariance of the Ob-An differences at points 1 and 2 may be written

$$\begin{aligned} & \text{Cov}[(\text{ob-an})_1, (\text{ob-an})_2] \\ &= \epsilon_o^2 \delta_{12} - 2 \epsilon_o \epsilon_a \rho_{a/o}(r) + \epsilon_a^2 \rho_{a/a}(r) \end{aligned}$$

where r is the separation between points 1 and 2

ob=observation error, with assumed zero mean and no spatial correlation,

an=analysis error, with assumed zero mean,

$$\epsilon_o = \text{rms}(\text{ob}),$$

$$\epsilon_a = \text{rms}(\text{an}),$$

$$\rho_{a/o}(r) = \text{correlation of observation and analysis error, and}$$

$$\rho_{a/a}(r) = \text{auto-correlation of analysis error.}$$

$$\delta_{12} = \text{Kronecker delta}$$

Considering only the isotropic component of the Ob-An covariance statistics, we may define the function b(r) by

$$b(r) = -[2 \epsilon_o \epsilon_a \rho_{a/o}(r) - \epsilon_a^2 \rho_{a/a}(r)]$$

This function of station separation can be fitted in a least squares sense by

a truncated series of Bessel functions of zero order. The truncated series expansion may be extrapolated to the origin to give an intercept $b(0)$.

We then have the relations

$$\epsilon_o^2 = [\text{rms}(\text{ob-an})]_{r=0}^2 - b(0) \quad 7.7$$

and

$$\epsilon_a^2 - 2 \epsilon_o \epsilon_a \rho_{a/o}(0) - b(0) = 0 \quad 7.8$$

Defining $\epsilon = \epsilon_a / \epsilon_o$ and $b = b(0) / \epsilon_o^2$, Hollingsworth and Lonnberg 1986b show that if the intercept b is negative then

$$1 - (1+b)^{\frac{1}{2}} < \epsilon < (-b)^{\frac{1}{2}}$$

while if the intercept is positive then

$$(b)^{\frac{1}{2}} < \epsilon < 1.$$

These bounds on the analysis error at observation points are instructive. The analysis error at observation points is smaller when the intercept is negative than when it is positive. The closer to zero the negative intercept, the smaller the analysis error at the observation points. Hollingsworth and Lonnberg 1986b use these results to estimate the analysis error at observation points. They also show that equivalent bounds on the spectrum of analysis error may be derived. For example Fig 10a shows the spatial correlation of their operational 500 hPa Ob-An differences in the height field. The negative intercept at the origin is quite different from what one sees with Ob-Fg differences (Fig 10b). The results in Fig 10a suggest that the analysis error variance at observation points is about one quarter of the observation error variance, at most.

c) Diagnosing the Efficiency of an Analysis

Apart from their value in providing bounds on analysis error, the spatial correlations of Ob-An differences provide a quick and powerful diagnostic of the efficiency of an analysis system. As an example consider Fig 11 which shows the correlation of the Ob-An differences for the vertical wind shear between 200 and 150 hPa over N America in Jan-Mar 1983. This figure is unambiguous evidence that the analysis at the time was not drawing effectively for the vertical wind shear information (HL/LH, Shaw et al 1984). The latter paper documents an extensive set of modifications to the system which were implemented in May 1984. Fig 12 shows the same results as Fig 11 for the

first winter after the implementation. There has clearly been a considerable improvement in the vertical resolution of the wind analysis near the tropopause, and there is still room for further improvement.

These methods enable one to determine if a three-dimensional analysis has used to the fullest extent the data available to it. They therefore provide a powerful diagnostic of the efficiency of an analysis by identifying those areas where the analysis is inadequate. We hope that systematic application of these ideas over the next few years will bring us to the stage where we can prove that our analysis algorithm has used all of the available data, to the full. At that point we will need the more explicitly four dimensional approach of the adjoint method to make further progress.

d) System Dependence of OSE Results

These analysis verification methods provide a useful tool for evaluation and interpretation of Observing System Experiments (OSEs), and of Simulated Observing System Experiments (OSSEs). It has often been found that the results of these experiments are system dependent (Gilchrist 1982, ECMWF 1984). This leads to difficulties in the interpretation of the results. The verification methods discussed here could be quite valuable in reducing the level of controversy. Experimenters should demonstrate that the analysis system used for the experiments is using all the information available to it from each of the observation systems, before beginning their experiments. If this cannot be demonstrated then the outcome of an OSE must be interpreted carefully, as the results will be system dependent.

7.4 Discussion

There is a satisfying symmetry about the way correlation studies can be used to study observation quality, forecast errors and analysis verification. Results of one-point correlations provide a rapid survey and summary of forecast and analysis performance. Two-point correlation studies of atmospheric fluctuations can be used to compare the performance of different observing systems, Wylie et al (1984). Equivalent studies of the forecast errors provide good correlation functions and good constraints for the analysis. Finally, similar studies of the analyses can determine if the analysis system has extracted all useful information from the observations.

We now discuss the areas of objective analysis where research is likely to be concentrated to exploit new technological and scientific possibilities. It has been estimated that machines which are 400 times more powerful than the Cray-1A will be available by 1990. Today's commercially available machines are about 10 times more powerful than the Cray-1A, ten years after its first introduction. We are therefore in a phase when developments in computers are accelerating rather than otherwise.

There is a steady improvement in telecommunications capacity which makes it possible to disseminate greater volumes of satellite data, much of which is not fully exploited in present operational systems. For example Julian (1985) and Cadet (1983) indicate that cloud top temperatures have information on humidity and divergence which is not exploited in current operational systems. Given that there are now very encouraging prospects for NWP in the Tropics, we can expect substantial developments in exploiting data which is 'available in principle' but not currently used.

New observational technologies are being introduced, such as the sounders which some see as replacements for the rawinsondes in some areas. It is expected that the Advanced Microwave Sounding System (AMSU), to be flown on operational satellites soon after 1990, will provide as much vertical resolution in cloudy areas as is currently available from the TIROS-N series in clear areas. The potential resolution of this data is of order 80km in the horizontal, and 200-300 hPa in the troposphere. In the early 1990s the microwave scatterometers on ESA's ERS-1 satellite, and the U.S. Navy's NROSS satellite will provide near global 50km resolution surface wind data. The need to exploit new opportunities will increase the pressure to improve our present methods, as well as raising quite new problems.

With these changes in mind, we may identify four fairly distinct growth areas for research in objective analysis in the next few years:

- i) High resolution analysis over land (8.1),
- ii) Synoptic scale analysis over mid-latitude oceans (8.2)

iii) Synoptic scale analysis over the Tropical Oceans (8.3) and

iv) Quality control of observations (8.4)

Each of the first three areas has different emphasis on the importance of particular types of data, and presents different problems in assimilation of that data. The topic of data quality control is vital for all three areas.

A common theme running through the discussions of each of these areas is the need for more effective use of the time dimension or time history in the analysis. A number of methods are under investigation, some more elaborate than others. They all indicate that a closer meshing of the model and the analysis is possible.

8.1 Analysis Over a Wide range of Spatial Scales

Short-range forecasts are still the most widely used products of the meteorological services. Since global models with grid-sizes of 50km are conceivable by 1990, models covering large regions with much higher resolutions are also conceivable. It has been recognised since the 1950s that objective analysis schemes must be capable of treating a range of spatial scales (Bergthorsson and Doos 1954, Cressman 1959).

The method of O/I has been used to prepare analyses for the 80km Nested Grid Model (Phillips 1979) at NMC Washington (DiMego 1985), but the assimilating model which provides the background field is a global model of much lower resolution. Meteorologie Nationale, Paris, runs a regional assimilation cycle where the assimilating model is the 35km resolution Peridot model (Pailleux, pers comm.). The method of continuous assimilation has been successfully adapted to run with an operational model having grid spacing of 75km at the UK Meteorological Office (Fugard 1985, pers. comm.).

Many questions are still open on what is the best way to approach the problem. If the concept of 'slow' modes is valid in any way at the high end of the spectrum, then Phillips' (1982) results suggest that O/I may be the best approach. As implied by Phillips, 'distinguishable' might be a better word than 'slow', since the main requirement for his results on the completeness of O/I is that the components of interest be distinguishable in some way from the

components which are not of interest. Even if O/I were the optimal approach, there are a number of sub-optimal approaches which might work quite as well in practice, such as continuous assimilation (Fugard 1984 pers.comm., Lyne 1982, DiMego 1984) or special forms of the Successive Correction Method (SCM) which converge to O/I (Bratseth 1986, Lorenc 1986).

a) Scale Dependent Constraints in O/I

The general formulation for the correlation functions of forecast error discussed in section 6 has many aspects that may be of value in developing an analysis system suitable for application over a wide range of spatial scales, particularly if the dynamical constraints to be imposed on the motion are strongly scale dependent. The spectral representation of the correlations permits all the properties of the correlation model to be scale-dependent. It is a simple matter to vary the geostrophic coupling as a function of scale. This would permit the enforcement of a tight geostrophic coupling at synoptic scales, and a zero geostrophic coupling at very short scales.

Similarly the wind-wind correlation model could permit more divergence at either very large or very small scales than might be desired at synoptic scales, by suitable determinations of the expansion coefficients in the $\langle \chi, \chi \rangle$ correlation. The coupling between the divergent and the rotational winds can be made scale dependent by setting the coefficients of the $\langle \phi, \chi \rangle$ correlations appropriately. Manipulation of the $\langle \chi, \phi \rangle$ correlation might also be useful in particular circumstances, since the correlation is observed to be non-zero.

There is solid empirical and theoretical evidence that the forecast errors are non-separable. Non-separability and anisotropy in the correlation functions may well become important as resolution is increased. It would be expensive to use a non-separable correlation model in operational work. Such a decision would have to be justified by evidence that the analyses were missing important features of the observations. The methods of analysis verification discussed in the last section would be of value in such a decision.

The spectral formulation requires the specification of a large number of disposable parameters, but it also provides the means of determining those

parameters, through spectral decomposition of the forecast errors. The spectral representation can also be useful in deciding if the additional refinements are necessary.

b) Analysis of Phenomena having Non-linear Balance

As model and analysis resolution is increased, it becomes more and more essential to provide an accurate description of strongly non-linear phenomena such as fronts. If the background forecast has a frontal position error of a few grid-points, linear analysis methods such as O/I may be incapable of moving the entire non-linear structure to the correct position. Similar comments may apply to phenomena where the non-linearity involves physical processes in a vital way, as in hurricanes.

