SCALE DEPENDENCE OF BOUNDARY LAYER FLUXES AND SUBGRID FORMULATIONS

L. Mahrt
Oregon State University
Corvallis, USA

Abstract

This note discusses the scale dependence of boundary-layer fluxes and other subgrid scale motions. Problems of formulating subgrid fluxes in numerical models are noted and recent investigations carried out at Oregon State University are briefly summarized.

1. INTRODUCTION

The parameterizations of surface fluxes in large scale models are generally based upon observations of the relationship between the local turbulent flux and the local mean gradient. Applying this relationship to estimate the surface flux over a grid area can break down for several reasons:

i). **The relationship between the grid area-averaged flux and the grid area-averaged gradient may be quite different than expected from point observations.** In fact the grid-averaged flux can be counter to the grid-averaged gradient. As one example, most of a grid area may be stable, corresponding to weak downward heat flux. However a small portion of the grid area may be unstable due to less cloudiness or drier soil conditions. The resulting large upward heat flux may dominate the grid-averaged heat flux which would then be counter to the area-averaged vertical gradient of potential temperature.

ii). The **subgrid scale flux** may include contributions from inertia-gravity waves, convective systems, topographically induced circulations and other mesoscale motions related to surface inhomogeneity. The transport due to such motions may not obey the same relationship to the mean vertical gradients as turbulence. In other terms, the nature of the subgrid scale flux depends on the horizontal resolution of the model as well as the vertical resolution. Cospectra, particularly in the upper part of the boundary layer, often show significant transport on a continuum of scales without a spectral gap between the turbulence and larger scales. Significant cospectra at larger wavelengths seems to develop most often for vertical transport of horizontal momentum and moisture even over relatively homogeneous water surfaces (e.g. Nicholls, 1985). Cospectra sometimes reverse sign with increasing wavelength (e.g., Greenhut, 1981; Wyngaard, et. al., 1978 and others) so that even the sign of the flux depends on the scales included.

iii). Most surfaces in the world are **inhomogeneous**. Even when surfaces appear to be homogeneous, variations in soil moisture can lead to significant variations of the surface energy balance and subsequent development of the boundary layer. Most inhomogeneities are not sharp so that well-developed internal boundary layers cannot be defined. A typical boundary-layer flow over land might therefore be constantly adjusting to the spatially varying surface conditions.

iv). **Horizontal fluxes** between grid boxes are generally neglected although generic diffusion is sometimes specified to control numerical features of the solution. Physically based formulations generally relate the diffusivity to the mean deformation (e.g., Lilly, 1967); the needed direct observational evidence is missing.

v). Subgrid scale flux induced by **surface topography** is particularly complicated and can assume several interdependent forms:

a). Topographically induced **gravity waves** can lead to significant momentum flux (Lilly, 1982; Brown, 1983). The weak point of existing approaches for large scale models appears to be the formulation of input stress at the lower boundary.

b). **Slope-induced circulations** lead to significant subgrid scale transport. Nocturnal cold air drainage occurs simultaneously on a variety of horizontal scales, the larger scales generally influencing deeper layers. These flows are often quite transient in nature often involving bursts of downslope flow even over gentle slopes. Overall downslope flows are even more chaotic than the terrain itself and one is tempted to consider their statistical effect in terms of *drainage diffusion* leading to both net horizontal and vertical transport. Under very stable conditions, the effective topography may be smoothed due to filling in of valleys with stagnant cold air sometimes referred to as *cold air lakes*. Since the vertical turbulent transport in the trapped air is very small, the effective amplitude of the terrain is reduced. Conversely, heated upslope flow could lead to an effective amplification of terrain effects requiring larger vertical displacement of the ambient flow.

c). With weak stratification, the surface stress due to flow over the topography is often viewed as a **form drag** related to the perturbation pressure field. The practice of enhancing the roughness length to accommodate such effects should not be used in the equations for surface heat and moisture flux since pressure fluctuations do not directly transport such quantities.

2. FORMULATION ATTEMPTS

Attempts to statistically include **subgrid scale surface variations** have specified a Gaussian distribution of a specific surface property (Sud and Smith, 1985; Mahrt 1987). One result has been a smoothing of the sharp decrease of the exchange coefficient as the stability changes from unstable to stable (Figure 1). Sud and Smith (1984, 1985) have pointed out the numerical advantage of this smoothing in terms of reduced "jolting" of the model from transitions between stable and unstable conditions.

