TECHNICAL MEMORANDUM

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Use of SATOB satellite winds in a variational analysis scheme

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PREFACE

by Jean Pailleux

The students of the French Weather Service (École Nationale de la Météorologie, Toulouse) finish their school period with a 6 month research study (small PhD) mostly in the French Weather Service. In December 1989, Laurent Perron was the first one to come to ECMWF to perform this study here.

Laurent was integrated in the team working on the development of the variational analysis, and he studied in more detail the aspects related to one observation type: the cloud winds or SATOB's. When he left in June 1990 he left a copy of his report (in French) which contains all the details of his work. This report is available at ECMWF (ask me). Laurent was so kind as to provide a much shorter report summarizing his work, which has been translated into English by Robert Bucher. I have edited this translation to produce the present Technical Memorandum. I want to thank both Laurent Perron and Robert Bucher for their work leading to this paper.

Abstract

The variational analysis (presently coded in research mode at the ECMWF and in the French Weather Service) requires the calculation of a cost function which measures the distance between the model's vector variable and observations. We have calculated the contribution of SATOB satellite winds (derived from cloud motions) to this function.

Two options were considered: The use of the wind components of the observed wind (in the form u, v) or the direction and force (dd, ff). The latter is interesting in that it affords the possibility to use just one of the two variables if the other one proves to be too poor. It also makes it possible to prepare ourselves for the use of future observations containing only the wind force, such as those that will be produced by a scatterometer flown on the ERS-1 satellite.

Finally, to minimize the cost function we have determined its gradient with respect to the vector variable of the model using the adjoint theory. Several minimization experiments were run, first in a 1 dimensional mode with just one observation, then in 3 dimensions on a 7x7 grid with 5 levels.

<u>Key words</u>: variational analysis, SATOB, cost function, adjoint.

1. INTRODUCTION

One of the principal aims of a data assimilation is to provide numerical forecast models with an initial state close to the real state of the atmosphere.

Currently, the data analysis systems provide the model with an initial state which is derived from observations and the first guess (last available numerical forecast). For this, empirical or statistical methods such as optimal interpolation are used. The limitations of the optimal interpolation technique are described in Pailleux (1989 a). To overcome these limitations, present research in data assimilation attempts:

- to make the analysis system less independent of the forecasting model;
- to use data that are closer to the genuine observed quantity measured by the instrument.

In order to improve present analysis systems, variational methods have been proposed by several teams, such as Le Dimet et Talagrand (1986), Lewis and Derber (1985) or Talagrand and Courtier (1987). The aim of the variational analysis is to calculate a cost function measuring the distance from the model vector variable to the observations and the first guess while taking account of physical constraints. By minimizing this cost function, it is possible to find the variable closest to the real state of the atmosphere at the desired moment, and this variable is in harmony with the model equations.

The following are the main benefits which numerical forecasting can gain from a variational analysis:

- the possibility to treat the analysis, the initialization and the model as one coherent ensemble enabling a fully four-dimensional variational assimilation;
- the disappearance of data selection algorithms and therefore the absence of discontinuities in the analysed fields;
- a better integration of physical constraints (such as geostrophy);
- a better use of new types of data, such as satellite data (radiances), at non standard observation times.

Since the variational analysis requires the calculation of a cost function measuring the distance between the model control variable and observations, it is necessary to calculate, from the model, values comparable to observed values. Here must be introduced the notion of the observation operator: this is, for a given observation type, the operation which produces from the forecast model a value similar to the observed value.

It is not sufficient to calculate a cost function, one must still be able to minimize it. Since all standard minimization schemes use the gradient of the cost function, in addition to its value, we need its gradient with respect to the control variable. This gradient cannot always be calculated analytically. Luckily, the adjoint of the observation operator makes it possible to determine this gradient through an entirely numerical approach (see Talagrand, 1989).

The study reported in this paper aims at calculating a cost function and its gradient for observations of the SATOB type (satellite winds). The simplicity of the observation operator in case of SATOB winds makes it possible to explicitly calculate the gradient of the cost function with respect to the control vector variable of the model, i.e. we can verify the results of the numerical calculation with an analytical calculation.

We will be treating two ways of representing an observed wind:

- the first way breaks it down in zonal and meridional wind;
- the second uses a direction and a speed.