Fine mesh analysis for these phenomena will always be faced with a problem of data scarcity. If a synoptician knows that a strong front is to be found in a given area, a few surface reports can often give him an accurate position for the front. To position a front accurately, while providing the forecast model with the full non-linear balance needed for a good forecast, may well require iterative non-linear methods using adjoint techniques (Le Dimet and Talagrand 1986). This approach may be more productive than trying to develop flow-dependent structure functions, where the mathematical formulation is much less clear cut.

c) Analysis near Mountains

The study of the effect of mountains on the atmosphere has been given a substantial impetus by the ALPEX experiment. Excellent reviews of the problem together with summaries of observational, theoretical, and modelling studies may be found in Speranza (1984) and WMO (1986). There have been studies of static fine-mesh analysis of the ALPEX data (Reimer, 1984, Buzzi 1984, McGinley 1984), but there is little published material on data assimilation with the ALPEX data with a high-resolution system. This suggests that the undertaking is not only expensive, but difficult as well.

The importance of high resolution models for forecasting near mountains such as the Alps is underlined by the results of McGinley and Goerss (1985) and Dell'Osso (1984). Operational analysis and assimilation systems to support

such models will need to consider special features of the flow such as blocking, with consequential in-homogeneity and anisotropy in the forecast errors. Other areas needing consideration will be the structure of the analysis grid, and the data selection strategies, which must also take account of the anisotropies.

8.2 Analysis Over the Mid-Latitude Oceans

Analysis over the oceans is probably the most difficult and important of all analysis problems, because of the scarcity of data over these enormous regions. Apart from the TOVS data, and a few radiosonde reports from ships and islands, all the remaining observation platforms provide information at only one or two levels.

a) Use of Single level data

The problem of how to spread out single-level data in the vertical is one for which there will never be a fully satisfactory solution. One does not know how much of the information in the observation originates in the barotropic component of the flow and how much in the baroclinic component. One possibility is to treat the information as a delta-function in the vertical, and not spread it out at all. Barwell and Lorenc (1985) have shown that this is not a good approach.

In the Australian operational system (Bourke et al, 1985) a barotropic interpretation for surface pressure data is adopted. Pressure changes are balanced geostrophically in mid-latitudes, and are applied uniformly in the vertical between 1000 hPa and 200hPa. McPherson et al (1979) and Lorenc (1981) use vertical correlation functions which mean that surface pressure data change the lower tropospheric temperature field. More recently HL/LH have shown that about half the variance in the ECMWF height forecast errors at 6hours over North America can be accounted for by components which are of very large scale in the horizontal, and have an almost barotropic structure in the vertical. The proportion of error accounted for by such deep structures probably varies between land and sea. Perhaps the choice of structure function should depend on data coverage.

An important point noted by HL/LH is that that it may be possible to get a more balanced analysis in mid-latitudes by analysing wind scaled by the sine of the latitude rather than the wind itself. This would give a more accurate treatment of the variation of the Coriolis parameter, and would probably be of most benefit in data sparse regions over the oceans.

b) Satellite Temperature Retrievals

Temperature soundings from polar-orbiting satellites provide the only truly global three dimensional observing system. This data is of crucial importance to operational forecasting, especially over the oceans, Uppala et al. (1985), Kashiwagi (1985). Considerable effort is devoted to improving the quality and utilisation of the data, McMillin et al (1979,1982), Smith et al (1985). In current operational practice the retrieval is based on statistical regression of the radiances on collocated radiosondes. Each stage of the processing from earth-location, through calibration and cloud clearing, to the production of the final retrieval is the subject of extensive development work as discussed in the reports of the International TOVS study Conferences (Menzel and Lynch, 1985).

Current questions of most direct concern to Numerical Weather Prediction are related to the vertical and horizontal resolution of the information in the retrievals. We need to know how the spatial correlation of the errors in the retrievals depend on the type of first guess used for the retrieval, either short-term climatology or a forecast model; and whether the retrieval should be one-dimensional or three-dimensional.

The operational resolution of the NESDIS retrievals is approximately 250 km, but only a subset of this, with a resolution of 500km, is currently available on the GTS. Direct read-out of the radiances can provide average resolution of 80km when the satellite is above the horizon. Examination of the radiance data (Kelly 1985) suggests that there is considerable information on quite small scales in the horizontal. However the vertical resolution of the data is rather low. In clear sky conditions about 4 independent pieces of information are available in the vertical in the troposphere, while in cloudy conditions this reduces to about 2. Historically, data users have demanded that the retrievals should look as much like radiosondes as possible, and the

data producers have responded by producing retrievals which appear to offer more vertical resolution than is justifiable. This trend is being reversed as data users become more aware of the real capabilities of the instrumentation. If thicknesses for deep layers are the only reliable data in certain areas, then that is all one should use.

Two critical problems in generating a detailed vertical retrieval are the estimate of the surface temperature, and the estimate of the pressure of the tropopause. Low level clouds or inversions complicate the estimation of the surface temperature, particularly in the presence of snow or ice. Independent estimates of land surface and sea surface temperature can be very useful in anchoring the temperature of the lower layers. Hamon and Bell (1985, pers comm.) have demonstrated how the use of a statistical retrieval method can give rise to a very poor estimate of the upper troposphere and lower stratosphere temperature structure, when the true tropopause is at a quite different level from the mean tropopause of the set of sondes from which the retrieval coefficients were derived. The mean radiosonde provides the first-guess for the retrieval. Lorenc and Eyre (1985) have argued that if both the retrieval and the first guess are known then one can improve the retrieval if a better first guess becomes available (say from a forecast model).

Much work is underway to establish more direct connection between the raw radiance data and the assimilating model. Suskind (1985) and others have used the assimilating model to provide the first guess for the radiance field in a physical retrieval method. There is a danger in this procedure that the whole assimilation may drift away from the truth in data sparse areas. Kelly and Pailleux (1986, pers. comm.) have seen some evidence for this. Chedin (1984) avoids much of this danger by only using the model's forecast of surface pressure and, optionally, surface temperature. The first-guess for the temperature in his approach is generated by an elaborate statistical procedure which picks a good first guess from a large library of observed atmospheric states. Durand (1985) has gone further than others in using the observed radiances directly in the O/I calculation. The first guess for the radiances is generated by a radiative calculation from the first guess temperatures. This approach amounts to a three-dimensional retrieval algorithm.

8.3 Analysis in the Tropics

The advent of operationally useful Numerical Weather prediction in the Tropics has been slow. There have been scientific and technical reasons for the slow rate of development. The availability of the FGGE data has provided a great stimulus to this area, resulting in many studies of the phenomenology of the tropical weather, and a deep study of the problems of objective analysis, initialisation and forecasting in the Tropics (National Research Council, 1985). These studies have had a beneficial effect on operational practice. Taken with improvements in modelling, we expect continued improvement in the skill of Numerical Weather prediction in the tropics.

a) Analysis and Data problems

Reed et al (1986) have examined the performance of the ECMWF system in August-September 1985 in the West African - Atlantic sector. They found that the analyses showed a reasonable level of skill when data was available. They also found many aspects of the observing system and analysis system which could be improved. The forecasts to two days showed good skill in the initiation, evolution and propagation of African Waves. Subjective evaluations of forecasts in the West Pacific between June and October 1985 (Pasch pers. comm. 1985, WMO 1986b) also indicate that, in some instances, there is useful forecast skill to 2 or 3 days at low levels and perhaps as much as 5 days at upper levels.

Gray et al (1986) report that the track-forecast is the most important element in operational forecasting for tropical storms. Reed et al (1986) report that forecasts for the vorticity maximum in tropical depressions and storms were frequently successful at two days when observational data was available. They also reported many cases when satellite imagery permitted an accurate fix of storm position, even though cloud track winds were unavailable or gave little indication of the presence of a storm. Kanamitsu (1985) reports that JMA use satellite imagery to generate synthetic ('bogus') data to represent tropical storms in a consistent way in their initial data. Extension of this approach to other areas would be worth investigation.

A companion paper in this volume (Puri and Gauntlett 1986) reviews recent progress on analysis of the mass and wind fields in the tropics. Lack of data

is the main difficulty in tropical analysis, but that is outside the control of operational centres. Of the problems that are within our ambit, the formulation of a suitable mass-wind relationship for analysis in the tropics is still unresolved, Daley, Cats and Wergen (1986). Phillips (1986) has calculated the statistics of an ensemble of slow modes in mid-latitudes, and shown that they provide a convenient vehicle to organise the statistics necessary for O/I. An equivalent calculation on an equatorial beta-plane yields an ensemble whose statistics are inherently non-homogeneous, and therefore difficult to use. Approaches using least squares Hough-function fitting also have difficulties (Halberstam 1985, Daley 1983). Aliasing between Rossby and Kelvin modes is therefore a continuing problem, Cats and Wergen (1982), Daley, Cats and Wergen (1986). The problems of analysing the large scale wind field, also discussed by Daley, Cats and Wergen (1986), are much reduced if one incorporates large scale terms in the correlation model to allow for the occurrence of forecast errors on very large scales (HL/LH).

Reed et al (1986) and Julian (1980) both discuss cases where the analysis of the divergent wind field in the tropics was defective. Two approaches to the problem have been used in one-level calculations. Julian (1985) uses outgoing long wave radiation as a proxy for the velocity potential field. Daley (1985) uses the method of O/I with a relaxed non-divergence constraint. Both methods show promising results, but have still to be tested in multi-level assimilation.

b) Humidity and Initialisation of Moist Processes

The definition of the humidity field is one of the most important and difficult problems in tropical analysis. The results of the analysis can be critical for the success of forecasts or the success of diagnostic calculations. The data coverage is as bad for humidity as for winds. Chen (1984) studied the humidity fields in the ECMWF and GFDL analyses of the main FGGE IIb dataset. He found substantial differences in the zonally averaged humidity budgets derived from the ECMWF and GFDL analyses. The differences were consistent with the known differences in the intensity of the Hadley circulation in the two sets of analyses, arising from the use of adiabatic normal mode initialisation at ECMWF but not at GFDL. More recently McGuirk et al (1986) have studied the phenomenon of 'moisture burst' in the eastern

Pacific, and has found good qualitative agreement between the FGGE analyses and independent satellite imagery not used in the analyses.

Apart from the radiosondes, the most widely distributed data sources are the humidity retrievals from the TOVS instruments; at best this can give two or three useful numbers in the vertical. As discussed by Cadet (1983), this data is not available in cloudy areas; he found it useful to introduce some bogusing of data to upgrade the FGGE analyses in areas of persistent cloudiness over the Indian Ocean.