Attempts to estimate the area averaged flux from observations must deal with the error of the flux estimate which often involves serious sampling problems. Existing **observational efforts** are of limited use for assessing typical transport by subgrid scale motions which are larger scale than the turbulence scales. Part of the problem is related to **sampling problems** associated with larger scales. To study the statistical stability of the computed fluxes as a function of record size, we partition the aircraft records from HAPEX and FIFE into N_j smaller

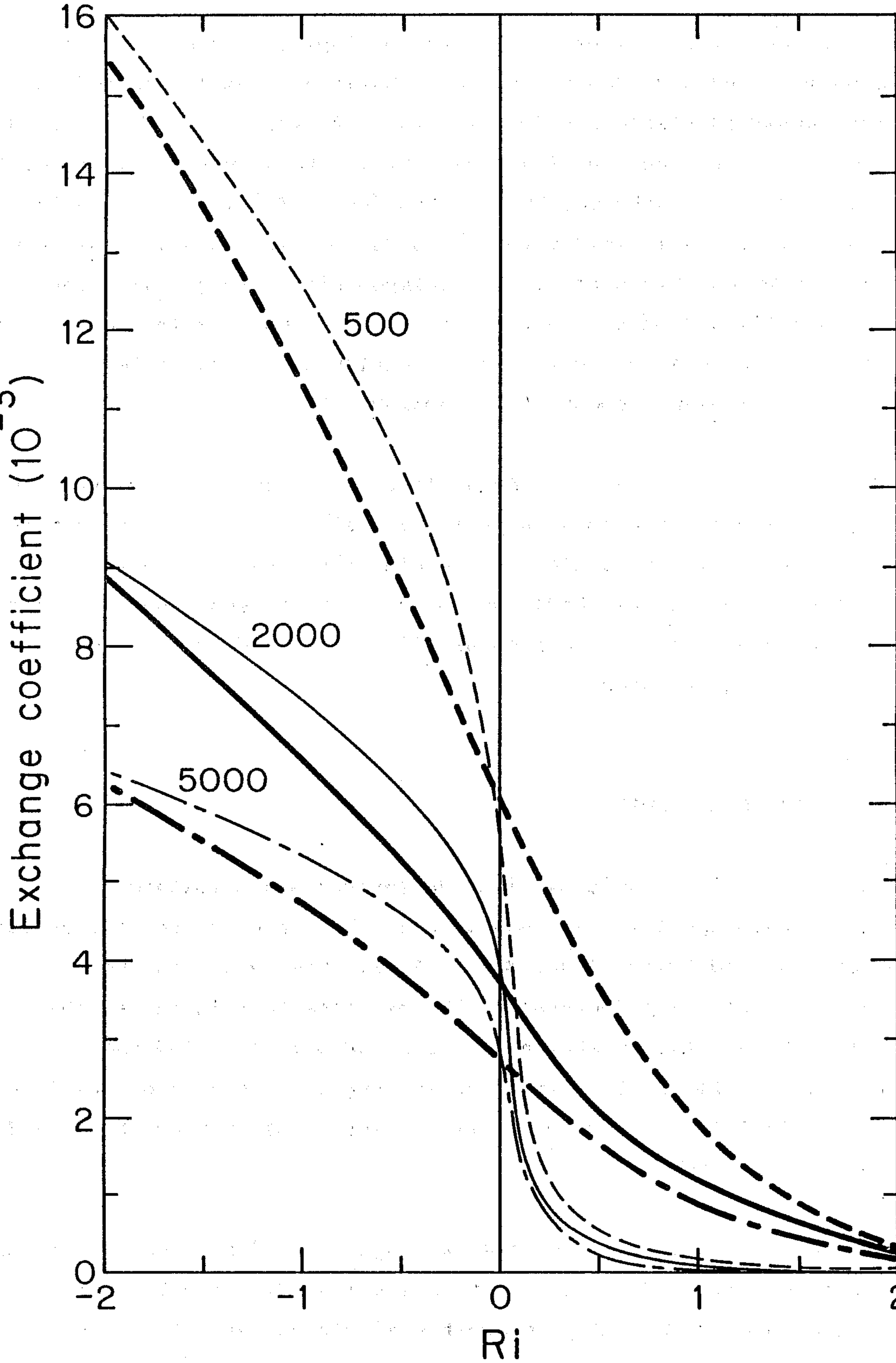

Figure 1. Dependence of the averaged exchange coefficient on the averaged Richardson number for three different values of z/z_o (thick lines) for a Gaussian distribution of the Richardson number with standard deviation of unity. The original relationship for the exchange coefficient, upon which averaging is performed, is from Louis (1979) and plotted with thin lines.