The latter makes it possible to use either the direction or the speed of the wind in the analysis if one of the two variables is not good enough. This possibility is not available in the optimal interpolation technique used at present by the ECMWF analysis. It is especially interesting for SATOB winds, which systematically underestimate the strength of the jet streams.

Once the calculation of the cost function and its gradient with respect to the control vector variable of the model is made, we apply an elementary minimization: in section 5 the model wind profile (in the vertical of a SATOB observation) is modified so as to minimize the distance between the model control variable and the observations.

In section 6 of this memo we move to a three-dimensional problem by minimizing the calculated cost function on a 7x7 grid, which makes it possible to validate the code developed (SATOB routine) in the framework of the IFS/ARPEGE project.

2. INTRODUCTION TO VARIATIONAL ANALYSIS

The present ECMWF data analysis system is based on a statistical optimum interpolation (OI) method. The ECMWF assimilation consists of an ensemble "analysis-initialization-forecast model" which is not as coherent as we would like it to be. The main reason for this lack of consistency is that the analysis is too independent of the model: the OI analysis does not know anything about the model equations (all it knows is a set of statistics on the 6 hour forecast errors); also the integration of physical constraints in the analysis is not very good (for example the mass/wind balance is insured in the analysis only through a crude geostrophic assumption on the increments through the OI statistics). Another weak point of the current OI analysis is that it can only use values which are linear functions of the model variables. This limitation becomes more and more important because of the development of new instruments observing new quantities which are further and further away from the model variables. Several teams have proposed to use variational methods to tackle these weaknesses (see for instance Le Dimet and Talagrand (1986) or Talagrand and Courtier (1987)).

The variational methods minimize the distance between the model and the observations. It is called "cost function" and written as J(X), X being the vector containing all the model's variables at the starting point of the assimilation period (see Pailleux, 1989 a).

For a given type of observation the process which produces from the model a value similar to the observed value is called "observation operator". The adjoint of this operator (Talagrand, 1989) makes it possible to calculate numerically the gradient of the cost function J (Grad J) with respect to the forecast model vector variable.

In practice, J(X) and Grad J are calculated, then passed on to a standard minimization algorithm as often as necessary to converge towards a minimum of J.

2.1 Four-dimensional variational assimilation

The advantage of the variational approach is that the information contained in the model dynamics is fully utilised, as we try to compute directly a model trajectory.

Figure 1 shows the model "trajectory", which we try to compute in order to minimize J.

2.2 Calculation of the cost function in practice

The cost function J consists of the following terms: J_o, the distance between the vector variable X of the model and the observations; J_g, the distance between X and the first-guess and J_c, representing the physical constraints on X (see Pailleux, 1989 b).

$$J = J_o + J_g + J_c$$

Assuming that the observation errors associated with two different types of observation are not correlated (as for optimal interpolation, the representativeness error is included in the term "observation error"), it is possible to break up J_o into independent terms $J_o = J_{synop}+J_{rs}+J_{airep}+J_{satob}+...$

For every type of observation, it is generally possible to separate the cost function into independent terms. Let us look at radiosonde data for instance: assuming that the observation errors of different radiosondes are not correlated, the contribution of each of them to J_{rs} can be calculated separately:

$$J_{rs} = J_{rs1} + J_{rs2} + J_{rs3} + ...$$

And you can go even further by separating:

$$J_{rsi} = J_{wind} + J_{geopotential} + J_{humidity}$$

for each radiosonde observation.