The quality of the TOVS humidity data has been the subject of a number of recent investigations (Johnson 1984, Illari 1986). Illari concluded on the basis of a variety of collocation studies, that the TOVS data were not as good as radiosonde data, but were at least comparable in quality. The TOVS humidity data is used in the French operational system since 1984, and also at ECMWF since early 1986. Some efforts to use other types of satellite data as proxy data for humidity, including cloud top temperatures and histograms of cloud-top temperature, have been reported. Kanamitsu (1984) reports a beneficial impact of the use of the latter data.

The balance between the dynamics and the diabatic forcing is critical to forecast success in the Tropics. The most elaborate and determined efforts to achieve this balance have been reported by Krishnamurti et al (1984). They used rainfall rate data and cloud-top temperature data to compile an estimate of the field of precipitation at the initial time of a forecast. They then made adjustments to the initial data in their model to ensure i) a balance between adiabatic warming and radiative cooling in the extensive subsidence areas of the tropics and ii) an initial rainfall rate in accordance with the observations in the rain areas. They found a substantial benefit from this initialisation of the diabatic forcing, a benefit which could be found in the forecasts well into the medium range. Illari (1986) also indicates that improvements in the humidity analysis have a distinct beneficial effect on tropical rainfall forecasts.

8.4 Data Quality Control

Improvements in Numerical Weather Prediction can come from improvements in technique across a wide spectrum of problems in analysis and modelling (Bengtsson 1985). A particularly critical problem is quality control of the observations presented to the analysis systems.

A striking example of the importance of quality control is discussed in the three-way analysis comparison in Hollingsworth et al (1985). In that study the same set of FGGE II-b observational data was presented to advanced analysis systems at ECMWF Reading, NMC Washington, and UKMO Bracknell. This produced three sets of analyses which were used to initialise forecasts with the models in all three centres, resulting in 9 sets of forecasts. In one case the forecasts with all models from one of the analyses produced by day 3 a spurious low in the North east Pacific which was not found in the verification, nor in any of the forecasts from the other analyses. The critical difference was traced to an error in the quality control of aircraft data in that analysis. Quality control is clearly of vital importance for forecast success.

a) The Variability of Operational Data Quality

Data can be corrupt for a wide variety of reasons. We have seen aircraft and ships reporting position at 30E over Africa when the true position was at 30W. Similar bugs can occur in earth-location of satellite reports. We have received 1-day old reports from a whole continent labelled as today's reports, because of software problems. Radiosondes can have serious biases in height or wind or both. Drifting buoys in isolated areas of the ocean can report constant pressures if the instrumentation or the telemetry fails. An operational centre has to develop a wide range of quality control tools to monitor the quality of the data, and to identify systematic problems as rapidly as possible. Random errors are always encountered, and real time quality control algorithms must be capable of dealing with them. Improvements in telecommunications are effective in reducing the number of errors due to corruptions in transmissions.

In operational practice, there are many different approaches to the quality control problem, as evidenced by the report of a recent workshop (ECMWF 1984). In some Centers there is a heavy reliance on manual intervention to detect and correct errors, while others rely entirely on automated methods. The last half hour before the main forecast run is a time of bustling activity in the first type of forecast office. Workers from that environment who visit an automated office just before run time can feel apprehensive at the eerie stillness.

b) Automated Hierarchical Quality Control Algorithms

Most operational quality control systems have a hierarchical sieve structure, which seek to identify and exclude data with successively smaller gross errors, until the data finally used in the analysis calculation has errors which are within the normal range of random error. The ECMWF system (Lorenç 1981) is typical in having such a hierarchical structure.

After a series of checks against climatology, the first-guess, and closely adjacent observations, the main check in the ECMWF system is against the combination of the other data and the first guess. The check begins with a preliminary analysis for the value of each datum, using all other data in the analysis volume except the data being checked. If the datum departs too far from this estimate it is flagged. The worst flagged datum is then eliminated and all the doubtful data re-checked. The process is repeated until all the data are judged to be mutually consistent. By a clever choice of optimisation criterion Lorenç (1981) was able to calculate all the preliminary analyses by making small corrections to the inverse of a single large matrix. The cost of the algorithm is therefore quite modest.

The cross-validation approach has proved to be a very effective device for deciding on rejection or acceptance of data. Wahba and Wendelberger (1980) used a similar algorithm, which they called generalised cross validation (GCV) for a related purpose. They examined the use of thin-plate splines which are generalisations of smoothing splines for analysis purposes. They used GCV to determine the setting of the observational noise and the setting of the 'roll-off' (curvature at the origin) of the analogue of the auto-correlation

function, in order to get the best fit to the data which minimised a penalty function on curvature. In a sense their algorithm and Lorenc's use the same method to proceed in opposite directions - one uses the structure functions to quality control the data, while the other adjusts the structure function to fit the data.

The tuning of quality control algorithms requires reliable statistics on the magnitude of first-guess errors, of analysis errors, and of observation errors for all observation types. This information is easily generated from archives of data assimilation results.

c) A Bayesian Approach to Quality Control

An inherent difficulty of a strictly hierarchical approach to quality control is that data which is rejected at an early stage in the processing is not re-considered at a later stage, when more extensive calculations might suggest that the datum is in fact free of gross error. There are many cases when it would be desirable to poll a set of tests of equal importance rather than to use the hierarchical approach (Gandin pers.comm. 1985).

Lorenc (1984), following Purser (1984), has proposed an approach to this problem based on Bayesian arguments, and using the principle of maximum likelihood. He begins by considering the probability distribution of the observation errors of a set of data where the errors may consist of two types. These are first the usual observation errors with Gaussian statistics and small variance. There is also a finite probability of gross error; the probability of such an error is constant across the range of climatologically possible states.

Lorenc developed a maximum likelihood analysis algorithm which is exactly equivalent to O/I when the statistics are Gaussian. This formulation can however deal with non-Gaussian statistics. When applied to the quality control problem, the Bayesian approach provides a rational method for polling the results of many independent tests. In particular it is possible to use the approach to pool, or ballot, the results of several individually inconclusive tests in order to form a soundly based decision on quality, as suggested by Gandin.

This review has considered current and projected work on objective analysis under three main headings: theory, verification and practice. It is necessary to mention also a basic requisite for practical work. As assimilation systems have become more complex, it has been necessary to develop extensive archives and diagnostic facilities to monitor the behaviour of the systems. At ECMWF we keep a complete record of the processing of every single datum. This archive has been an essential raw material, and a vital stimulus, for the development of new diagnostics of assimilation system performance.

Recent developments in the theory suggest that there are close links between all analysis methods based on optimality principles. Moreover, simpler analysis methods such as successive correction can be modified to converge to an optimal method in simple circumstances. Applications of the theory to four-dimensional analysis using the adjoint approach is likely to be an area of great activity. The basic job of any analysis algorithm is to filter the observation error from the data and interpolate the filtered data to a regular grid. We have presented a new viewpoint on O/I which shows the algorithm in a simple light with an intimate connection between the filtering and interpolation capabilities. The abundance of new ideas, coupled with the general awareness of the importance of analysis accuracy for NWP, augurs well for productive work in the next few years.

Verifications of observations, forecasts, and analyses have been reviewed, using statistics on one-point and two-point correlations of climatic fluctuations, of forecast errors, and of the differences between observations and analyses. The two-point correlations are inter-related through the kinematic equations of two-dimensional turbulence; in the limit of vanishing spatial lag they degenerate to the one-point correlations.

The kinematic diagnostics offer a means to compare the error characteristics of different observing systems. They already indicate some sampling problems with cloud-track wind data, and further work remains to be done. When used to verify short-range forecasts, the diagnostics provide a complete description of the structure of the forecast errors. A simple theory of forecast errors can reproduce many of the features of the empirically determined structures. New results indicate that there may be a need for anisotropic and non-separable correlation functions for O/I analysis.

The correlation diagnostics offer new and rigorous ways to estimate analysis error at observation points. They provide a quick and easy measure of the efficiency of an analysis system. They will prove useful in identifying the reasons for system dependence in the results of observing system experiments. In time they ought to enable one to prove that a three-dimensional algorithm has used all the data to the fullest extent of its capability. At that point we shall need the more explicitly four dimensional adjoint method if progress is to continue.

The review of current practice considered first the special problems of analysis in data rich areas over land and near mountains. The problems of analysis over the mid-latitude oceans were considered next, and then the problems of tropical analysis. Extensions of O/I offer new approaches to these problems which will be explored in the near future. Some problems will not yield to this approach because of their inherent unhomogeneity. In the longer term, new developments in the theory of 4-dimensional analysis may well supersede current methods. Finally we considered developments in data quality control, an area of great importance for all other developments. New developments here, using Bayesian methods, offer promise.

The field of objective analysis for numerical weather prediction is enjoying rapid development as a result of progress in observing and computing technology, and of theoretical advances in analysis method. The area is interesting and challenging, as it is the meeting point of many different areas of study: observational technology; statistical approximation theory; initialisation theory; numerical methods; dynamics; and the parameterisation of physical processes. The importance of accuracy in the analyses for forecasting and for diagnostic studies adds zest to the work.

The application of quantitative method to all aspects of assimilation has produced encouraging results. We may confidently expect progress to continue. The growing recognition that objective analysis is as exciting a challenge as any of the more glamorous areas of atmospheric science will help attract new workers for the problems to be faced over the next decade.

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APPENDIX

FILTERING AND INTERPOLATION IN STATISTICAL INTERPOLATION

As shown in Section 4 we may write the vector of analysed values at the observation points as

$$[a(r_i)] = \underline{P} \cdot (\underline{P} + \underline{D})^{-1} \cdot [d(r_i)] \quad 4.2$$

so that Eq 4.1 may be rewritten as

$$a(r_k) = [p(r_k, r_i)]^* (\underline{P})^{-1} \cdot [a(r_i)] \quad 4.3$$

Thus the analysis calculation may be thought of as a linear filter (Eq 4.2) of the observed values to produce the analysed values at the observation points, followed by an interpolation (Eq 4.3) of the filtered observations to the analysis points.

a) The Filter

The nature of the filter in Eq. 4.2 is most conveniently discussed in terms of the ratio of observation error to prediction error. Both \underline{P} and \underline{D} are symmetric and positive definite. \underline{P} may then be expressed in the form

$$\underline{P} = \underline{E} \{ \lambda \} \underline{E}^*$$

where \underline{E} is an orthogonal matrix whose rows or columns are the eigenvectors of \underline{P} , $\{ \}$ denotes a diagonal matrix and the λ_i are the corresponding eigenvalues. Define \underline{Q} by

$$\underline{Q} = \underline{E} \{ \lambda^{-\frac{1}{2}} \}.$$

Since \underline{D} is positive and \underline{Q} is non-singular it follows that $\underline{Q}^* \underline{D} \underline{Q}$ is symmetric and positive definite, and may be written in the form

$$\underline{Q}^* \underline{D} \underline{Q} = \underline{R} \{ v \} \underline{R}^*$$

where the eigenvalues v_i are all positive and \underline{R} is orthogonal. \underline{P} and \underline{D} may now be diagonalised simultaneously:

$$\underline{R}^* \underline{Q}^* \underline{D} \underline{Q} \underline{R} = \{ v \}$$

$$\underline{R}^* \underline{Q}^* \underline{P} \underline{Q} \underline{R} = \{ 1 \}.$$

With a little manipulation it follows that

$$[a(r_i)] = (\underline{R}^* \underline{Q}^*)^{-1} \left\{ \frac{1}{1+v} \right\} (\underline{R}^* \underline{Q}^*) [d(r_i)] \quad \text{A.1}$$

This shows clearly the nature of the filtering. The data undergoes a non-singular (but, in general, a non-orthogonal) change of basis $\underline{R}^* \underline{Q}^*$, the j^{th} component of the resulting vector is damped by the amount $\frac{1}{1+v_j}$ and the change of basis is reversed.