subrecords of width L_j and define the *estimated error of the flux* calculation as

$$\sigma_{flux}(L_j,\lambda) = \{(1/N_j)\sum[F_{ij}(\lambda)-F(\lambda)]^2\}^{1/2} \qquad (1)$$

where $F_{ij}(\lambda)$ is the i^{th} flux estimate for a subrecord of size L_j, $F(\lambda)$ is the flux computed from the entire record and the sum in (1) is over all of the subrecords of size L_j. Noting that for a given filter length and subrecord width, the sum of fluxes $F_{ij}(\lambda)$ over the subrecords is equal to the flux for the entire record, and the above error estimate (1) becomes simply the standard deviation of the subrecord flux. Nonzero values of (1) are due to errors associated with sampling problems and also due to natural variability. The standard deviation (1) can be viewed as a typical error when estimating a "representative" flux from a given subrecord length.

Figure 2 shows how the estimated error, for a given filter length, increases with decreasing record length. The unwritten rule that the record length should be at least 10 times larger than the largest scales included by the filter seems reasonable, although the length scale of the main transporting eddies and the scale of inhomogeneity also enter the problem. For example for the data in Figure 2, most of the of the flux occurs on scales of a kilometer or less so that increasing the filter length to 20 km has only a secondary effect and the factor of 10 rule no longer needs to be satisfied. This can be seen in Figure 2 where a broken horizontal line indicates where the estimated error would equal 20% of the averaged flux. In other terms, the needed record length depends on the minimum of the cutoff filter length and the main scale of important transport. For a filter length of 5 km, which safely includes the most important transporting scales for the present data, the estimated error normally appears to be smaller than 20% when the record length is ten times greater than the filter length (this requires extrapolation of results in Figure 2 for some cases).

The boundary layer depth in the above cases is typically 1500 m so that the factor of ten rule requires that the record length be about 30 times longer than the depth of the boundary layer. This requirement is more lenient than the rule of 100-10,000 longer than the depth of the boundary layer suggested by Lenschow and Stankov (1986). However in the

