

THE PARAMETRIZATION OF MOMENTUM FLUXES IN THE FREE ATMOSPHERE

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For the proceedings of this seminar a distinction has been made between the fluxes of heat and moisture due to subgridscale processes and fluxes of momentum and vorticity. Although this distinction is somewhat artificial, there are basic theoretical arguments which indicate that the larger-scale flow responds on distinctly different space and time scales to mechanical or thermal forcing. Much work on geostrophic adjustment, data assimilation theory and model forcing experiments has confirmed the importance of mechanical forcing in atmospheric circulations on a variety of scales, and recent studies of mesoscale phenomena such as squall lines, rainbands etc. underline the need for improved parametrizations of their momentum as well as thermodynamic fluxes. A good reference for some of the theoretical appreciation of atmospheric response to mechanical/thermal forcing can be found in Wergen (1983). He applied data assimilation theory to extend geostrophic adjustment results to P.E. models, and emphasizes the importance of both the horizontal and vertical scales of the forcing as well as its latitude. Both Wergen (1983) and Mohanty (personal communication of recent results) showed that model systematic errors were reduced in forcing-type experiments either with thermal or mechanical forcing but were most reduced by a combination of both.

This paper will attempt to discuss the principal physical processes which effect dynamical transports on scales which generally require parametrizing and the extent to which our understanding enables us to design parametrization schemes. Where possible numerical model results will be used to demonstrate the impact/sensitivity.

An outstanding problem in the whole research area of dynamical subgridscale transports is the enormous difficulties of observational verification. While difficulties also exist for heat and moisture fluxes there are at least observable integrated quantities such as precipitation.

1. BASIC PROCESSES

The basic primitive equations suitably averaged for a large-scale model grid contain horizontal and vertical flux divergences of subgridscale fluxes. In general all horizontal derivatives of subgridscale terms are treated as 'horizontal' diffusion and the vertical derivatives are parametrized as a number of processes. Clearly the horizontal divergences

of momentum contribute nothing to a global integral whereas the vertical term provides the total surface stress. For the corresponding kinetic energy equation, all subgridscale terms represent sinks of KE i.e. dissipation, provided they are sensibly formulated. The possibility that convective transports might be a net source of KE will be referred to later.

An approximately chronological list of momentum flux parametrizations as included in numerical models is listed below:

- Surface friction
 - 'Horizontal diffusion'
 - 'Vertical diffusion'
 - Convection
 - Orographic gravity waves
 - Other gravity wave sources
 - Slantwise adjustments
 - 3-D potential vorticity
- } Current research

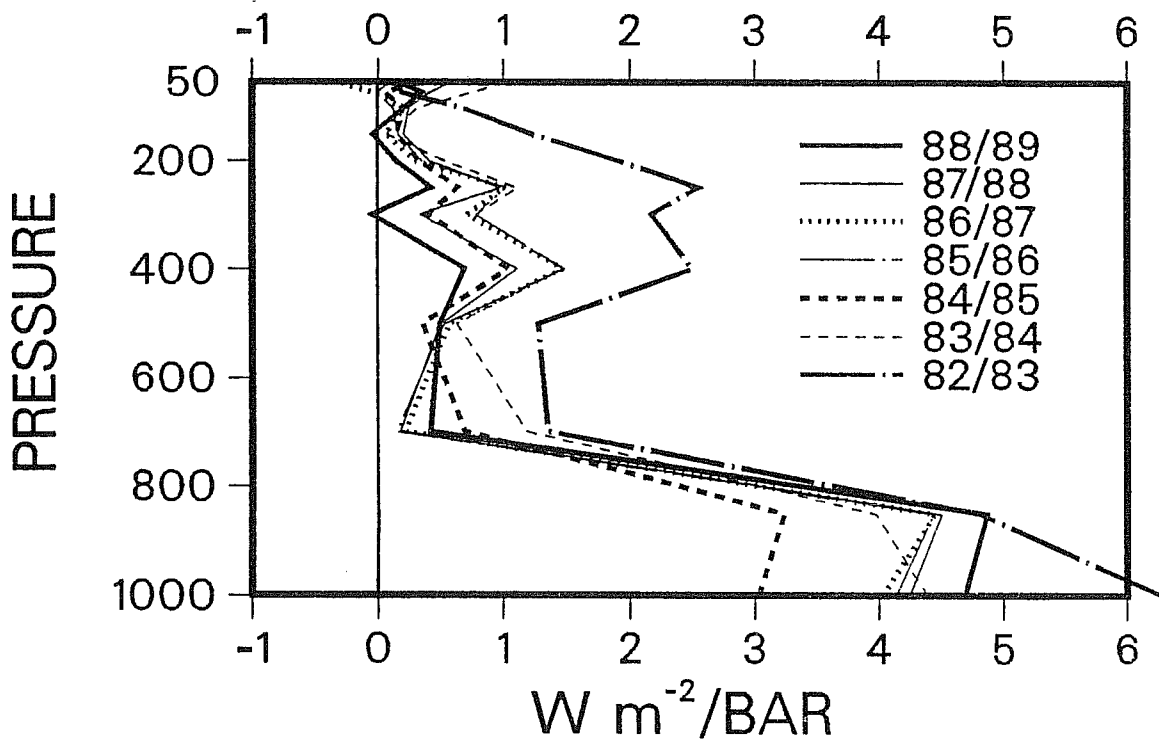
Historically surface friction and diffusion were included as fundamental for numerical stability. The more recent inclusions are the result of increasing model resolution and sophistication encouraged by the undoubted sensitivity of models to momentum flux parametrizations.

Before considering the various parametrization schemes we can examine some evidence of global dissipation from budget studies. Fig. 1 shows the vertical profiles of KE dissipation deduced as a residual for recent winters using ECMWF analyses. It is clear that while there is evidence of a dissipation maxima in the upper troposphere, the inter-annual variability is large with a tendency for less free atmosphere dissipation most recently. To what extent the assimilating model used in the analyses influences these results is not well understood but would seem to be significant. It is worth commenting that although the global dissipation rates are only a few percent of thermodynamic sources/sinks, they are comparable when normalized by the appropriate energy itself. In the present operational model the global dissipation is apportioned roughly two-thirds to vertical diffusion (including surface friction) and the other third shared between cumulus 'friction' and horizontal diffusion (with GWD contributing ~ 5% to the total).

2. GRAVITY WAVE DRAG

The subject of the momentum fluxes and pressure forces associated with orographically excited gravity waves (so-called 'gravity wave drag' (GWD)) has been a particularly popular one in the last few years. It has of course a very much longer history and the problems

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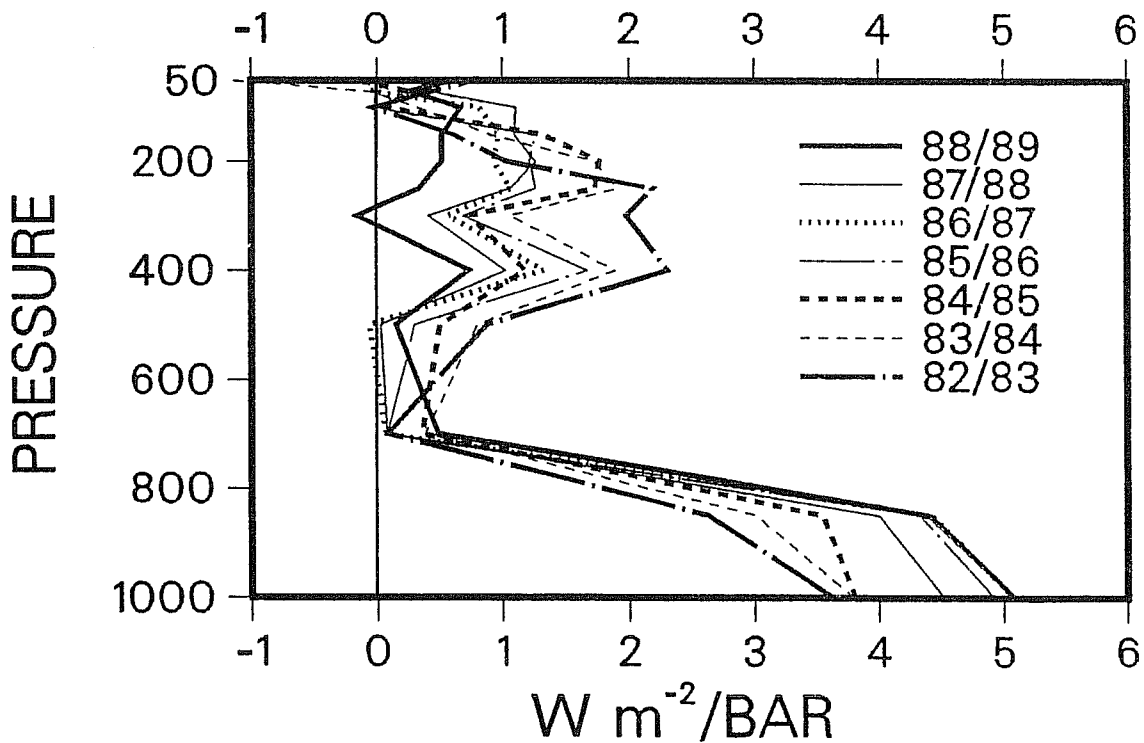


Fig. 1 Budget estimates of global dissipation rates as a function of height for the D+10 operational forecasts (top) and the analyses for the past seven winters.

and prospects of including in models parametrizations of the subgridscale fluxes due to gravity waves were discussed by Sawyer (1959), Bretherton (1969) and Lilly (1972). They showed that the GWD might be a significant component in the atmospheric momentum budget and locally a very much larger stress than the frictional one.