If \underline{P} and \underline{D} are commutative with eigenvalues $[\lambda_i]$, $[\sigma_i]$, and a common set of eigenvectors \underline{E} , then $\underline{R} = \underline{I}$, the identity matrix, and $v_i = \frac{\sigma_i}{\lambda_i}$. It follows that

$$[a(r_i)] = \underline{E} \left\{ \frac{1}{1 + \sigma/\lambda} \right\} \underline{E}^* [d(r_i)] \quad \text{A.2}$$

In the simplest case of uniform uncorrelated observational error, $\underline{D} = \sigma \underline{I}$ and then the interpretation of v_i as a ratio of observational error in a given component to forecast error in that component is quite obvious, as the damping on the i^{th} component is

$$\frac{1}{1 + \frac{\sigma}{\lambda_i}}$$

If the ratio of observational error to forecast error in a given component is small the response is close to 1 and such components of the Ob-Fg differences are well analysed; if the ratio of observational error to forecast error is large then such components of the Ob-Fg differences are heavily damped. This simple result suggests that in the general case, when \underline{P} and \underline{D} are non-commutative, it is also reasonable to interpret v_i in Eq. A.1 as a ratio of observation error in a given component to prediction error in that component.

b) Interpolation

The interpretation of Eq. 4.3 as an interpolation follows immediately from a standard result. The function $f(x)$ which interpolates a set of N data points F_n with a set of N functions $g_n(x)$ is given by

$$f(x) = \sum_{n=1}^N c_n g_n(x)$$

where the coefficients c_n satisfy

$$[c_n] = \underline{G}^{-1} [F_n]$$

and the matrix \underline{G} is defined by

$$\underline{G}_{ij} = g_i(x_j)$$

A familiar example of this result is the Lagrange interpolation formula, where a clever choice of the functions $g_n(x)$ ensures that $\underline{G} = \underline{I}$.

c) Filtering and Interpolation

To see the intimate connection between the filtering and the interpolation, consider the interpolated value $a(r_k)$ in Eq 4.3 when the filtered data vector $[a(r_i)]$ corresponds to one of the eigenvectors \underline{e}_j of \underline{P} , the prediction error correlation matrix. The function

$$\mathcal{E}_j(r_k) = [p(r_k, r_i)]^* \underline{P}^{-1} \underline{e}_j$$

then interpolates the eigenvector exactly and we may call it an 'eigenfunction' of the matrix, where ' ' indicates that we use the term in a very special sense. The relation between the filtering and the interpolation is now very simple if the matrices \underline{P} and \underline{D} commute. This assumption includes many cases of practical interest such as the case $\underline{D} = \sigma \underline{I}$, where the observation errors are uniform, random, and independent. For then if the original vector $[d(r_i)]$ is expanded in eigenvectors of \underline{P} as

$$[d(r_i)] = \sum_1^N c_j \underline{e}_j \tag{A.3}$$

then the analysed values at the observation points are given by

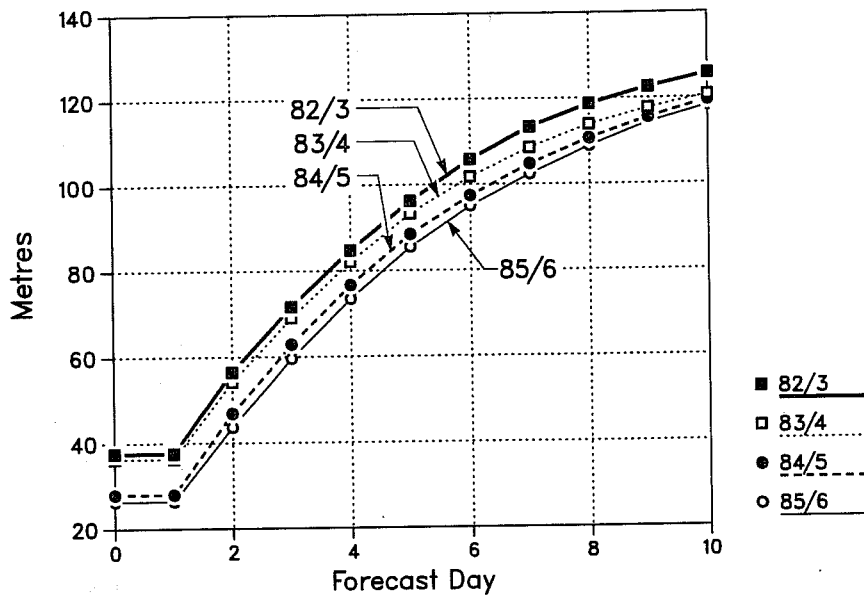
$$[a(r_i)] = \sum_1^N c_j \frac{1}{1+v_j} \underline{e}_j, \tag{A.4}$$

where v_i is the ratio of observational error to prediction error in the i^{th} component. The analysed value at the point r_k is given by

$$a(r_k) = \sum_1^N c_j \frac{1}{1+v_j} \mathcal{E}_j(r_k) \tag{A.5}$$

where the function \mathcal{E}_j interpolates the vector \underline{e}_j according to Eq. 4.3.

Southern Hemisphere 500mb RMS Errors 1982/3 1983/4 1984/5 1985/6 Jun-Apr (11mo)



Northern Hemisphere 500mb RMS Errors 1982/3 1983/4 1984/5 1985/6 Jun-Apr (11mo)

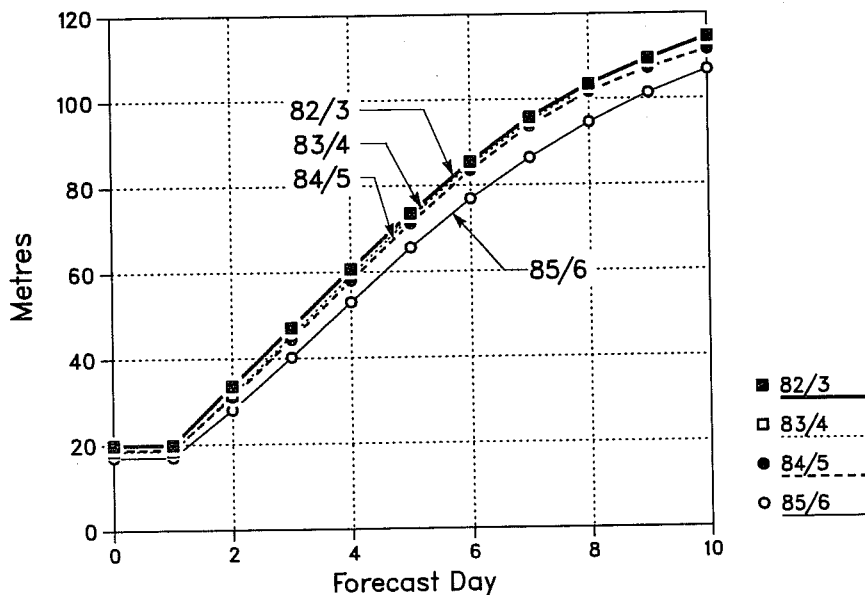


Fig 1 RMS 500mb height forecast errors as a function of forecast range for the Southern hemisphere (a) and the Northern hemisphere (b). The results are shown for the months of June through April in 1982/83, 1983/84, 1984/85, 1985/86. The one day forecast errors are plotted twice (at day 0 and day 1) to show clearly the effect of successive system changes. Between the first and second periods the model changed from N48 to T63 with envelope orography; between the second and third periods there were substantial changes to the analysis; between the third and fourth periods the model changed to T106 with extensive changes in the physics. See text for further details. (from Shaw et al 1987)