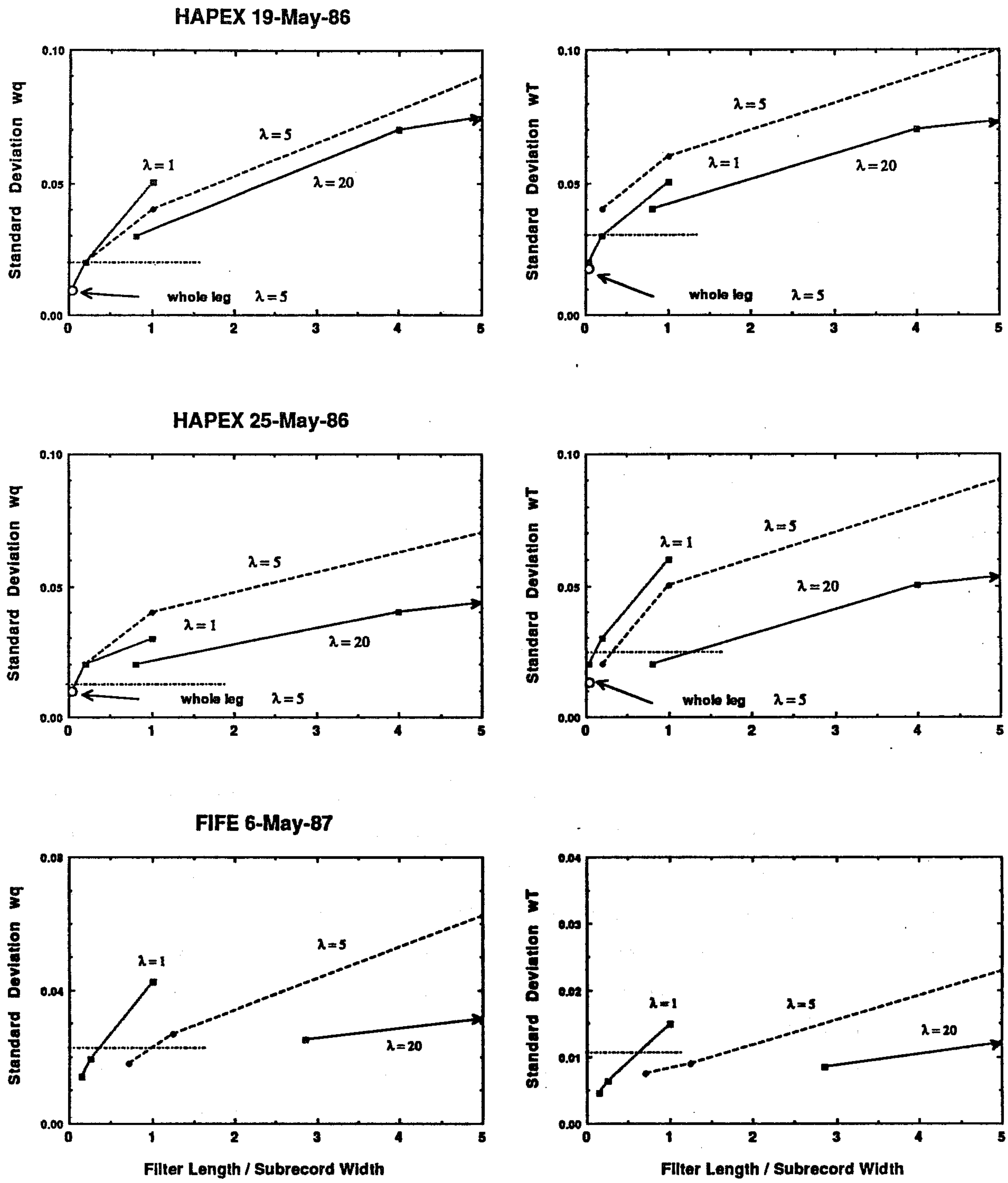

Figure 2. The estimated error (standard deviation) of the flux as a function of the filter length divided by the record length. Various record lengths were created by arbitrarily partitioning the original records. The cutoff wavelength for the high pass filter (λ) is indicated on the graph. The horizontal broken line indicates 20% of the mean flux as averaged over all of the legs. The open circles indicate the estimated error associated with entire flight legs as compared to the flux averaged over all of the flight legs.

present study, the vertical fluxes are based on flights only 100 m above the ground where vertical motions and associated fluxes are confined to smaller scales. In the middle and upper part of the boundary layer, important fluxes extend to much larger scales and may include significant transport on scales larger than turbulent scales. Aircraft flights would normally have difficulty obtaining a record length of 10 times the largest subgrid scale width without including significant nonstationarity or inhomogeneity. Problems with tower data using Taylor's hypothesis would typically be even more severe.

A sampling strategy might be to estimate the largest important scales of transport, choose the appropriate cutoff filter, then study the trend of the estimated error with increasing subrecord size. If the estimated error appears to be unacceptably large for a given flight length, one might consider reducing the error with repeated legs over the same track and flight level. Then one can study the variability of the accumulated flux with systematic addition of records.

Use of existing **mesonet data** to study subgrid variations of surface air conditions is complicated by the fact that near surface values are often representative of only the immediate microclimate on a scale much smaller than the spacing between stations. This problem is most severe under nocturnal conditions with weak winds where conditions at 10 m may be representative of a horizontal scale of less than 100 m. Existing meso-networks have tended to emphasize hilltop locations leading to a regional prejudice. Nonetheless considerable mesonet data is available for constructing examples of subgrid scale variations.

The study of Mahrt (1987) attempted to look at transport by subgrid motions by considering a mesoscale numerical model with topography to be a grid box of larger scale model where surface fluxes were parameterized with Louis (1979). These numerical results indicated that the greatest net contribution due to subgrid scale variations was during stable conditions while different flux contributions due to subgrid scale variations tended to cancel in the unstable case; however the generality of these results are not known.

3. HORIZONTAL FLUXES

Horizontal fluxes are normally thought to be unimportant because their variation occurs over length scales which are large compared to the boundary-layer depth leading to unimportant horizontal divergence of the horizontal flux. This assumption breaks down in regions of significant local variation of surface conditions. Usually surface inhomogeneity is studied in terms of sharp discontinuities between uniform surface areas where chances of physical understanding is greatest. Many surfaces consist of continually changing surface conditions forced by variations of soil conditions even when the vegetation appears to be uniform. The constantly perturbed boundary-layer flow may never achieve equilibrium with the local surface conditions. These perturbations together with transient motions such as inertial gravity waves or cloud-induced circulations lead to motion variance and fluxes on a variety of scales larger than the main turbulent eddies. From a modelling point of view, the horizontal flux divergence may not be negligible and the vertical flux may include more than just turbulent contributions.