However it was not until this decade that both Canadians and the UKMO separately included a parametrization scheme of GWD. Since then many other modelling centres have followed suit. [This activity followed the related work on envelope orography (e.g. Wallace et al. 1983)]. There are many papers now published describing all aspects of GWD parametrizations and their impact in models; a list of references and discussion of problems and philosophy can be found in Miller et al. (1989). In particular these authors show how the improving horizontal resolution of NWP models has amplified the problem of systematically excessive midlatitude westerly flow in the Northern Hemisphere winter. They argue that the need for GWD therefore only emerged when the larger scale eddy momentum fluxes were adequately described by the models (resolutions \geq T42 for example). By studying the vertically-integrated momentum budget, Palmer et al. (1986) argued that larger surface torques were required to balance the observed eddy momentum fluxes either by increasing the frictional stress or the mountain contribution. Fig. 2 shows an example of the mean winter circulation with a standard mean orography and the observed 'climate'. Also shown is the corresponding result with an envelope orography and with this combined with a GWD parametrization scheme. Previous references describe the details of such schemes and these will not be repeated here. The basic issues however are the formulation of the gravity wave stress itself and its vertical distribution. Fig. 3 summarizes the principles of the scheme in use at ECMWF up until May 1989, based on Palmer et al. (loc. cit) with enhancements described in Miller et al. (loc. cit), labelled 'old'.

With the inherent difficulties in measuring the spatial distribution and magnitude of gravity wave momentum fluxes over mountain complexes, the possibility of using very high resolution models to simulate the detailed flow field over mountainous area has been studied by several workers (e.g. Carissimo et al., 1988; Hoinka and Clark, 1990; Clark and Miller, 1990). Of particular relevance is the resolution study of Clark and Miller (loc. cit.) who computed the net drag of the Alpine complex for a range of horizontal resolutions down to 5 kms. They showed that the drag at 80 km resolution was about one half of the value at the highest resolutions (Fig. 4). This 'unresolved' component of the drag is what is currently incorporated via a GWD parametrization scheme. They also showed the vertical profiles of momentum flux for a specific case and concluded that a significant fraction of the momentum fluxes is absorbed at low levels. The interrelationship between resolved and parametrized drag is shown in Fig. 5 provided by A. Simmons (personal communication). It can clearly be seen how the two contributions complement each other at differing resolutions.

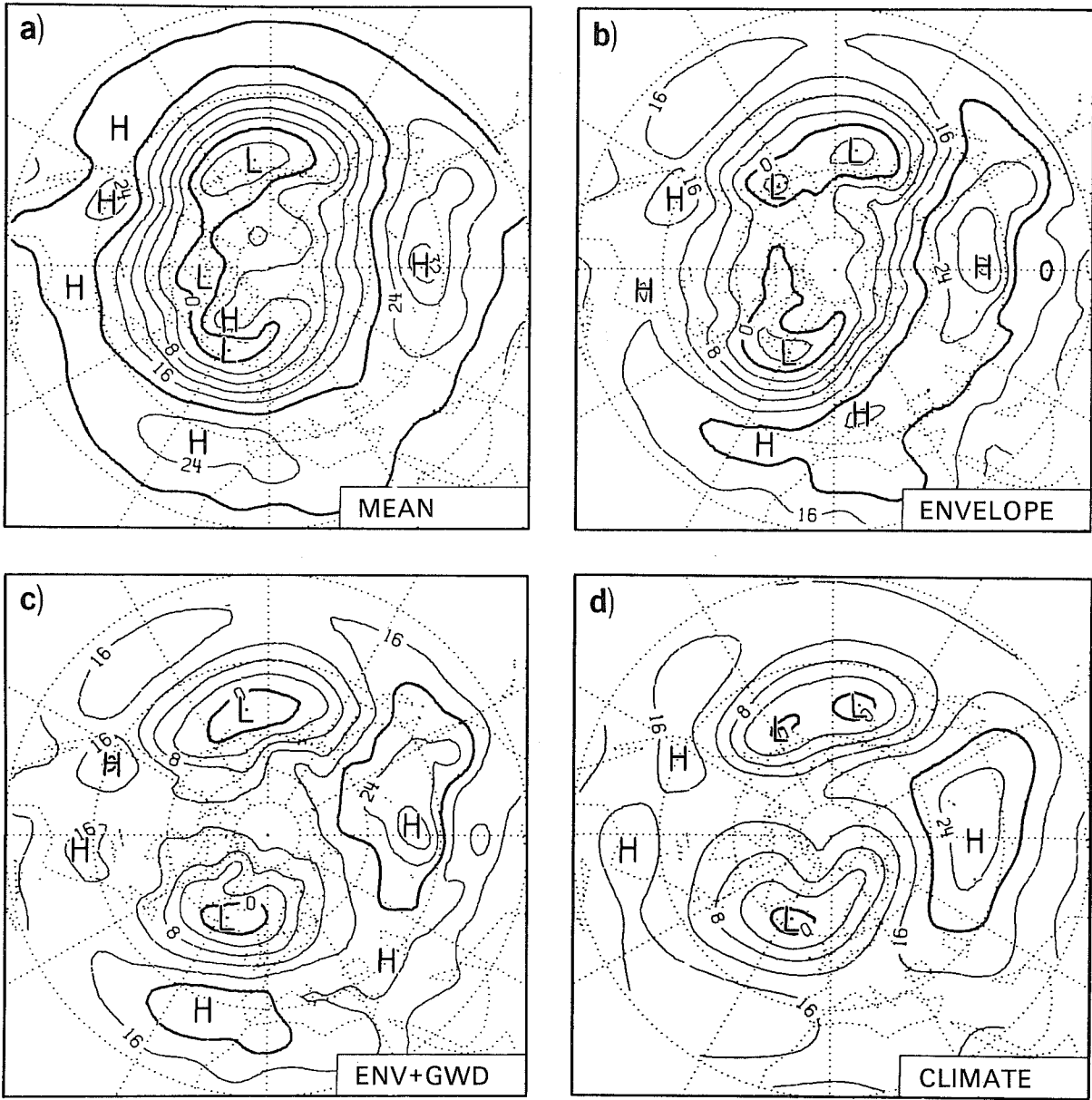


Fig. 2 T42 90 day integrations (initial date 1/12/83) and the corresponding climate at 1000 mbs. a) using mean orography; b) envelope orography; c) envelope orography plus GWD, d) climate.

Gravity Wave Drag Scheme and Revised Gravity Wave Drag Scheme

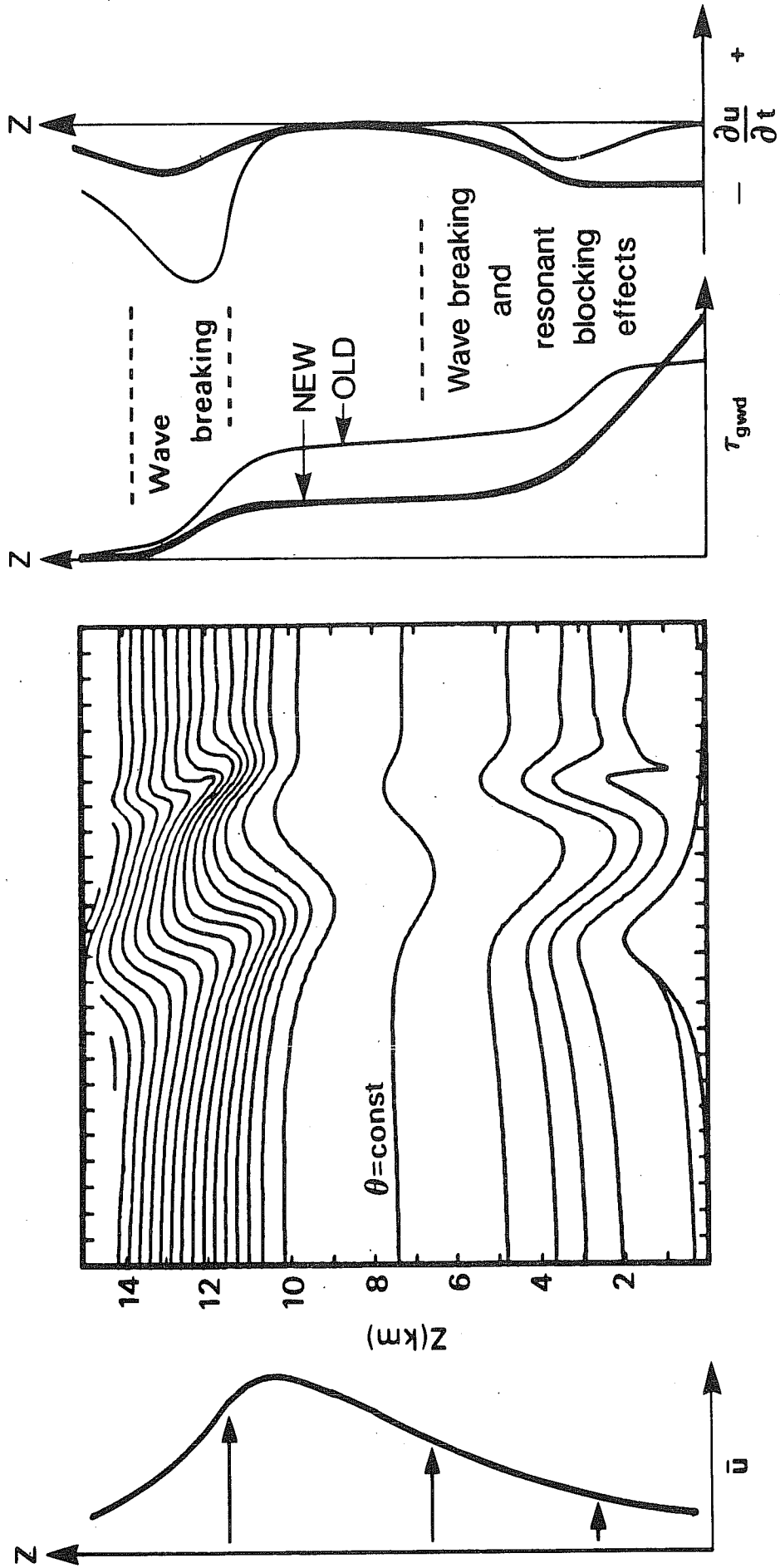


Fig. 3 Schema of original GWD scheme (marked OLD) and revised scheme (marked NEW).