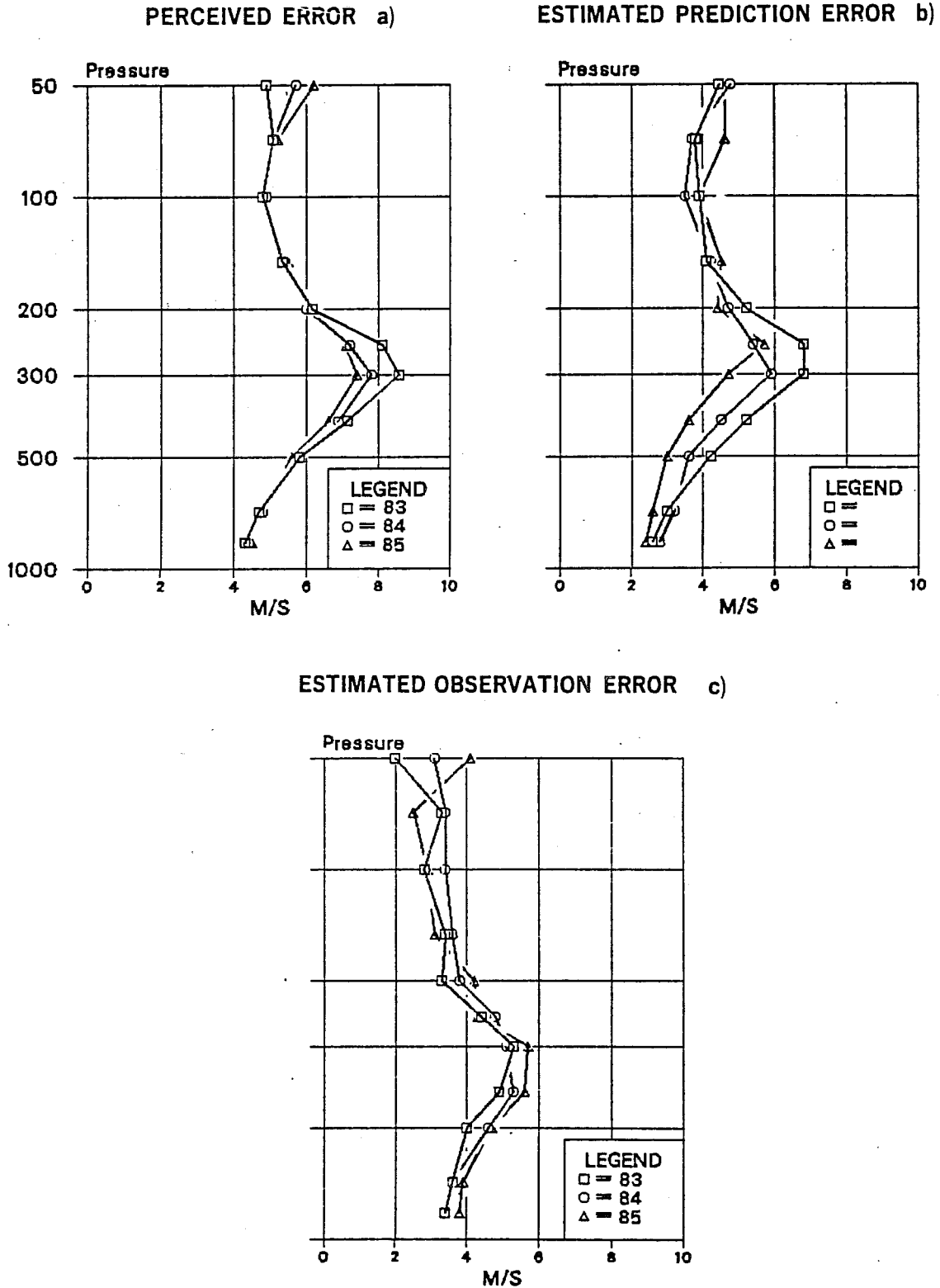


Fig 2 Perceived wind forecast error (panel a) for 6-hour forecasts verified against 1200 UT North American rawinsondes for Jan-March 1983, Jan-March 1984, and December 1984-February 1985. The perceived forecast error for each year was partitioned into (spatially correlated) prediction error given in panel b, and (spatially uncorrelated) observation error in panel c. Note the stability of the estimates of observation error from year to year, and the steady reduction in the prediction error. (from Shaw et al 1987)

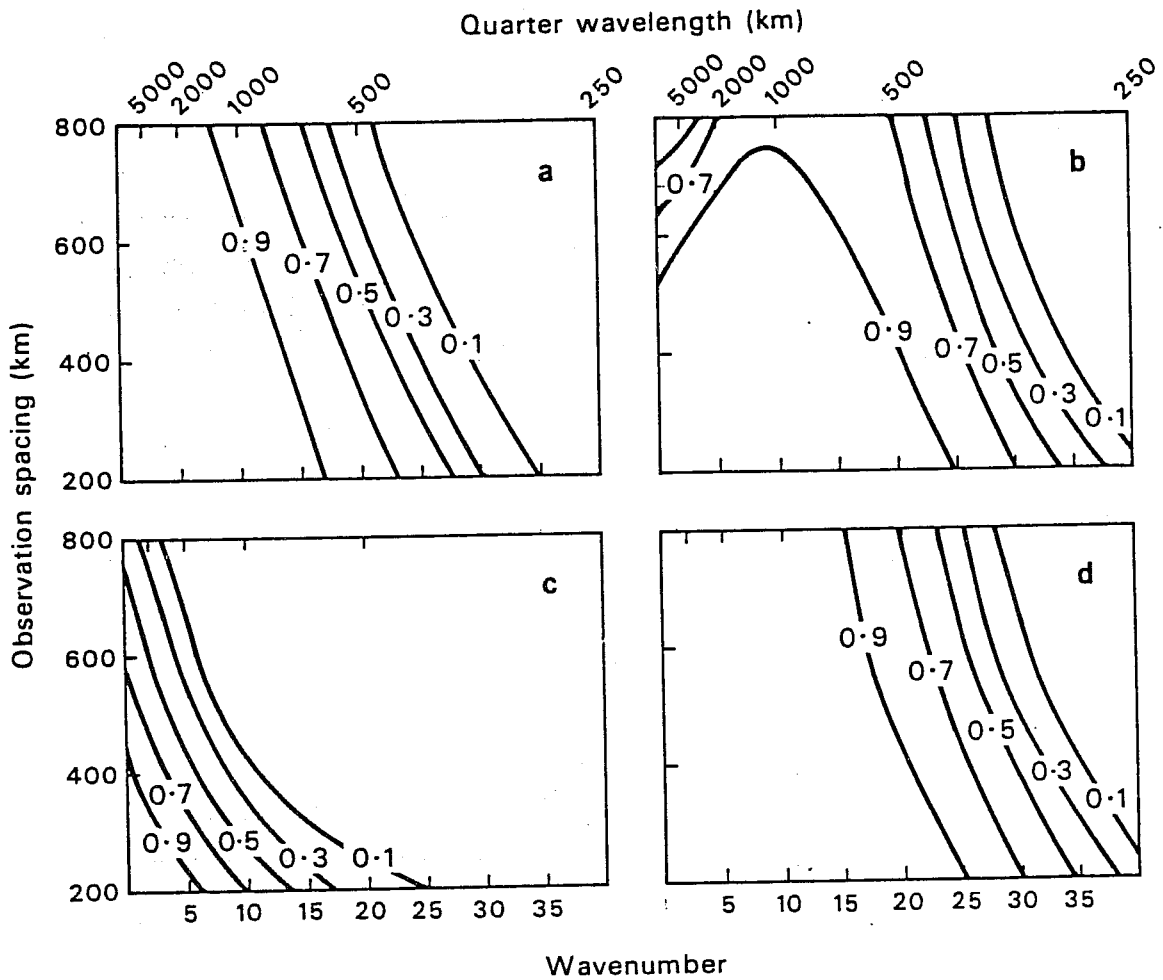


Fig 3 Response diagrams illustrating response of a box-type analysis to regularly spaced observational data, as a function of input wavenumber (abscissa) and observation spacing (ordinate). On the top of each diagram is also shown the quarter wavelength (km). a) geopotential response to geopotential observations. b) wind response to non-divergent wind observations. c) wind response to irrotational wind observations. d) mass, or wind, response to mass and non-divergent wind observations. The auto-correlation functions are modelled by a gaussian with a 600km length scale, and the non-dimensional observation error is 0.25 in all cases (from Daley 1985).

HIGH LEVEL WINDS

VECTOR CORRELATION

Auto correlation

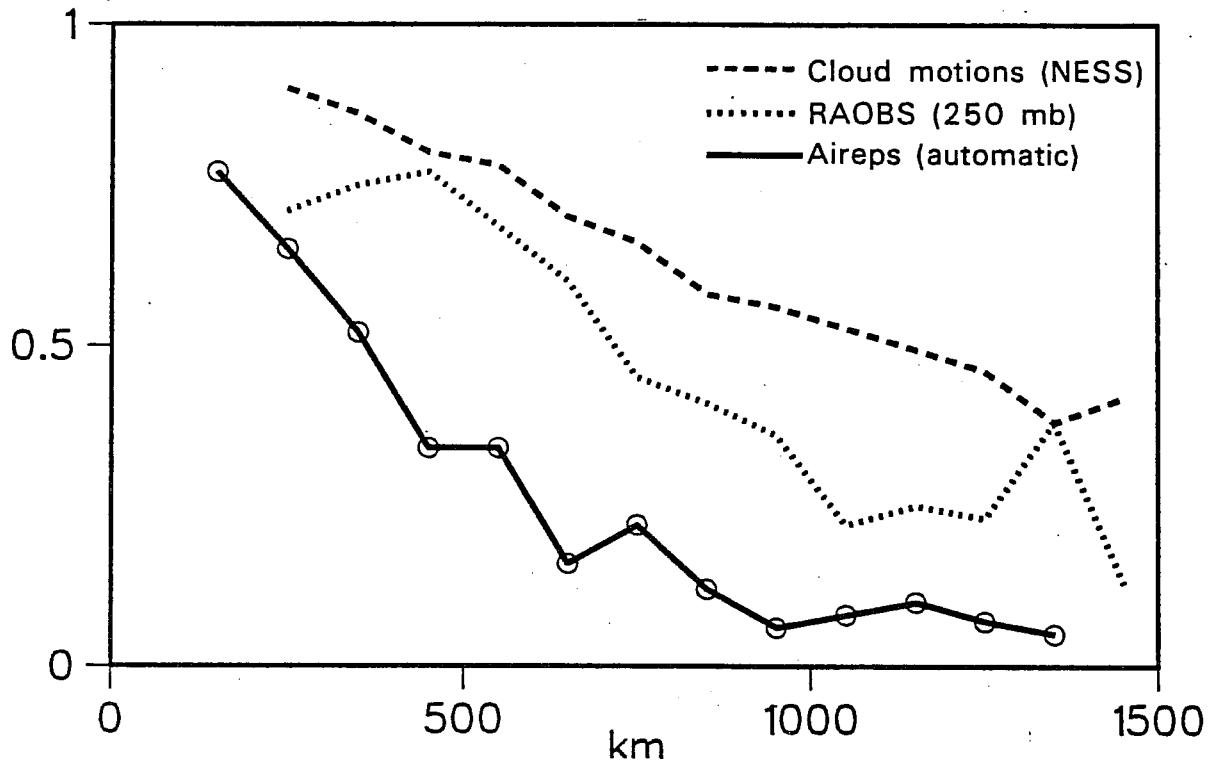
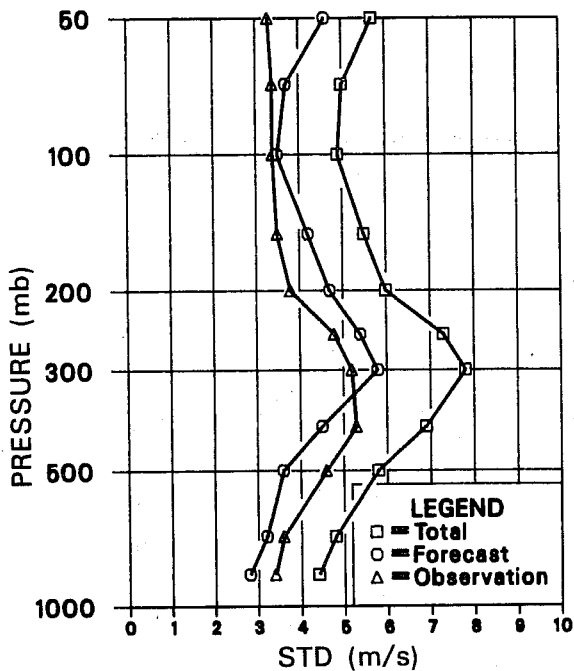
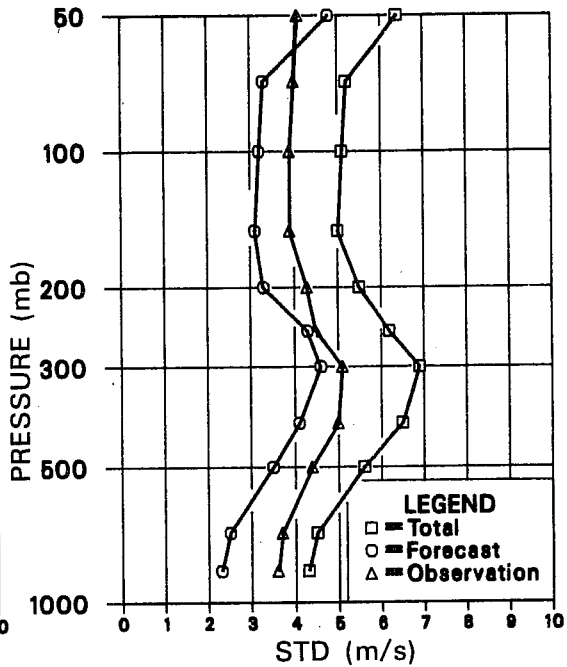