The horizontal flux divergence can be neglected if the ratio

$$[\delta(v'\phi')/\Delta y]/[\delta(w'\phi')/\Delta z] \qquad (2)$$

is small, where Δy and Δz are the grid spacings, δ indicates a change of the flux between grid points or grid levels and f is some variable such as temperature, specific humidity or one of the velocity components. In the boundary layer, the vertical flux divergence is typically on the order of the surface flux divided by the boundary-layer depth and not sensitive to the vertical resolution. However the magnitude of the horizontal flux appears to be more sensitive to the choice of the horizontal resolution. Computations based on HAPEX aircraft data indicate that horizontal fluxes are large in the daytime boundary layer. However much of the horizontal flux is related to vertical velocity fluctuations acting on the mean shear and thus producing fluctuations of v' and ϕ' which are systematically correlated. The horizontal divergence of such fluxes occurs on the scale of the horizontal variation of the mean vertical gradients and therefore may not lead to significant local changes.

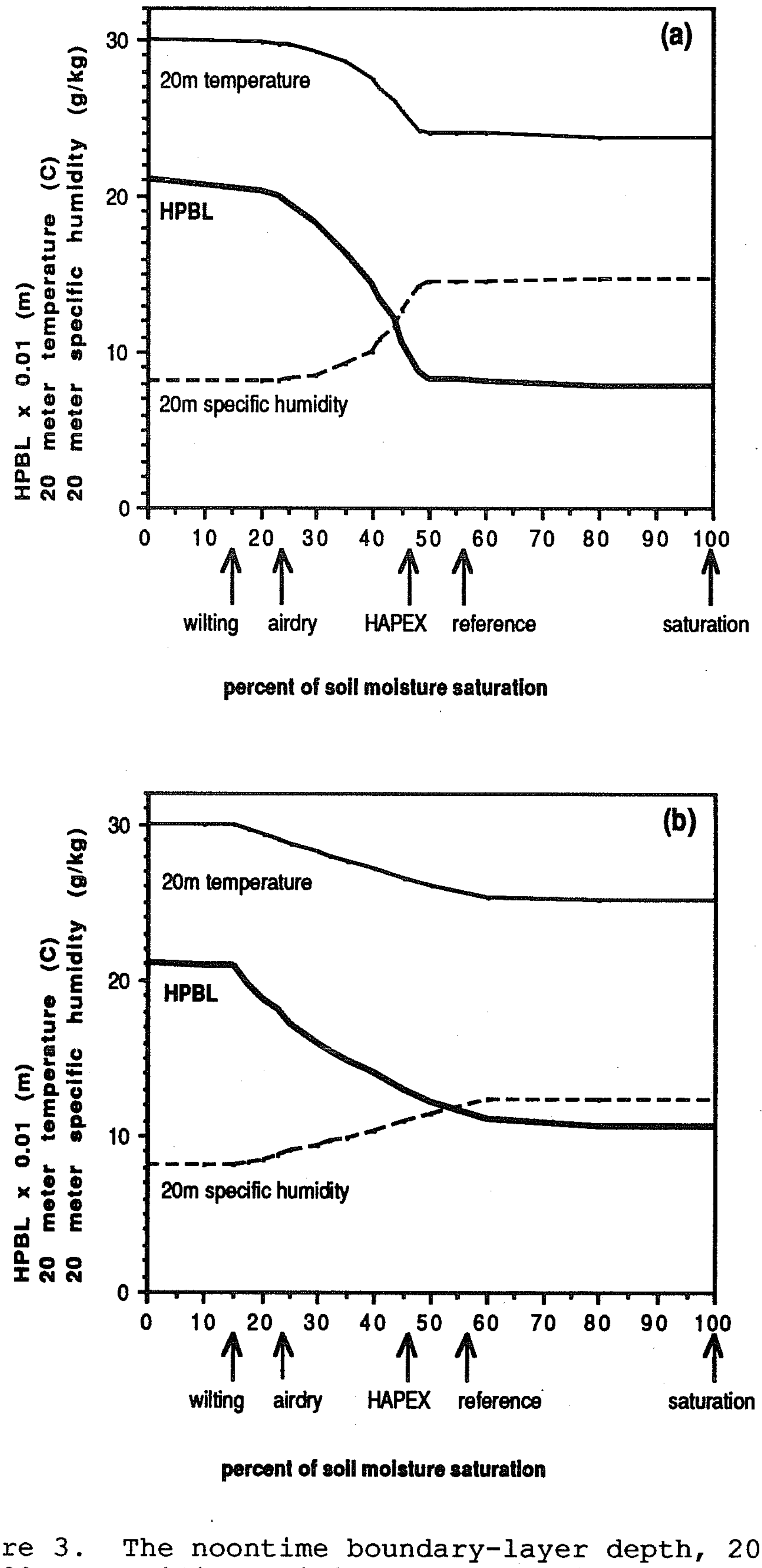

Figure 3. The noontime boundary-layer depth, 20-m temperature, and 20-m specific humidity as a function of soil moisture content for a one-dimensional model simulation of bare soil conditions (a) and complete canopy cover (b).

The inducement of inhomogeneity due to spatial variations of surface evapotranspiration has been generally overlooked. Figure 3 shows the sensitivity of surface air temperature, specific humidity and boundary-layer depth to soil moisture content in the model of Pan and Mahrt (1987) with other conditions specified to be those of 19 May 1986 during the HAPEX. For intermediate ranges of the soil moisture, the modelled evapotranspiration is sensitive to changes of soil moisture so that significant spatial variations of evapotranspiration might be expected.

4. CONCLUDING REMARKS