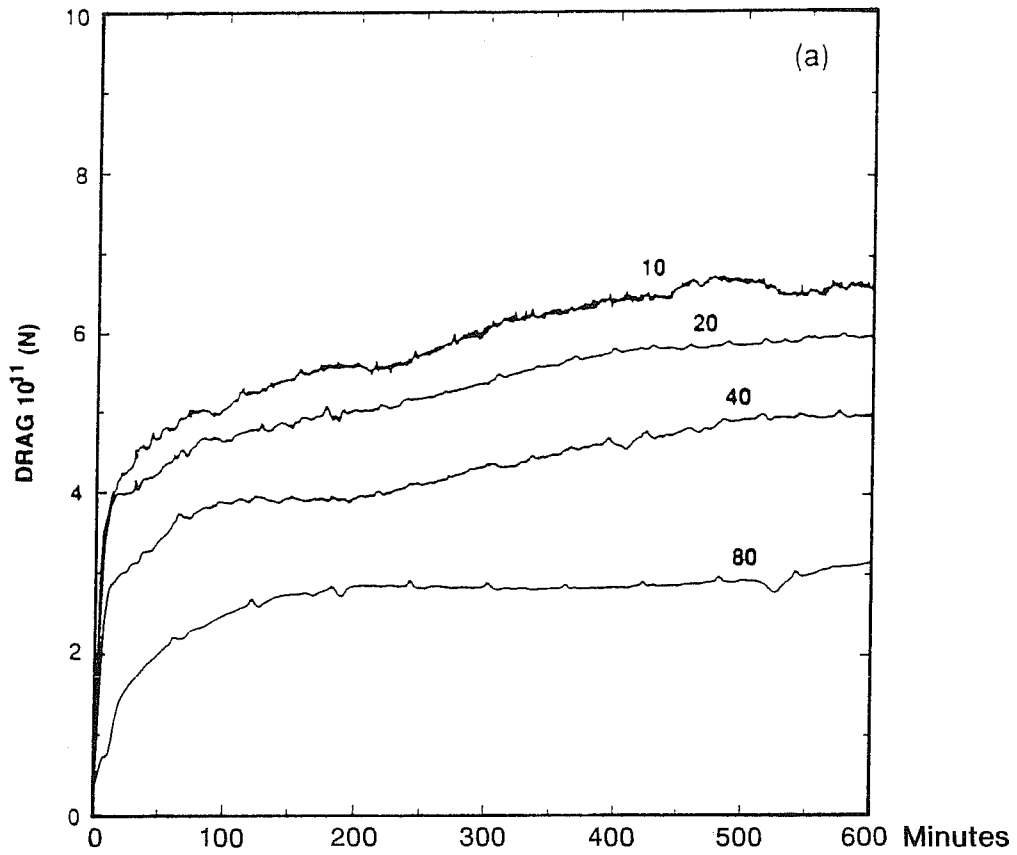


Fig. 4 Evolution of the net drag of the Alps for model resolutions of 80, 40, 20 and 10 kilometres (from Clark and Miller, 1990).

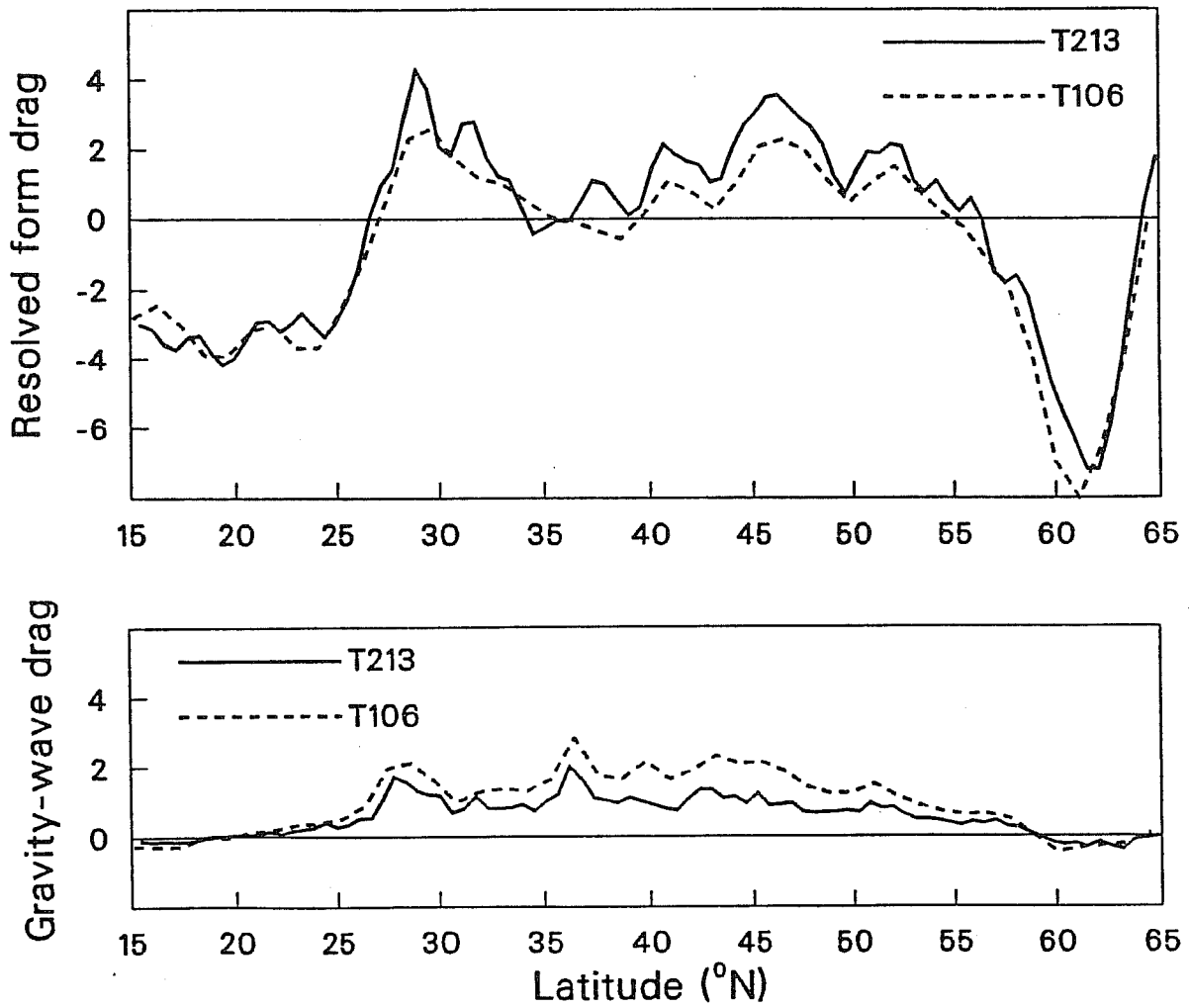


Fig. 5 Zonally averaged drag (resolved form drag and gravity wave drag) for T106 and T213 resolutions.

2.1 The impact of GWD parametrizations

Inclusion of a wave drag scheme in forecast or extended integration models has a marked effect on the mean zonal flow and the stratospheric temperature distribution. These improvements lead to much better baroclinic eddy statistics and Fig. 6 from Palmer et al. (1990) shows the reduction in eddy momentum flux errors due to the inclusion of GWD. Medium-range forecasts show a very systematic improvement in forecast skill with virtually all forecast experiments showing some positive impact, Fig. 7 shows the ensemble scores for a set of forecasts spread through the year, and hence including some summer cases when GWD is a minimum. An example of the way in which improving the upper tropospheric flow can dramatically affect baroclinic development can be found in Simmons and Miller (1987). The dramatic impact of GWD on 'climate-type' simulations has already been seen in Fig. 2. Many other similar examples can be found in the literature e.g. (Slingo and Pearson, 1987). These show the direct benefit of reducing the westerly bias which results in extension of the winter storm tracks deep in to the continents, and poor modelling of low frequency variability. The impact of including the GWD scheme at ECMWF on model systematic errors can be found in these proceedings (Arpe).

2.2 Recent improvements in GWD parametrization

Despite the benefits obtained from the inclusion of a GWD scheme in the Centres model, longer (90 day) integrations indicated that with this version of the scheme the wave drag at upper levels of the model was excessive. This was confirmed more recently by the one-timestep budget calculations of Klinker and Sardeshmukh (1987).

At least two processes are known to be important in confining wave drag to the lowest few kilometres of the atmosphere. Firstly, under realistic flow conditions, gravity wave activity could be trapped below a steep 'inversion' in the Scorer parameter. The lower atmosphere would act to duct wave energy downstream where it would (ultimately) be dissipated (Bretherton, 1969). In this case, drag would not necessarily arise through convective instability of the gravity wave. Such an inversion in the Scorer parameter appears to be a climatological feature of observed extratropical flow over orography. Secondly, in situations where the Froude number (Nh/U) is $O(1)$, nonlinear gravity wave resonance can give a massive amplification in low level drag (e.g. Peltier and Clark, 1979).

Indeed, even in situations in which gravity-wave radiation may be small, the drag on the atmosphere associated with buoyancy forces can be substantial. For example, for sufficiently steep mountains, semi-geostrophic theory predicts upstream damming of cold air which is subsequently released as a 'weir' across the lee side (Shutts, 1985). Local dissipation of energy in this weir would again be responsible for low level drag.