Fig 4 Tropical 250mb vector wind correlation of fluctuations about the climate mean, as observed during FGGE by three observing systems. The aircraft are sampling much shorter scales than the radiosondes, and the cloud track winds are sampling much larger scales. (from Wylie, Hinton Howland and Lord, 1985).

a) WIND / NORTH AMERICA



b) WIND / EUROPE



c) WIND / TROPICS

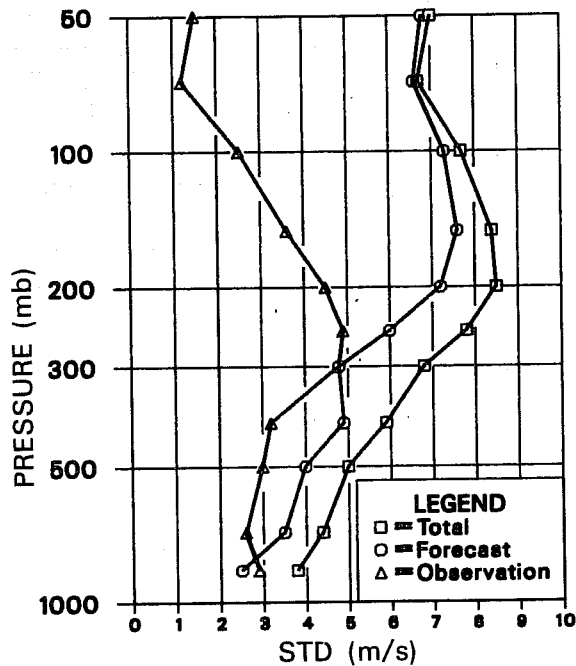


Fig 5 For each of the three areas North America (top left), Europe (top right), and the tropics (bottom), the plots show the total perceived wind forecast error for the vector wind, and the calculated observation error and prediction error as indicated in the legend. The calculations are based on 6-hour forecasts for the first quarter of 1984 with the ECMWF system; the unit is m/s. In all three areas the prediction error and observation error are comparable in the troposphere. (from Hollingsworth et al 1986).

a) HEIGHT / NORTH AMERICA

b) WIND / NORTH AMERICA

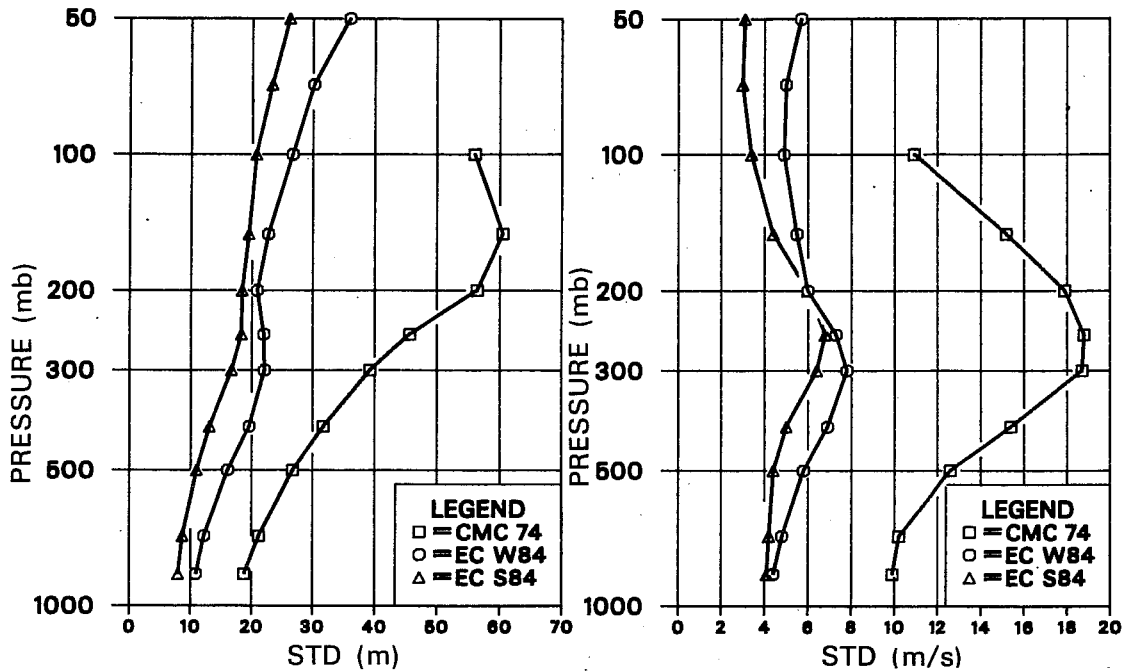


Fig 6 The plots show the perceived forecast errors for height (left, unit m) and wind (right, unit m/s) for verifications against North American radiosonde data. The curves labelled CMC74 are from Hollett (1975), and the ECMWF forecasts for Jan-Mar 1984 and Jun-Aug 1984 are indicated by EC W84 and EC S84 respectively (from Hollingsworth et al 1986). The substantial reductions in short range forecast errors indicate the strides that have been made in data assimilation over the last decade.

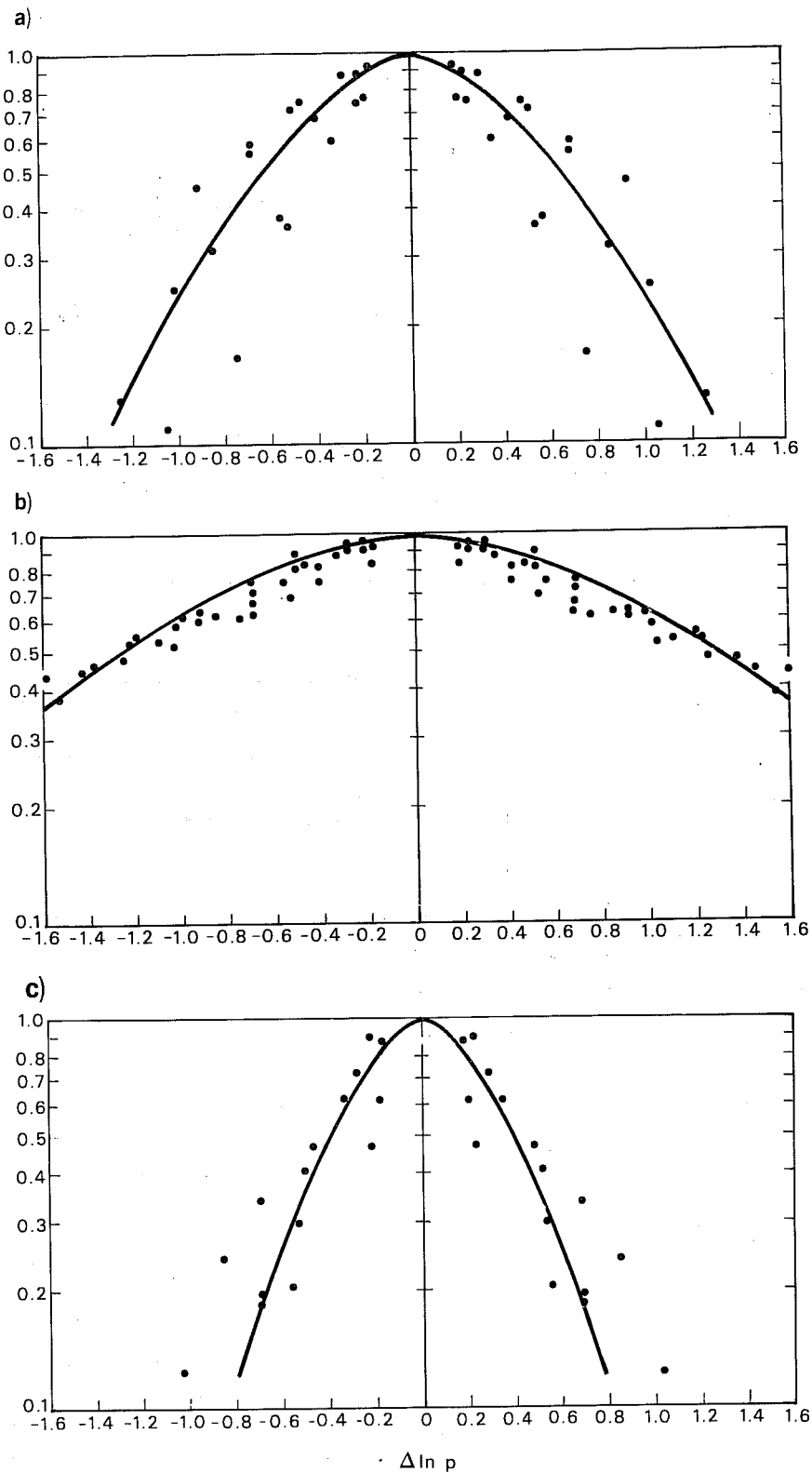


Fig 7 The plots show the vertical correlation (on a log scale) of
a) synoptic scale height forecast errors
b) large scale height forecast errors
c) synoptic scale wind errors

as functions of the difference in the logarithms of the pressures for standard levels below 150mb, using 6-hour ECMWF forecasts verified against radiosondes over North America for the first quarter of 1983. If the errors were separable and geostrophic then the curves a and c would be identical. Panel b shows the nearly barotropic nature of the large scale forecast error in height. (from Hollingsworth and Lonnberg 1986, Lonnberg and Hollingsworth 1986).