The formulation of subgrid scale fluxes must necessarily be crude partly because of the neglect of transport by subgrid scale motions which are larger than turbulent scales. As a result, use of sophisticated similarity relationships for the dependence of fluxes on stability does not seem justified. Our HAPEX work will attempt to distinguish between fluxes due to the main turbulent eddies and transport related to surface inhomogeneity and other subgrid scale circulations which occur on scales larger than the main turbulent eddies.

REFERENCES

Brown, P. R. A. 1983: Aircraft measurements of mountain waves and their associated momentum flux over the British Isles. *Quart. J. Roy. Meteor. Soc.*, **109**, 849-865.

Greenhut, G. K., 1981: Analysis of aircraft measurements of momentum flux in the subcloud layer over the tropical Atlantic ocean during GATE. *Bound.-Layer Meteor.*, **20**, 75-100.

Lenschow, D. H., and B. B. Stankov, 1986: Length scales in the convective boundary layer. *J. Atmos. Sci.*, **43**, 1198-1209.

Lilly, D. K., 1967: Techniques for the study of mountain waves. *Facilities for Atmospheric Research*, Boulder, Colo., **4**, 12-16, 1967.

Lilly, D. K., J. M. Nicholls, R. M. Chervin, P. J. Kennedy, and J. B. Klemp, 1982: Aircraft measurement of wave momentum flux over the Colorado Rocky Mountains. *Quart. J. Roy. Meteor. Soc.*, **108**, 625-642.

Louis, J.-F, 1979: A parametric model of vertical eddy fluxes in the atmosphere. *Bound.-Layer Meteor.*, **17**, 187-202.

Mahrt, L., 1987: Grid-averaged surface fluxes. *Mon. Wea. Rev.*, **115**, 1550- 1560.

Nicholls, S., 1985: Aircraft observations of the Ekman layer during the Joint Air-Sea Interaction Experiment. *Quart. J. Roy. Meteor. Soc.*, **111**, 391-426.

Pan, H.-L. and L. Mahrt, 1987: Interaction between soil hydrology and boundary-layer development. *Bound.-Layer Meteor.*, **38**, 185-202.

Sud, Y. C., and W. E. Smith, 1984: Ensemble formulation of surface fluxes and improvement in evapotranspiration and cloud parameterizations in a GCM. *Bound.-Layer Meteor.*, **29**, 185-210.

Sud, Y. C., and W. E. Smith, 1985: The influence of surface roughness of deserts on the July circulation (A numerical study). *Bound.-Layer Meteor.*, **33**, 15-49.

Wyngaard, J. C., W. T. Pennell, D. H. Lenschow, and M. A. LeMone, 1978: The temperature-humidity covariance budget in the convective boundary layer. *J. Atmos. Sci.*, **35**, 47-58.