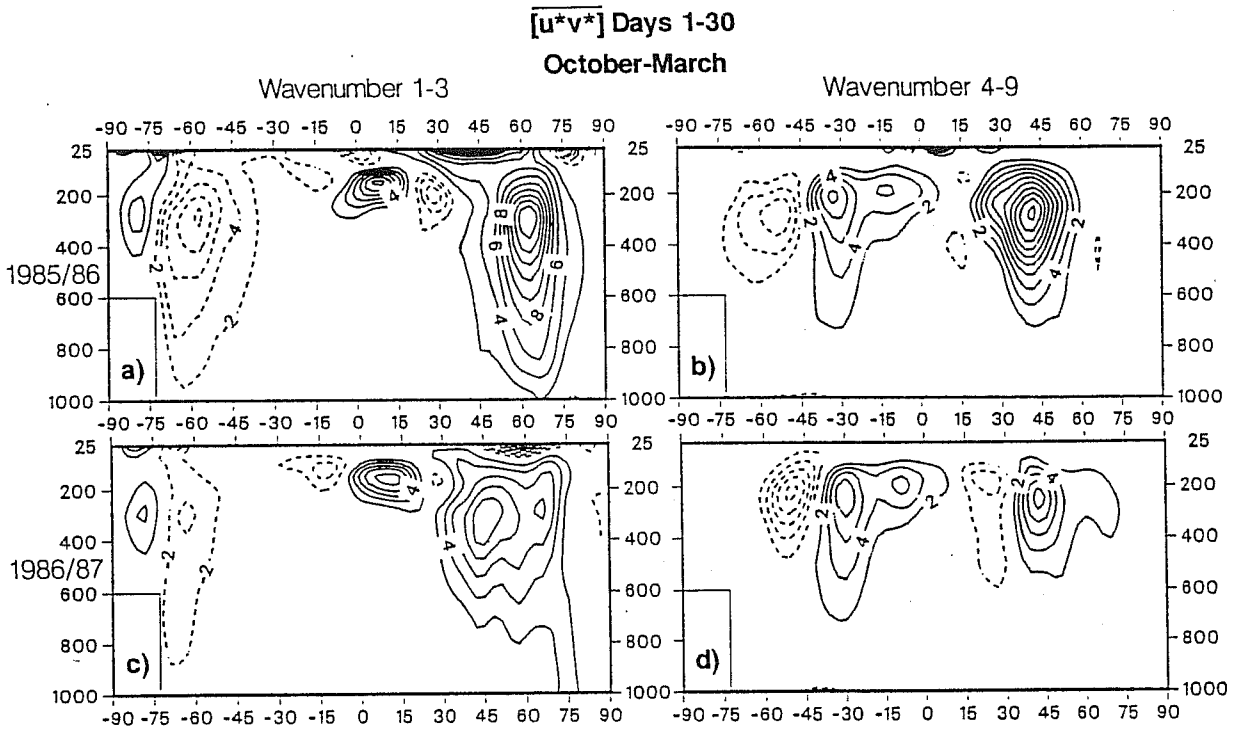


Fig. 6 30 day mean zonal cross sections of horizontal momentum flux error of 30-day T106 forecasts from the winters of 1985/86 (no GWD in model) and 1986/87 (GWD included). (From Palmer et al., 1990).

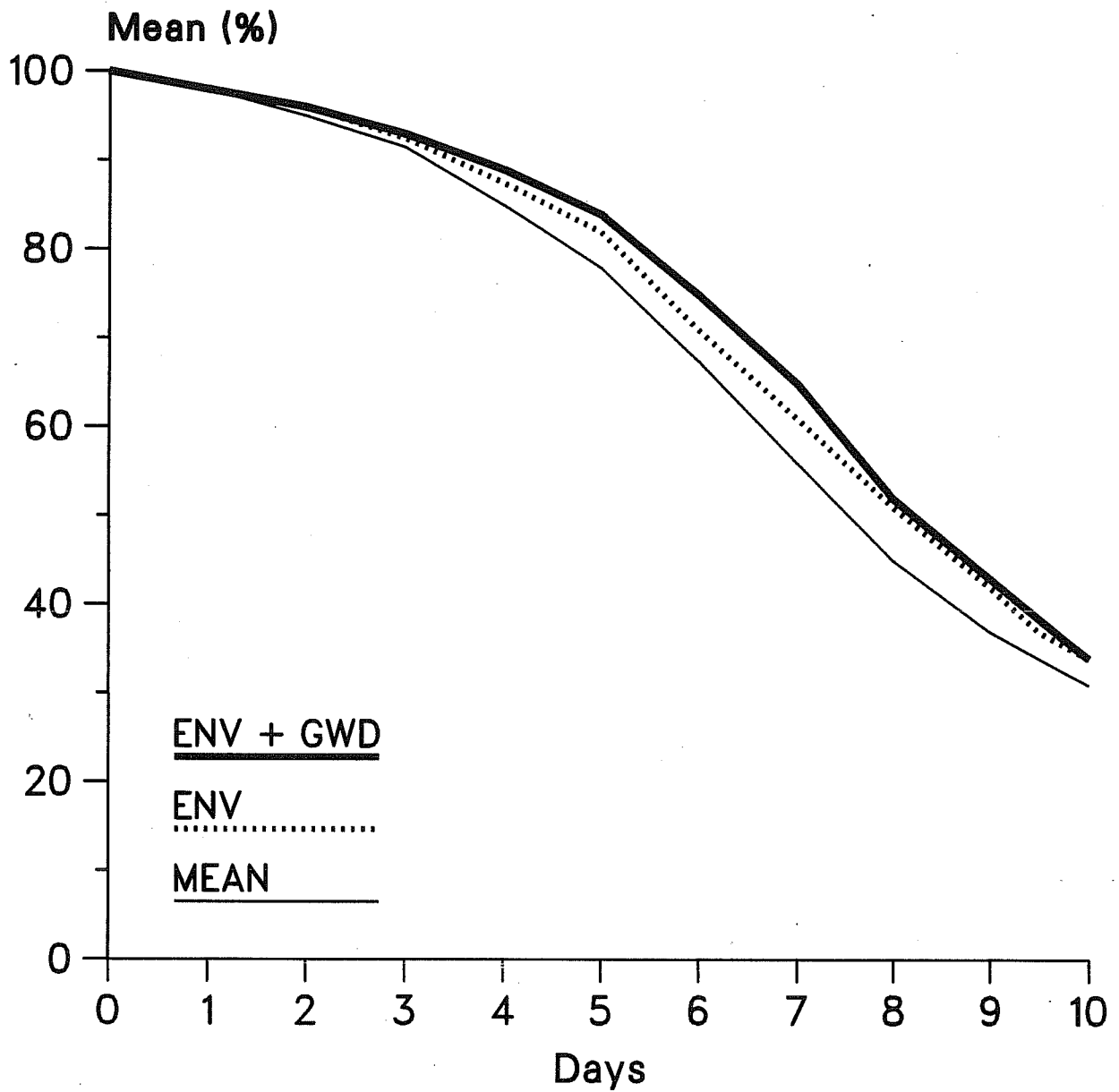


Fig. 7 Mean anomaly correlation of 500 mb height for the N. Hemisphere for 12 T106 forecasts spread through the year and with mean or envelope orographies and also with envelope plus GWD.

Whilst it is clear that further research is needed to address these issues satisfactorily, the scheme has been modified to take cognisance of these processes.

The two main modifications are the inclusion of additional stress term when the low level Froude number is large and revisions to the lower tropospheric stress profile to ensure that a larger fraction of the stress is absorbed at low levels. Fig. 3 summarizes this change schematically.

The modified scheme showed further systematic (but more modest) improvement in medium-range forecast skill and larger impacts on extended integrations (e.g. Fig. 8).

3. VERTICAL DIFFUSION

Vertical transports of momentum (and heat etc.) due to turbulence resulting from instabilities of various kinds in the free atmosphere undoubtedly exist and there is a considerable literature documenting the occurrence of turbulence detected by aircraft and radiosondes. Some of these papers try to evaluate the magnitude of the fluxes others the frequency of occurrence, but inevitably the global frequency and extent of 'free-atmosphere' turbulence are poorly appreciated. In particular the importance of 'clear-air' turbulence not linked to underlying orography or convection is unclear. The mechanisms of Kelvin-Helmholtz and inertial instability certainly generate very thin, possibly large (in the horizontal), regions of vertical mixing which have been parametrized only very crudely.

Until January 1988 the PBL vertical diffusion scheme operated not only in the PBL but anywhere in the model. Because of this and the manner in which the diffusion coefficients were formulated in stably stratified conditions, large vertical momentum (and heat) fluxes were modelled in the upper troposphere and stratosphere with excessively large dissipation rates. Significant erosion of the jets occurred and considerable loss of eddy kinetic energy on all scales resulted. The scheme could generate diffusion coefficients of several hundred metres squared per sec. whereas aircraft measurements were typically 5-10 times smaller. Since January 1988 the scheme has been revised to restrict the vertical diffusion to below a diagnosed PBL top with vertical diffusion above this only if static instability is generated. The absence of upper vertical diffusion is in accord with some other models (e.g. UKMO, but see later) and the sensitivity seen in our experimentation would suggest that a proper dynamically-based representation of clear-air turbulence might benefit the models.