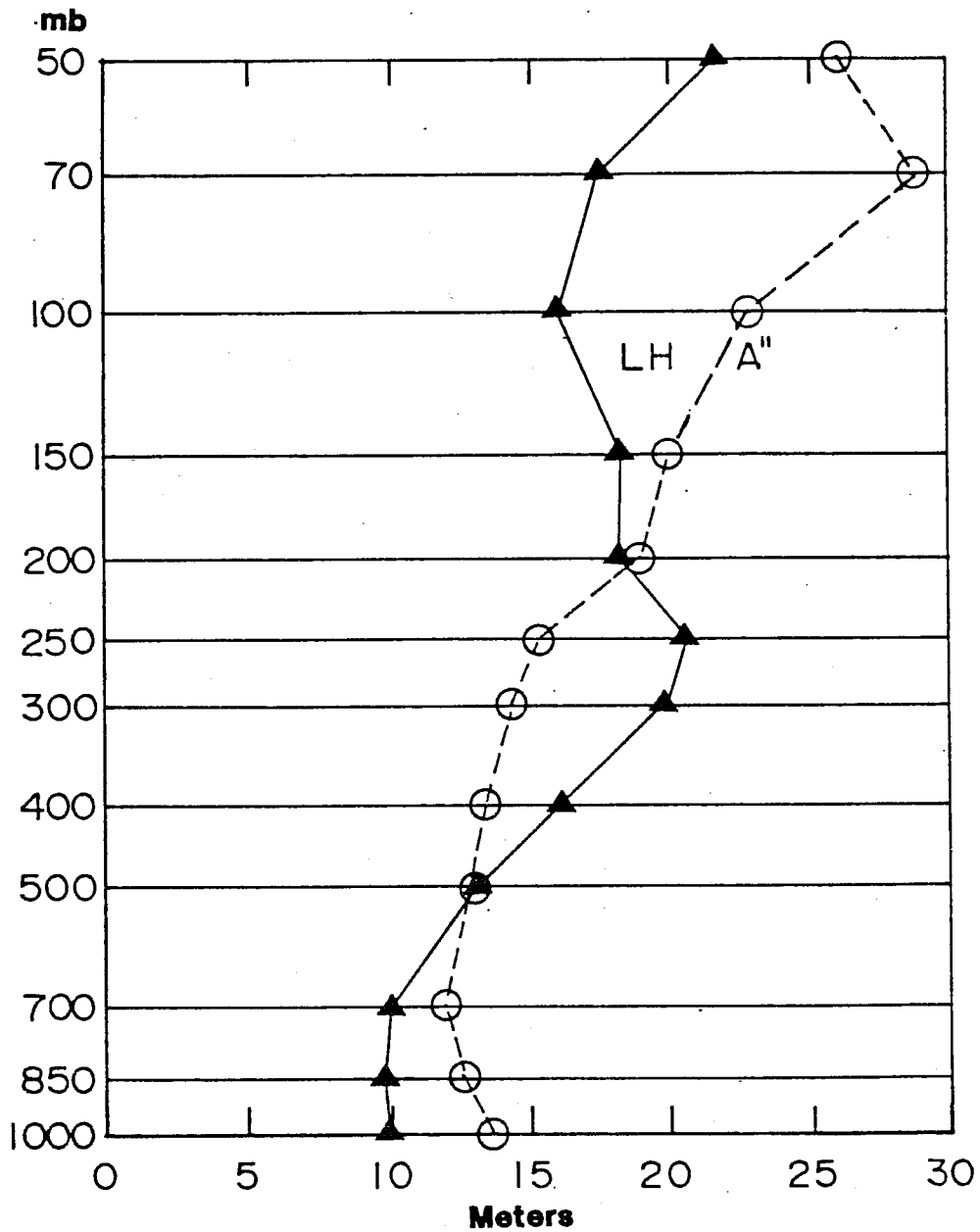
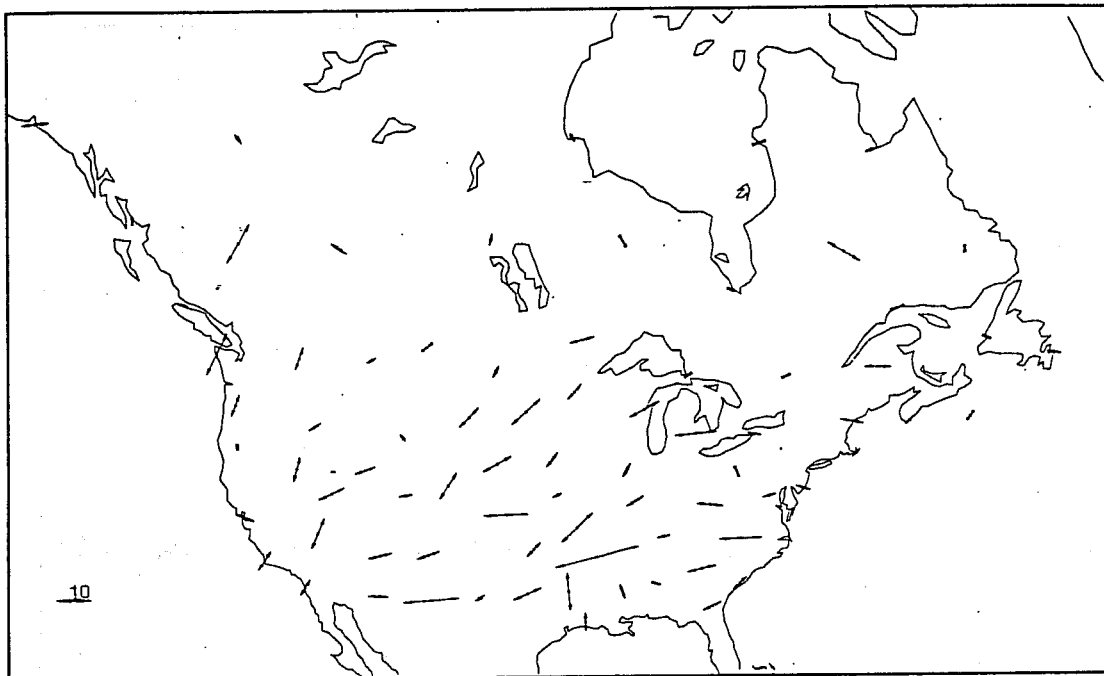


Fig 8 Distribution of square root of height variance as determined for the ECMWF forecast errors by Lonnerberg and Hollingsworth, 1986, (LH), and for an ensemble of idealised slow modes (A''). The overall similarity of the curves indicates that the idealised ensemble can reproduce some properties of the empirical results. (from Phillips 1986).