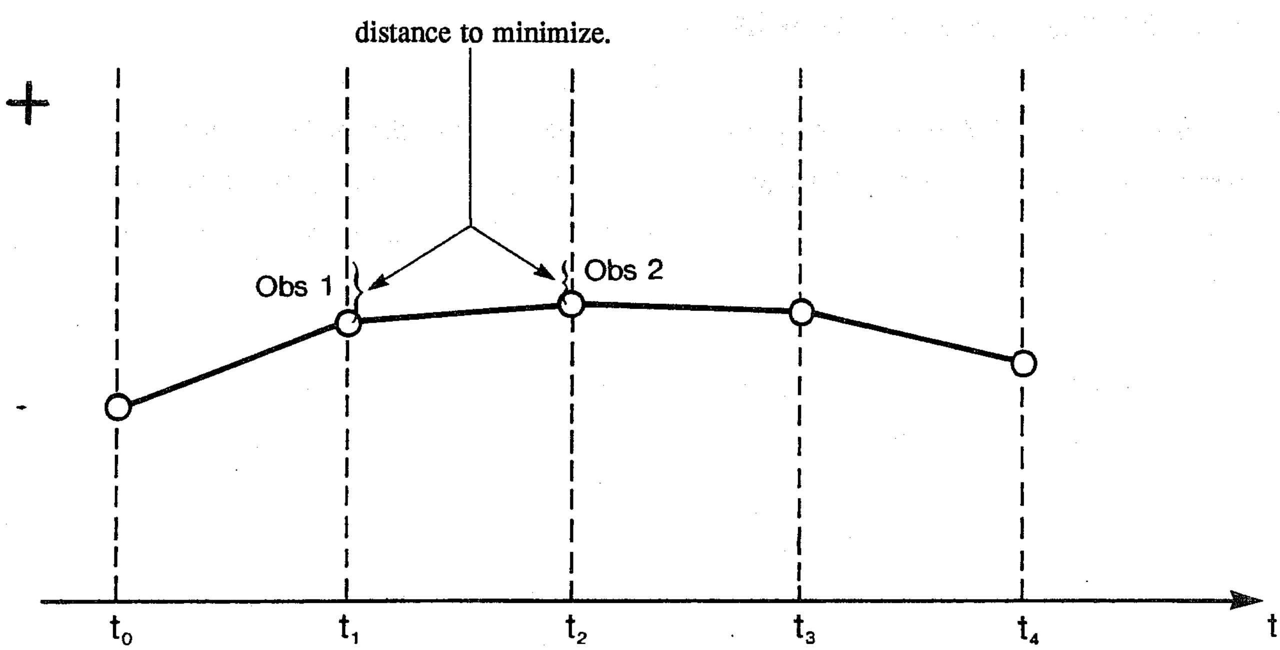

Fig 1 Schematic representation of the 4D assimilation problem.
X: analysis vector variable

3. PRINCIPLES OF A STANDARD MINIMIZATION ALGORITHM: A FEW TESTS

3.1 Introduction

The experimental variational analysis which is currently coded at ECMWF requires the minimization of a cost function measuring the distance from the model variable to the observations and the first guess. Currently we rely on standard minimization algorithms developed outside meteorology by numerical research groups and used for different applications.

3.2 The minimization module M1GC2

The minimization software used in our experiments is called M1GC2. It was developed by INRIA from work of A. Buckley. The module M1GC2 solves the optimization problems of the following type: find the vector X which minimizes J(X).

It is necessary to provide M1GC2 with the value of the function J and its gradient with respect to the vector variable X: the computation of J and its gradient are called "simulation" in optimal control language.

The aim of M1GC2 is to make the best use of the computer (depending on the memory available) to minimize J.

If the user allocates sufficient memory space, M1GC2 calculates at every iteration an approximation of the inverse of the Hessian matrix

$$\left(\frac{\partial^2 J}{\partial xi \cdot \partial xj}\right)_{i,j}$$

This is the quasi-Newton method.

If memory is insufficient, M1GC2 starts by an approximation of the quasi-Newton techniques. At every iteration an approximation of the Hessian matrix is updated. One continues with the algorithm of the conjugate gradient preconditioned by this matrix. This method is likely to be used for all the "real size" experiments performed with the variational analysis.

3.3 Routines to be written by the user of M1GC2

Before calling the minimisation routine M1GC2, two routines must be declared in EXTERNAL:

- SIMUL — calculates the value of the function to minimize and its gradient. Each calling of SIMUL is called "simulation".
- PROSCA — calculates the scalar product of two vectors. This scalar product is not necessarily Euclidean as a better choice accelerates the convergence.

These two routines must be written by the user following the documentation on M1GC2.

3.4 Acceleration of the convergence

The enormous size of the variable of the numerical forecast model (it would be around 10^6 with the present ECMWF operational model) does not allow for a great number of iterations (probably less than 10). Therefore the convergence must be accelerated by passing the ad hoc scalar product routine to precondition properly the minimization.

If M is a symmetrical matrix close to the secondary derivatives of the function to be minimised J, one can use the scalar product

$$(a, b) = a^t M b$$

without forgetting to feed M1GC2 with $M^{-1}\nabla J$ instead of ∇J.