3.1 Impact of Jan 88 revision

Confining the vertical diffusion to the PBL had a marked impact on the models systematic errors as well as individual forecasts. Figs. 9-11 complement the discussion on systematic

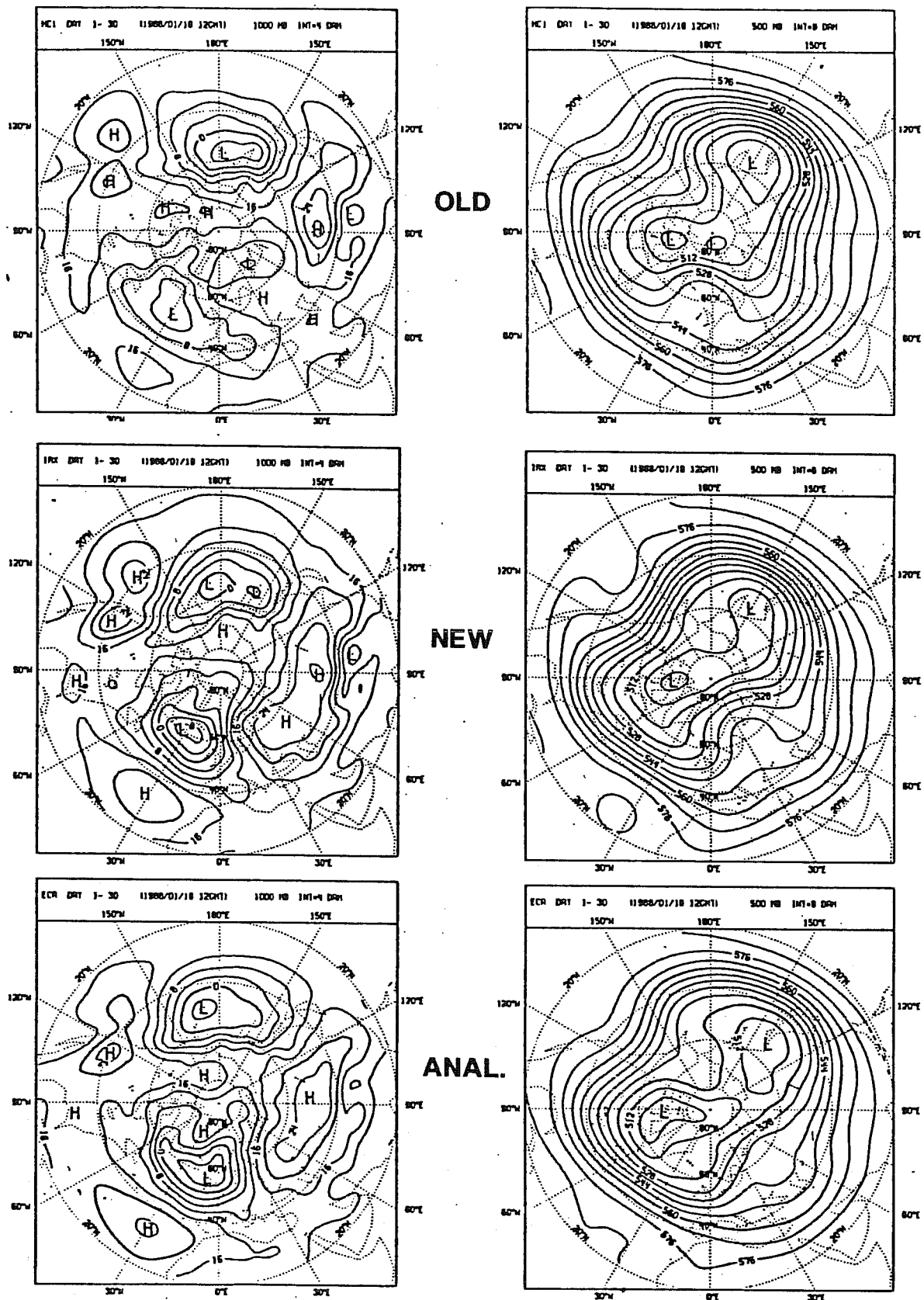


Fig. 8 30 day mean maps of 1000 and 500 mb heights from two 30 day T63 winter simulations with the original and new gravity wave drag schemes plus mean EC analyses.

errors by K. Arpe elsewhere in this proceedings. All three figures refer to results from the Centre's 30-day forecast programme described in Palmer et al. (1990) from which these figures are reproduced. Because of the nature of this forecast programme the results do not represent a totally 'clean' impact of the VDIFF change but this was the only significant change during the winter of 1987-88. (Note that the 30-day forecasts for Oct-Dec 1987 included the VDIFF change).

Fig. 9 demonstrates the marked improvement in levels of eddy KE resulting from this change; and Figs. 10-11 show that there is a corresponding reduction in systematic errors on a geographical basis. Although inter-annual variability influences these results, the magnitude of the error reduction is such as to leave little doubt as to the positive impact of the VDIFF revision.

Further clear evidence of the benefit of this revision can be seen in Fig. 12. This shows the zonal mean momentum budget residual for January 1987 and January 1989. (See Klinker and Sardeshmukh 1987 for details of this diagnostic technique). Of particular note is the virtual removal of large errors in the tropical stratosphere and upper troposphere.

Although the reduction of vertical diffusion in the upper troposphere and stratosphere has proved highly beneficial, it also exposed a numerical 'noise' problem previously controlled by the vertical diffusion. At these high levels, particularly in the Tropics, two gridlength 'waves' of locally large amplitude distort the fields. A similar problem was noted in the UKMO forecast model and controlled by the inclusion of a simple vertical diffusion restricted to Tropical latitudes (Bell and Dickinson, 1987).

4. CONVECTIVE MOMENTUM TRANSPORTS

The preceding discussions on GWD and vertical diffusion have pointed out that there are some uncertainties in both the physical mechanisms and parametrizations. In the case of convective momentum transports the levels of uncertainty both in dynamical/physical terms and parametric terms are such as to ensure a high degree of controversy. As discussed earlier, there is potential sensitivity of small-scale systems in the Tropics to momentum forcing, however the fundamental difficulties (particularly with the pressure field) in determining from observations whether convective momentum transports are important have meant that no definitive evidence has been forthcoming. In the Tropics there have been several studies which compute the convective momentum fluxes for subsynoptic scale systems as a non-negligible term, whereas no clear evidence has been produced for extra-tropical systems. Global scale budget studies have concluded that these convective fluxes are not a significant term in large-scale momentum or vorticity balances. The possibility of diagnosing cloudscales momentum fluxes directly from very high resolution non

KE Days 1-30 October-March

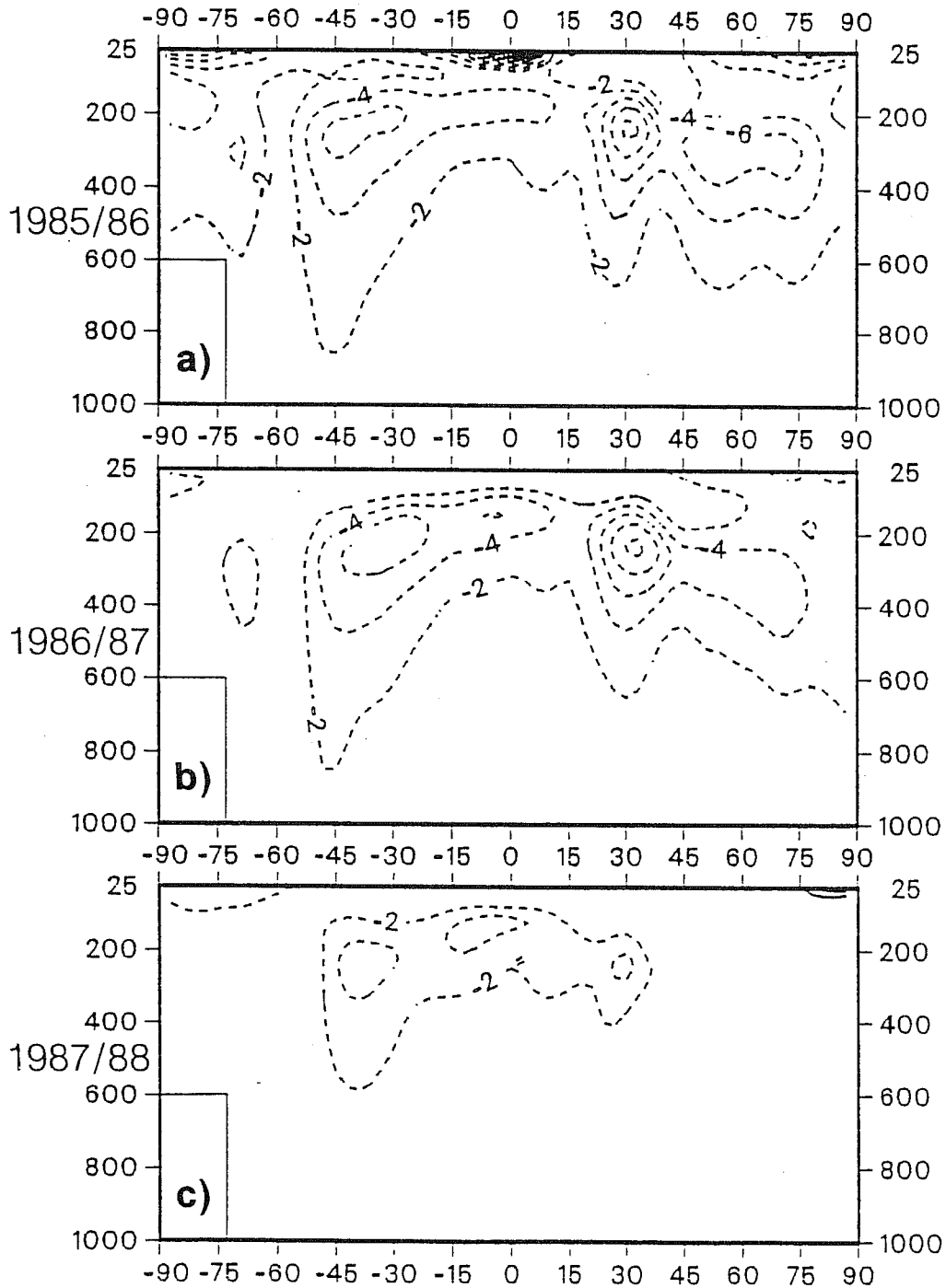


Fig. 9 As for Fig. 6 but for eddy kinetic energy error, for three winters 85/86 and 86/87 both with original VDIFF and 87/88 with revised VDIFF.