ANISOTROPY(OB-FG) VECTOR WIND 250 HPA

DECEMBER 84 - FEBRUARY 85 12 GMT



ANISOTROPY(OB-FG) VECTOR WIND 250 HPA

DECEMBER 84 - FEBRUARY 85 12 GMT

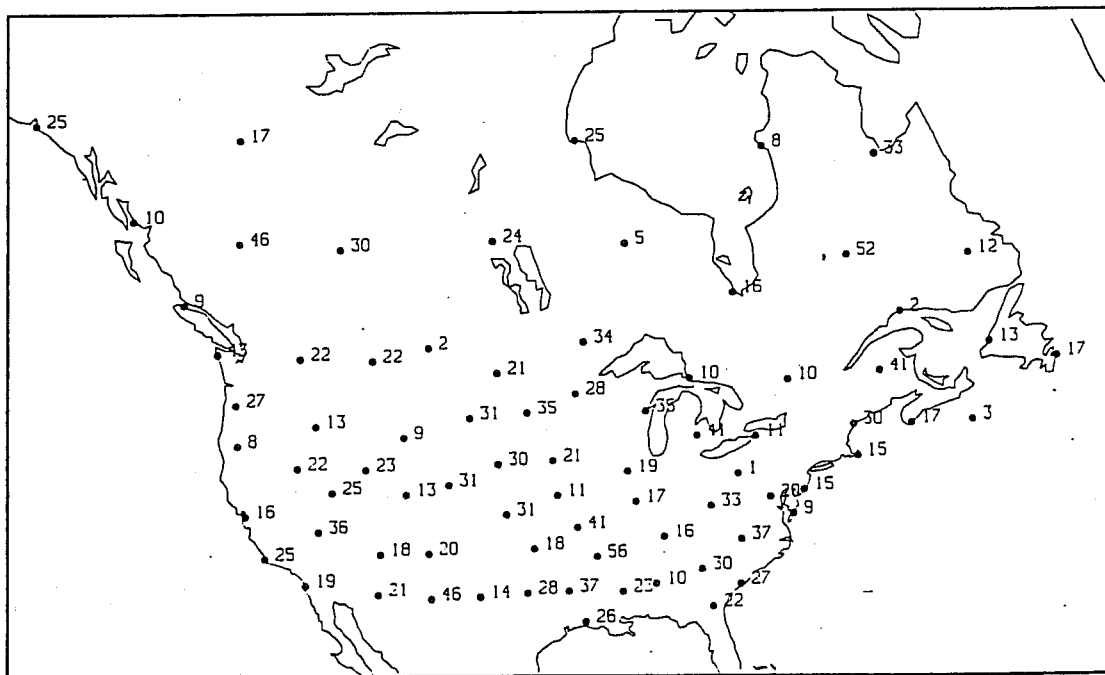
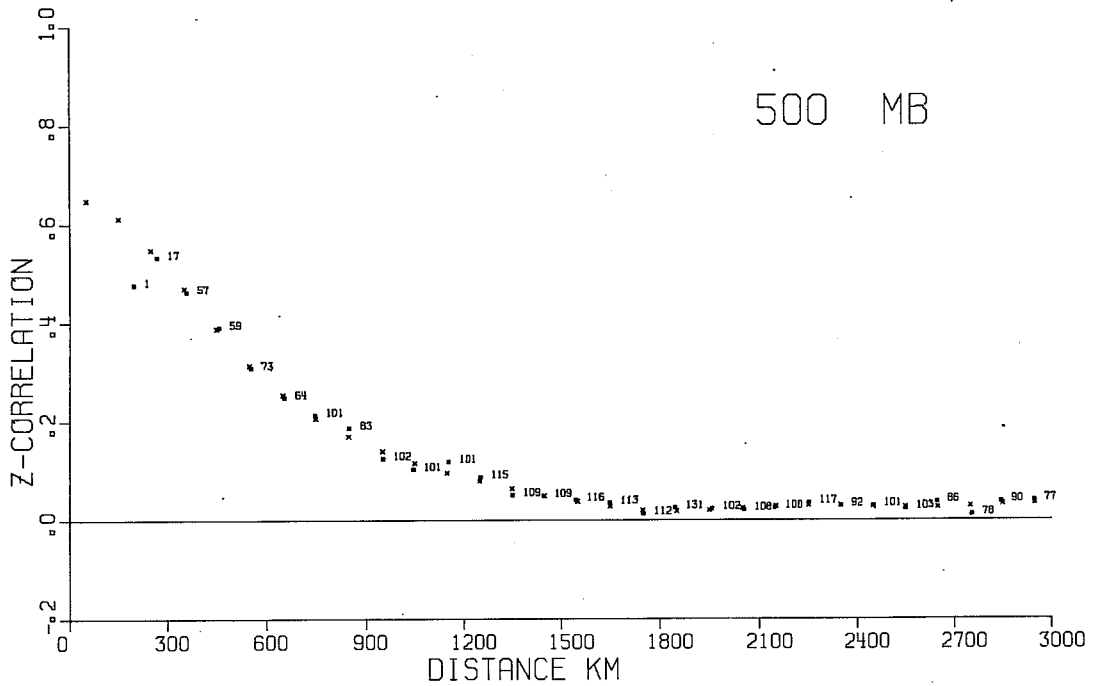
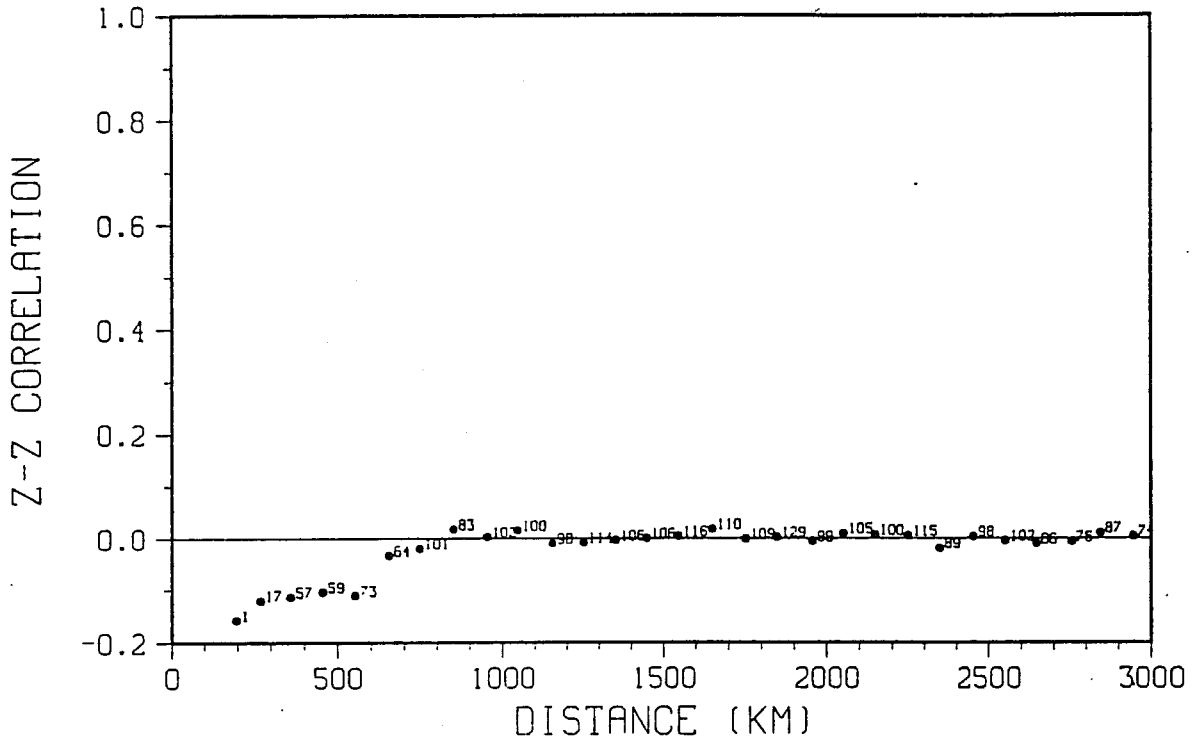


Fig 9 a) The anisotropy vector [magnitude $(M^2+N^2)^{\frac{1}{2}}$, and angle $\frac{1}{2} \tan^{-1}(N/M)$ from latitude circle] of ECMWF 6-hour wind forecast errors verified against North American radiosondes for 1200 UT in December 1984- February 1985; the unit is $(m/s)^2$. The coherent patterns and the magnitudes of order $25(m/s)^2$, indicate that there is substantial anisotropy in the forecast errors. b) The ratio $\alpha = (M^2+N^2)^{\frac{1}{2}}/K$ for the vectors shown in a), plotted as a percentage. If half of K is observational error, then these numbers may be doubled, indicating that anisotropy is as significant for the forecast errors as it is for the atmospheric fields.

ANALYSIS ERROR 500 HPA



F/C ERROR CORR. 60 - 30 N 140 - 50 W JAN - MAR 1983 12 GMT

Fig 10a) The spatial correlation of Ob-An differences in 500mb height for 1200 UT radiosonde data over North America in December 1984-February 1985. The empirical data, averaged over bins of 100km, are shown by squares, and a least squares fit to the data is shown by 'x'. The figures indicate the number of station pairs in each bin. Each pair had a minimum of 60 reports for the calculation. The negative intercept can be used to put an upper bound on the analysis error at the grid points. b) As a) for the Ob-Fg differences for the first quarter of 1983. (from Lonnerberg and Hollingsworth 1986).

ANALYSIS ERROR 200 - 150 HPA

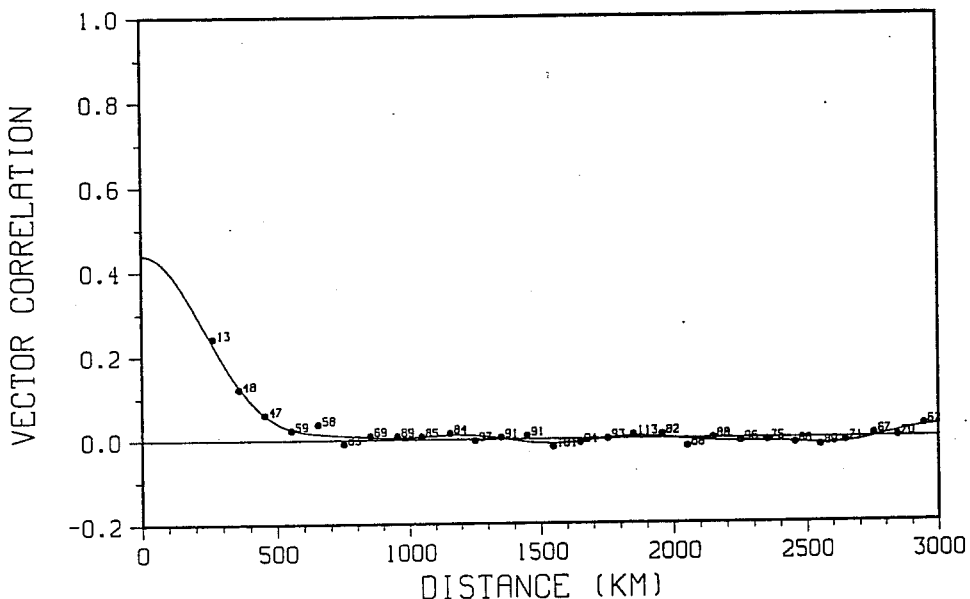


Fig 11 The vector correlation of Ob-An differences for the wind shear between 150mb and 200mb for 1200 UT radiosonde data over North America in the first quarter of 1983. The empirical data, averaged over bins of 100km, are shown by squares, and a least squares fit to the data is shown by 'x'. The figures indicate the number of station pairs in each bin. Each pair had a minimum of 60 reports. The positive intercept indicates that the data has not been fully used. The positive intercept can be used to put a lower bound on the analysis error at the grid points.

ANALYSIS ERROR 250 - 150 HPA

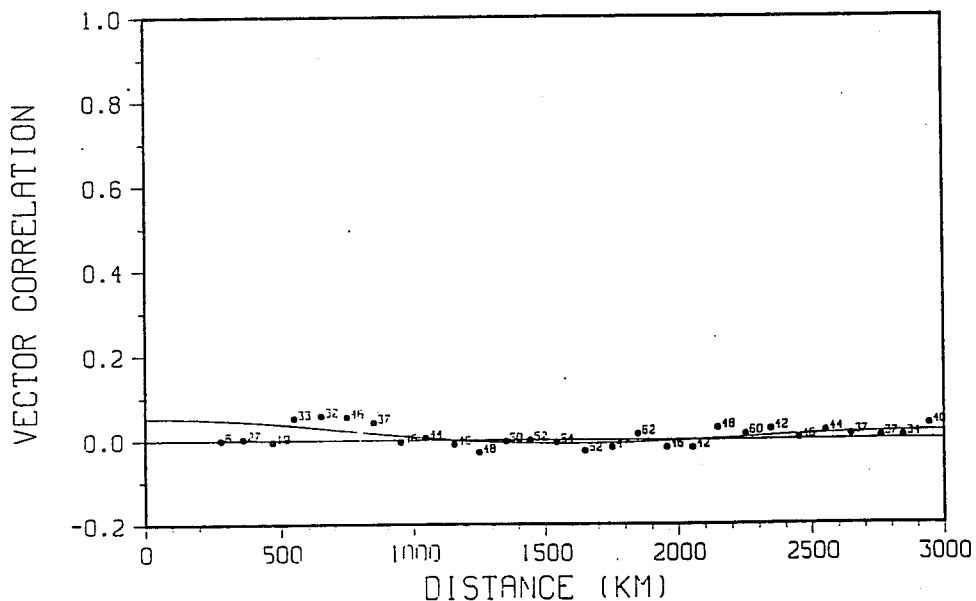


Fig 12 The vector correlation of Ob-An differences for the wind shear between 150mb and 200mb for 1200 UT radiosonde data over North America in December 1984-February 1985. The empirical data, averaged over bins of 100km, are shown by squares, and a least squares fit to the data is shown by 'x'. The figures indicate the number of station pairs in each bin. Each pair had a minimum of 60 reports. The lower values near the origin, in comparison with Fig. 11, indicate that lower bound on the analysis error was much lower in 1984/85 than in early 1983.