In this case you have indeed $dJ = \nabla J^t X$:

$$dJ = (M^{-1}\nabla J, MX) = (M^{-1}\nabla J)^t MX = \nabla J^t X$$

and $M^{-1}\nabla J$ is close to the Newton direction.

3.4.1 Choice of the scalar product

For defining our scalar product, the ideal solution would be to use the matrix of the second derivatives (Hessian matrix) of the function to minimize. Unfortunately the introduction of a Hessian matrix dependent on the model variable would need too much computing time and memory (in the above example a $10^6 \times 10^6$ matrix). In order to accelerate the convergence with reasonable computing times one can ask oneself whether it is sufficient to pass to M1GC2:

a) the approximation of the Hessian matrix diagonal at the initial point of the minimization.

b) the approximation of the Hessian at the initial point of the minimization.

The use of the diagonal of the Hessian matrix alone does not necessarily accelerate the convergence towards the minimum: this has been shown by performing simple numerical tests, with simple cost functions in a low dimensional space (Perron, 1990).

What matrix can we take to define a scalar product capable of accelerating the convergence towards the minimum of the cost function J? The ideal one would be the inverse of the variances/covariances matrix of the analysis error (as it is the Hessian of J), but it is not available before the minimization. It is possible to use that of the preceding analysis, assuming that this matrix varies little from one analysis to the other, but with changing observation networks, this idea is not very good either.

To tackle this problem, we can use the matrix of variances/covariances of the forecast error P, which is supposed to be close to the covariance matrix of the analysis error. The cost function to minimize is broken down as follows:

$J = J_o + J_g + J_c$ (see section 2) with $J_g = (X-X_g)^t P^{-1} (X-X_g)$. P^{-1} is already used in the calculation of Jg, it is by construction a symmetrical matrix defined positive definite which can be used for the definition of the scalar product. P^{-1} will have to be tried for preconditioning the minimization routine via the scalar product in the hope of accelerating the convergence towards the minimum of the cost function J. This is by no means ideal as the observations are then not used.

The importance of the initial point of minimization varies according to the type of function for which one wants a minimum. If the function is quadratic, the initial point is of little importance and convergence will be reached in one iteration as long as M1GC2 is fed with first and secondary derivations of the function. If on the other hand the function is not quadratic, the initial point can be of great importance: it is crucial to start as close as possible to the final solution. Several academic numerical experiments, illustrating these aspects are available in Perron (1990).

4. CALCULATION OF A COST FUNCTION AND ITS GRADIENT FOR THE SATOB WIND OBSERVATIONS

4.1 Use of the observed wind in the form u, v

We treat the case where the observed wind is broken down into zonal and meridional wind in the form u, v. The two components cannot be treated separately: it would not be logical to use only the zonal or meridional component of an observed wind.

4.1.1 Calculation of a cost function and its gradient with respect to the variables u, v and Ps

We now calculate the contribution of SATOB observations to the cost function. Each SATOB wind observation must be compared to the model's wind interpolated at the observation point. The observation operator is therefore very simple and permits an explicit calculation of the gradient. The gradient calculated with the adjoint operator can thus be verified.

Observation Operator:

From the model wind profile at the vertical of the observation a vertical interpolation (linear in pressure) is performed (by the routine PPUV in the set of ECMWF postprocessing routines). This interpolation produces the interpolated wind at observation point.

Cost Function for one observation

$$J_o = \left(\frac{u_o - u_p}{s_u}\right)^2 + \left(\frac{v_o - v_p}{s_v}\right)^2$$

where

u_o observed zonal wind

v_o observed meridional wind

u_p interpolated zonal wind at observation point

v_p interpolated meridional wind at observation point

s_u standard deviation of the zonal wind observation error

s_v standard deviation of the meridional wind observation error.