\bar{Z} 500 mean error
30-day mean October-March

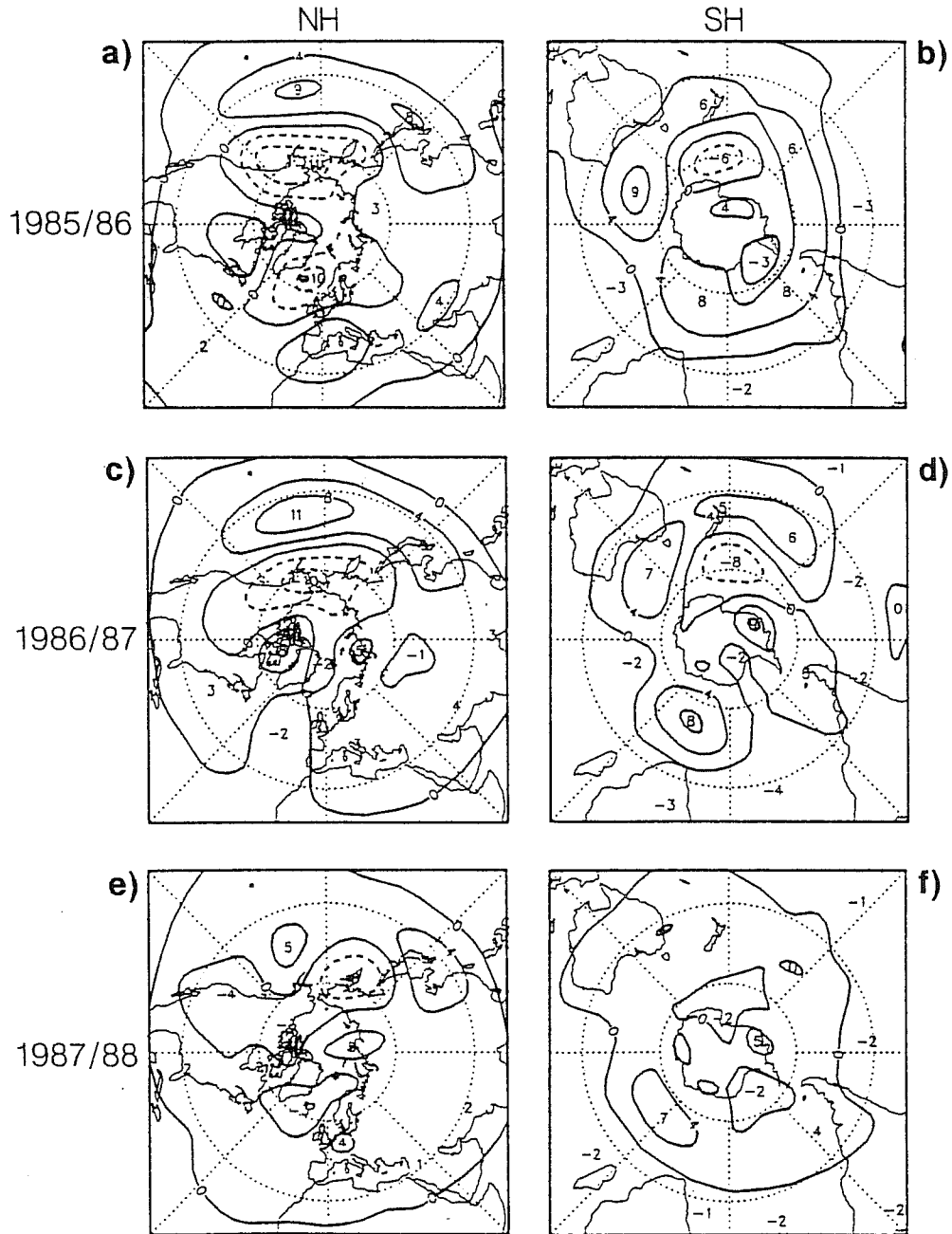


Fig. 10 30 day mean error of 500 mb geopotential height in the extratropics. Same set of forecasts as Fig. 9.

\bar{Z} 500 error ratio
Days 21-30 mean; October-March

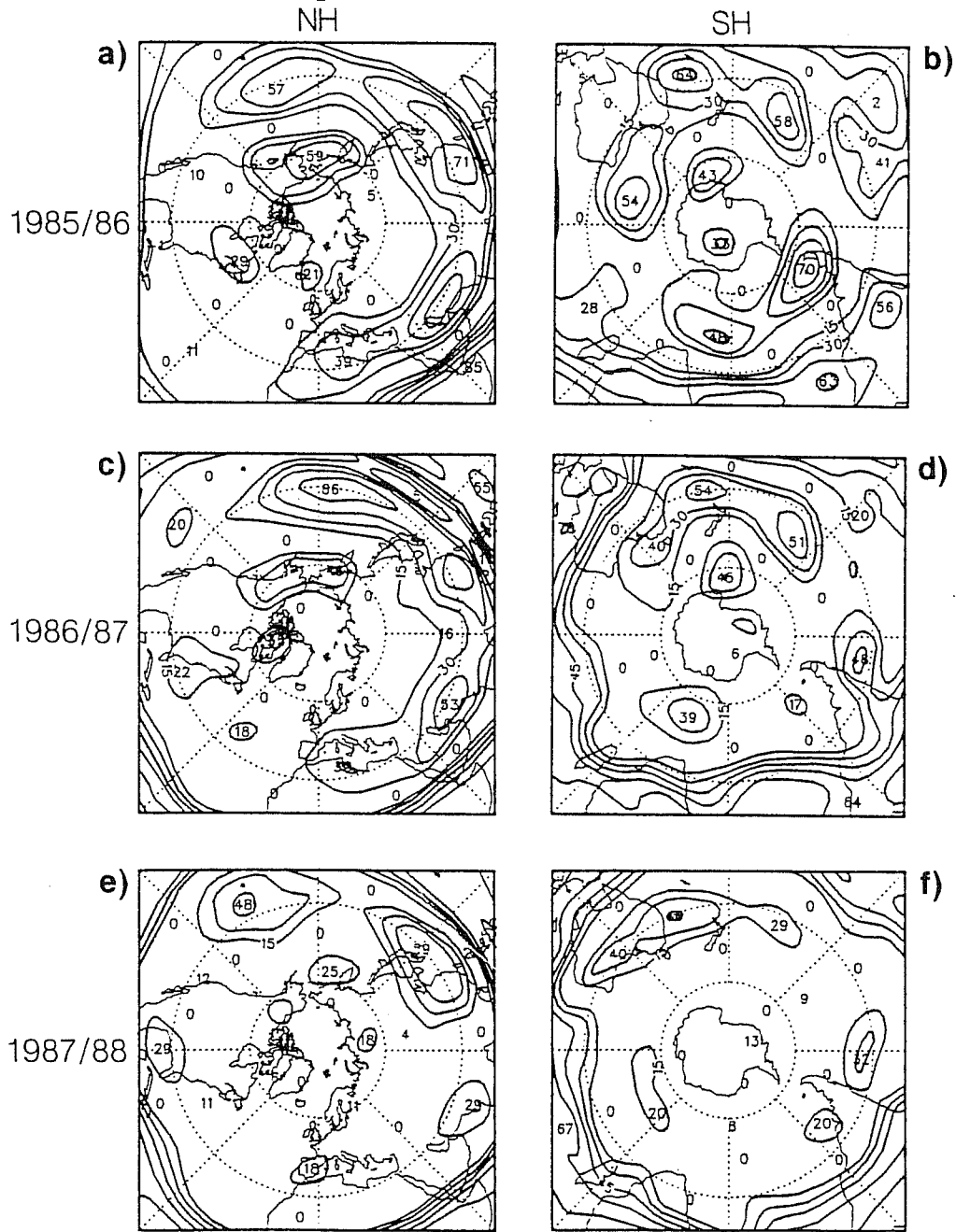


Fig. 11 Ratio of systematic to daily mean square error of 500 mb geopotential height in the extratropics. Computed using days 21-30 of the forecasts as in Fig. 9. (Contours every 15%).

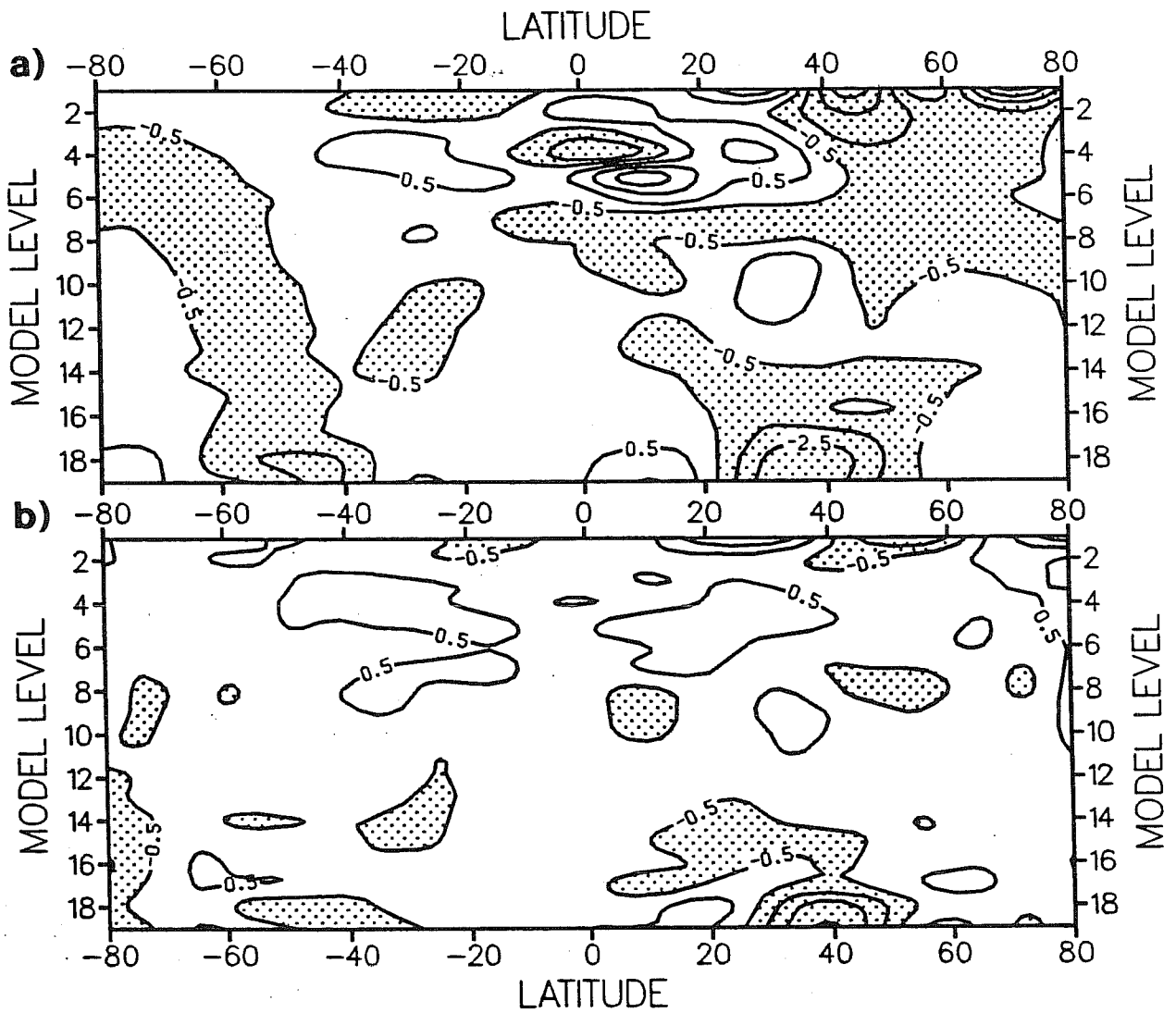


Fig. 12 Zonal mean momentum budget residual for a) Jan 1987 (original VDIFF), b) Jan 1989 (revised VDIFF). (See Klinker and Sardeshmukh, 1987 for details of calculations).

hydrostatic models is an attractive possibility only just becoming practical. Gregory and Miller (1989) used a two-dimensional non-hydrostatic model of clouds with grid length of about one kilometre to examine a tropical cloud field in a 256 km domain. By studying simulations with and without vertical wind shear they were able to diagnose the terms in the momentum budget and the extent to which these were parametrizable. They concluded that the cloudscale momentum fluxes were relatively small and difficult to represent in conventional parametric forms. One such simple parametric form was formalised by Schneider and Lindzen (1976), who, by analogy with thermodynamic parameters, assumed that the horizontal momentum of a parcel entering through cloudbase was conserved (or only modified by entrainment). The relevant momentum fluxes can then be computed. Attempts to verify the validity of this over-simplification have had mixed results. Clearly momentum is not a conserved quantity and evidence from cloud models shows that the horizontal velocity of air parcels in the cloud rapidly adjust towards the environment values through the perturbation pressure field. Gregory and Miller (loc. cit.) found that in-cloud parcels had horizontal velocity differences of only a few metres/sec at most. It can readily be appreciated that this must be so in strongish vertical wind shear otherwise air parcels cannot remain in the cloud during their ascent in the updraught. Also no account is taken of cloud motion and updraught slope in such a simple approach. Unfortunately, more sophisticated dynamically-sound theories for convective momentum transports as described by Moncrieff (e.g. 1985) do not yet provide any usable parametric formulations for a general range of convective situations.

Windshear and stability often organize the convection into streets or rolls, or mesoscale clusters and squall-lines. When this occurs, fluxes of all quantities can be significantly different from those of the 'statistically random' concept of cloud fields. This is especially so for momentum fluxes and no account is currently taken of this type of large-scale modulation in convective behaviour.

For slantwise convection the conservation properties of the so-called 'absolute momentum' enable a more dynamically sound formulation for convective momentum transport, this implies a mutual adjustment of mass and wind via semi-geostrophic theory, however the extent to which such an approach can be applied to a wider range of convective phenomena is not known.

4.1 Recent experience at ECMWF

With the introduction of a mass-flux scheme (Tiedtke, 1989) to replace the Kuo scheme, a Schneider and Lindzen formulation for momentum transports was incorporated. By running forecast experiments and analyses with and without these convective transports and also

using budget residual techniques, it has been possible to study their impact. The following conclusions can be drawn:

- (i) As formulated in the ECMWF model these fluxes can sometimes have a significant impact on both the forecast quality and model systematic errors.
- (ii) This impact can be beneficial or detrimental depending on area, season, scale etc. Figs. 13-15 show examples of this. As expected, tropical storms are sensitive to the momentum fluxes, and the tropical upper tropospheric flow is also sensitive at least in N.H. summer. Similar or larger impacts on the tropical mean flow can also be achieved by changing the diabatic heating, and it remains unclear to what extent the apparent beneficial impact is for the right reasons.
- (iii) As parametrized, the convective momentum fluxes are large not only in convectively active tropical regions but in the extra-tropics where the windshears are generally much larger. For extra-tropical medium-range forecasts there is considerable variability in impact from case to case which adds to the problems of understanding the role of these momentum fluxes.

5. HORIZONTAL DIFFUSION

Finally in this discussion of subgridscale momentum fluxes is the subject of horizontal or more correctly 'quasi-horizontal' fluxes. Their inclusion in models is demanded by numerical stability requirements and the need to control the build-up of energy at the smallest resolved scales through aliasing and the non-linear energy 'cascade', but a physical basis for the form of the parametrized motions is generally lacking particularly in larger-scale models. Typically an harmonic or bi-harmonic diffusion term with constant diffusion coefficient is used, at best a coefficient dependent on local deformation or estimate of turbulent KE. The coefficients are often different for different variables such as vorticity and divergence. Studies of the sensitivity of models to the formulation and magnitude of "eddy-diffusion" terms have been carried out at many centres. In general as model resolutions have improved, the relative importance of such terms has diminished, although many models have uncomfortably large implied damping rates of the smallest resolved scales. At lower resolutions a well-tuned diffusion-type formulation can significantly improve a models overall characteristics (e.g. Laursen and Eliassen, 1988) who used a form of coefficient which only damps wavenumbers higher than a chosen threshold (Boer et al., 1984).

Medium-range forecasting experience at ECMWF does not indicate a significant systematic impact due to the choice of horizontal diffusion parametrization, however cases of marked

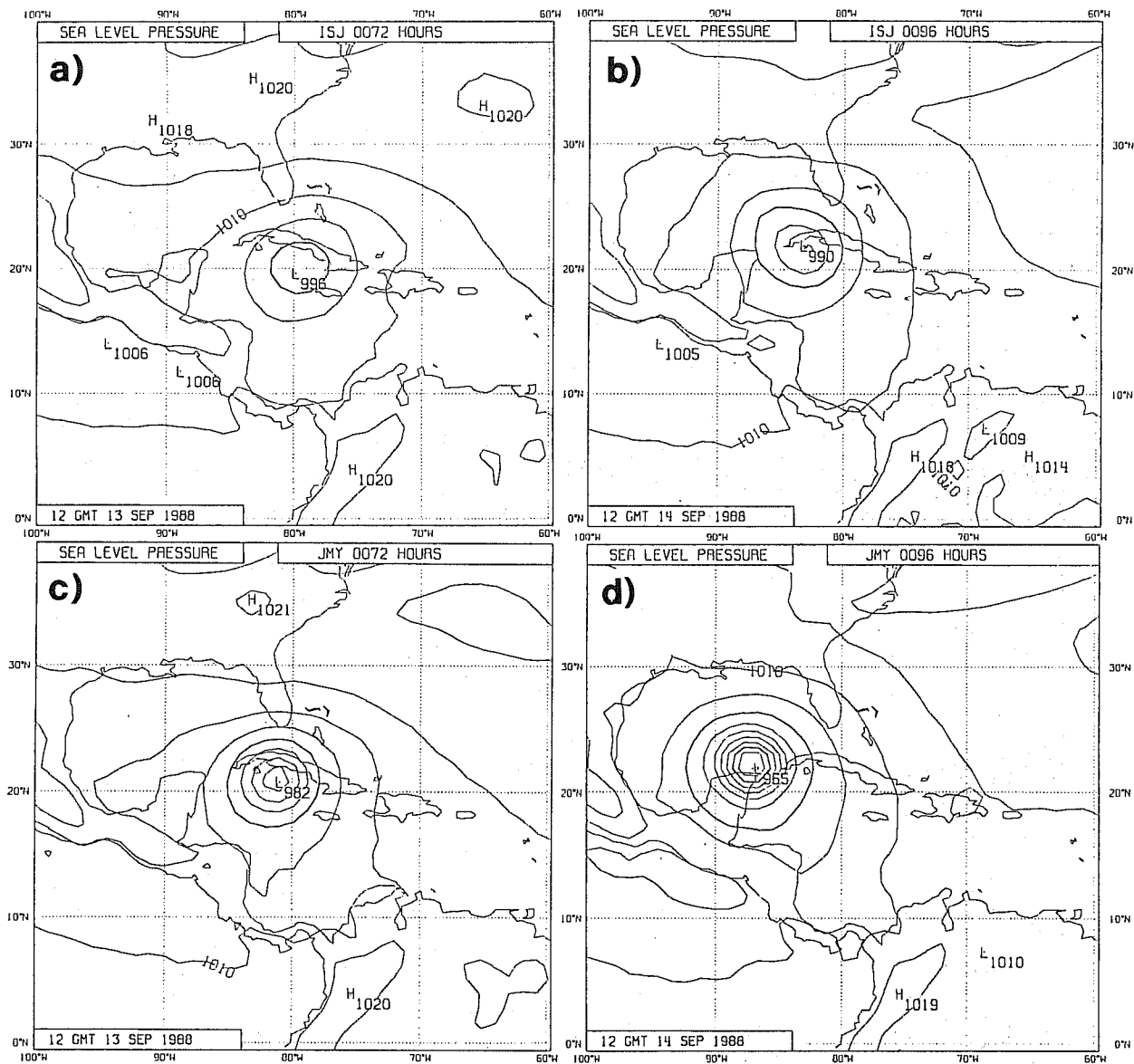


Fig. 13 MSLP maps at D+3 and D+4 for forecasts of hurricane Gilbert with cumulus momentum transport (a,b) and without cumulus momentum transport (c,d).

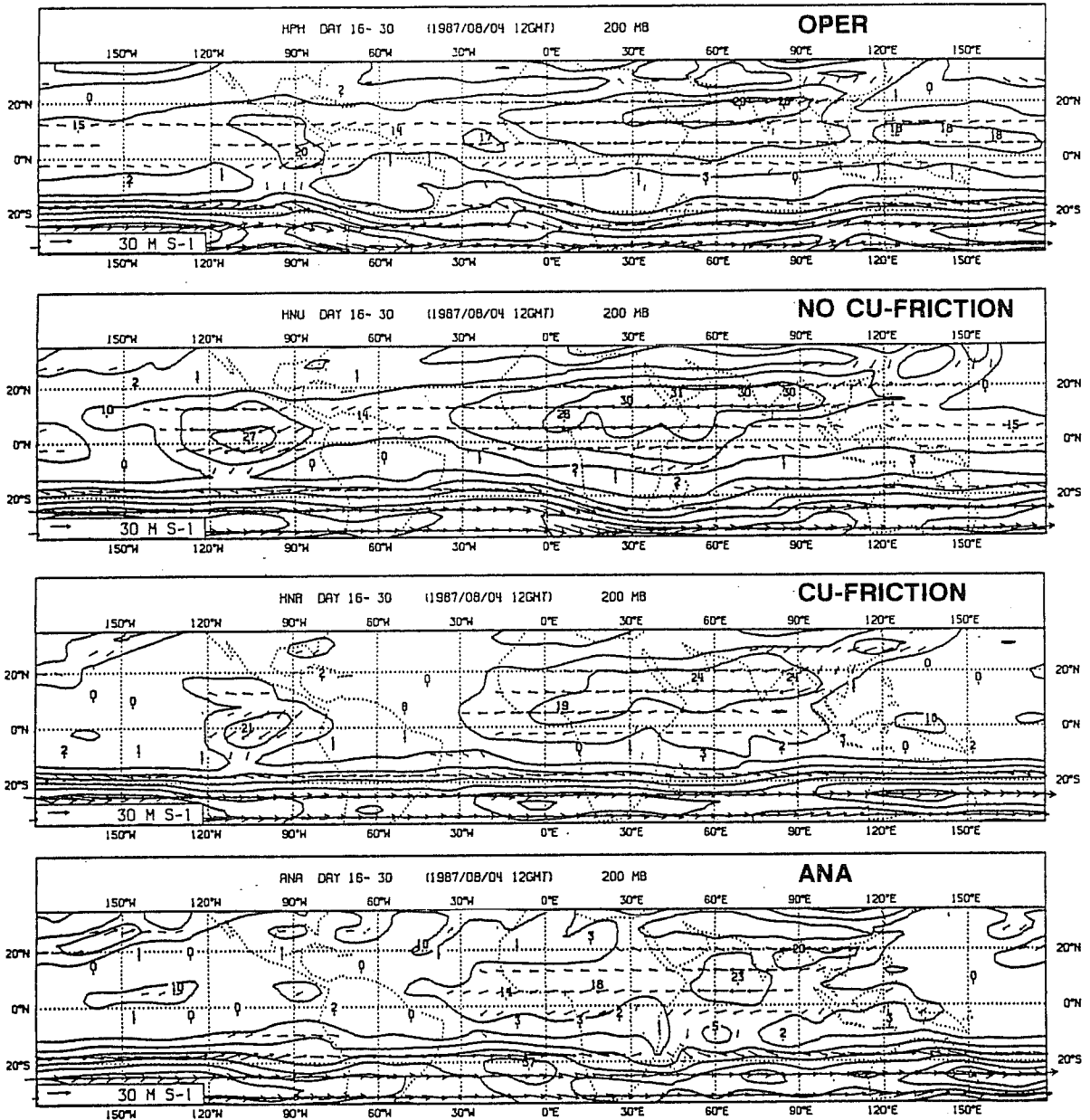


Fig. 14 (16-30) day mean tropical flow at 200 mb for T63 integrations from 19.7.87, 12Z with operational model, massflux scheme without cumulus friction and with cumulus friction plus analysed mean flow for same period.

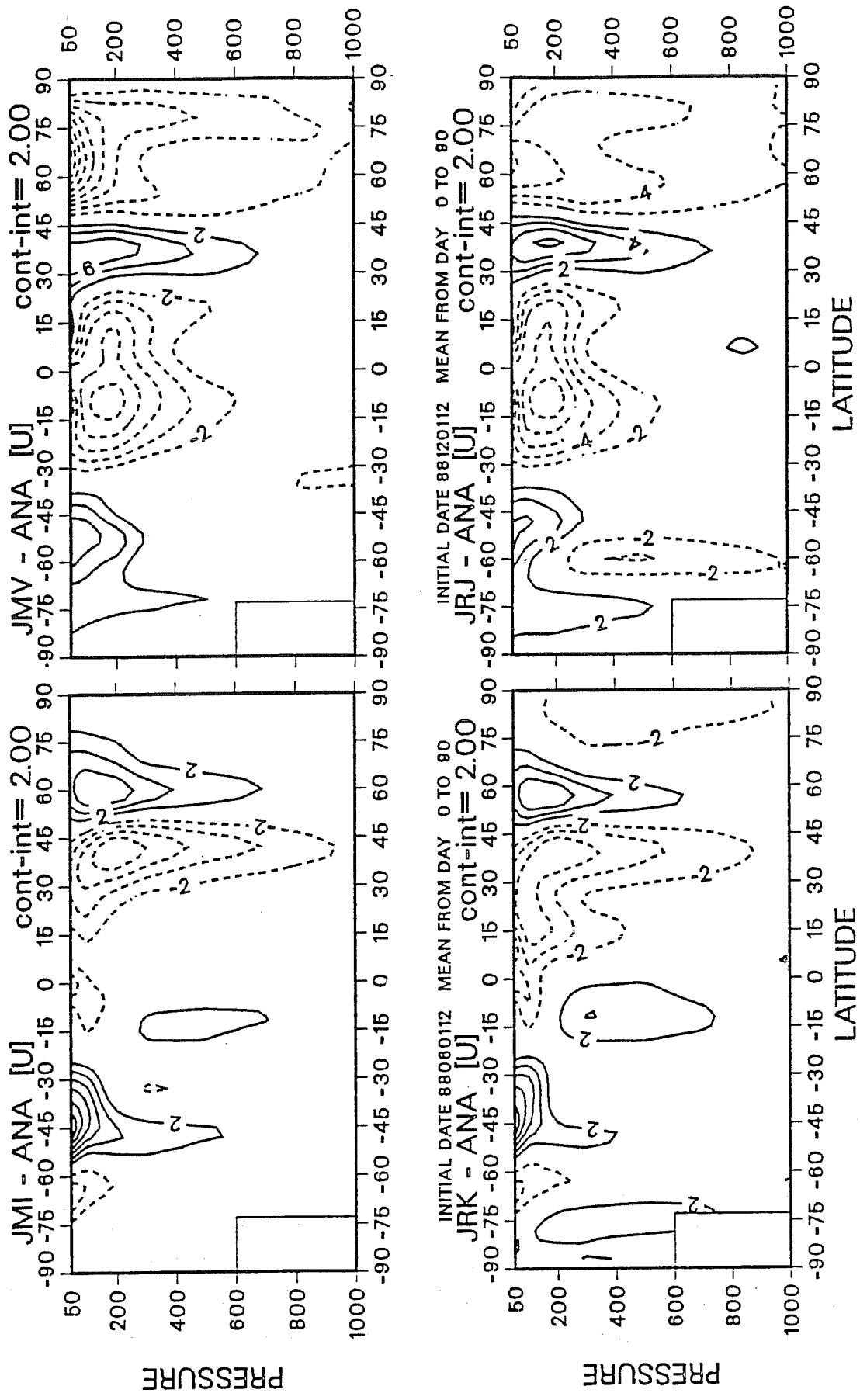


Fig. 15 Zonal mean zonal wind errors for a summer (left) and winter (right) T42 90 day integration with (upper) and without (lower) cumulus momentum transport.

forecast sensitivity have been documented as well as the study of 'noise' control due to orography or convection. It is certainly true that varying the damping rates on the 'horizontal' diffusion of divergence can substantially modify the 'spin-up' characteristics of the model and its associated vertical velocity and precipitation maxima. A particular problem occurs with 'horizontal' diffusion terms in the vicinity of steep orography. Since the terms are usually applied on terrain-following coordinate surfaces the parametrized fluxes are no longer horizontal. There are well-documented discussions in the literature on the problems of spurious orographic rainfall due to this, but the equivalent problem of spurious momentum fluxes does not appear to have been addressed.

Although not strictly 'horizontal' diffusion, the common tendency to use large time-filter coefficients (Asselin, 1972) with leapfrog time integration implies substantial damping/diffusion rates not originally intended.

Following the impetus gained in "potential vorticity thinking" (e.g. Hoskins et al., 1985) there has been interest in the formulation of subgridscale fluxes of potential vorticity (PV). The questions addressed include whether these fluxes should be down the local PV gradient and along isentropic surfaces. Rossby wave breaking in the upper troposphere produces regions of PV 'mixing' and so-called 'PV barriers' (McIntyre, 1987); should subgridscale processes erode these or not? A recent paper by Thorpe and Rotunno (1989) shows that downgradient fluxes of heat and momentum do not necessarily imply a downgradient PV flux.

This is an area of active work at present which may well lead to an improvement in our parametrization of 'horizontal' subgridscale fluxes and the modelling of extra-tropical systems.

6. CONCLUDING REMARKS

In the past decade, the marked sensitivity of GCM and NWP models to mechanical/dynamical forcing, both resolved and subgridscale, has been demonstrated and increasingly appreciated. This paper has tried to discuss the major ways in which the subgridscale momentum sources/sinks have been represented in 'state-of-the-art' models. Because of the fundamentally difficult problems which exist in observing, and hence verifying, momentum fluxes and balances on all scales, there are major uncertainties not only in the formulation and magnitude of most of the parametrized processes but even of their mechanism and validity. Nevertheless continuing progress in this area will undoubtedly lead to improvements which should further benefit medium-range weather forecasting and through continuing reduction of systematic errors enable valuable extended-range information to be obtained.

Acknowledgements

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