Gradient with respect to u_p:

$$\frac{\partial J_o}{\partial u_p} = -2(u_o-u_p)/s_u^2$$

Gradient with respect to the variable u:

- obtained with the adjoint interpolation routines:

The adjoint of the observation operator is the adjoint of the interpolation routine (PPUV). It is sufficient to feed it with the gradient of J_o with respect to interpolated wind as an input argument to obtain as output the gradient of J_o with respect to the zonal wind at all model levels (in the vertical of the observation).

$$\frac{\partial J_o}{\partial u_p} \rightarrow PPUVAD \rightarrow \frac{\partial J_o}{\partial u} \qquad \text{(at all levels)}$$

- Explicit calculation:

- If the observation is below the model top level (level 1):

Let k and k-1 be the model levels bordering the SATOB observation,
p_k and p_{k-1} their pressure in the vertical of the observation,
u_k and u_{k-1} their zonal wind in the vertical of the observation, then

$$\frac{\partial J_o}{\partial u_k} = \frac{\partial J_o}{\partial u_p} \cdot \frac{\partial u_p}{\partial u_k} = \frac{\partial J_o}{\partial u_p} \cdot (p-p_{k-1})/(p_k-p_{k-1})$$

$$\frac{\partial J_o}{\partial u_{k-1}} = \frac{\partial J_o}{\partial u_p} \cdot \frac{\partial u_p}{\partial u_{k-1}} = \frac{\partial J_o}{\partial u_p} \cdot (p_k-p)/(p_k-p_{k-1})$$

Since the other levels are not used in the interpolation, the corresponding gradients are zero.

- If the observation is above the model top level (linear extrapolation to obtain u_p) the gradient equations are similar to the previous ones (with k = 2).

Gradient with respect to the variable v:

The calculation is similar to the one of the gradient with respect to the variable u.

Gradient with respect to the variable p_s:

The surface pressure at the vertical of the observation is connected to the interpolated wind at observation point through the position of the model levels. This feature is generally neglected in the analysis schemes. However, it is easy to take it into account in the present context. Neglecting it would lead to an approximation on the gradient computation which would make it impossible to check.

- Gradient obtained with adjoint interpolation routines:
  Let us see in which way the surface pressure P_s is related to the interpolated wind at observation point (u_p, v_p):

  From p_s we calculate the model level pressures with the routine PPPRES. Then we initialize the tables necessary for interpolation with PPINIT. Then we interpolate with PPUV.

  All we need to do is to call the adjoint routines in the opposite order to obtain the gradient of J_o with respect to p_s:

$$\left\{\begin{array}{l}\frac{\partial j_o}{\partial u_p}\\ \frac{\partial J_o}{\partial v_p}\end{array}\right. \rightarrow \textit{PPUVAD, PPINITAD, PPPRESAD} \rightarrow \frac{\partial J_o}{\partial p_s}$$

- Explicit calculation:

$$\frac{\partial u_p}{\partial p_s} = (u_k - u_{k-1})\left(B_{k-1}(p-p_k) + B_k(p_{k-1}-p)/(p_k-p_{k-1})\right)^2$$

  Where: k and k-1 are the levels bordering the SATOB observation, p_k and p_{k-1} the pressure of these levels, B_k and B_{k-1} the coefficients defining the vertical coordinate of the model:

$$p_k = A_k + B_k \cdot p_s$$

4.1.2 Test of the calculation of the cost function and its gradient by the subroutine SATOB

The computation of the observation cost function for one SATOB, as developed in the previous section, is involving one of the simplest examples of observation operators. Because of this simplicity it was possible to do the full analytical calculation of the gradients, and to verify the results against the results provided by the chain of adjoint routines: PPUVAD, PPPRESAD, ... This analytical computation has also been in this case a way to validate these routines (routines which will be used in many other places of the variational analysis code).

For most of the observation types, the observation operator is more complicated, and it is impossible to validate the adjoint code by a full analytical computation. However, there is a standard technique to verify the gradient (which has already been developed in the variational analysis code): it is using the Taylor formula, and it has been also applied to our SATOB operator.

H being the observation operator and X the control variable, the Taylor formula can be written

$$H(X + \varepsilon\delta X) = H(X) + H' \cdot \varepsilon\delta X + O(\varepsilon)^2$$

The function

$$A\ (\varepsilon) = \frac{H(X + \varepsilon\delta X) - H(X)}{H' \cdot \varepsilon\delta X}$$

must be linear and must tend towards 1 when ε tends towards zero and when δX is proportional to H', if H' is the correct gradient. We have verified our SATOB gradient by computing the function A(ε) for different values of ε varying from 10^{-6} to 10 (logarithmic variations) as shown on the following table:

ε	A
0.1 E -05	0.99999920
0.1 E -04	0.99999212
0.1 E -03	0.99992123
0.1 E -02	0.99921232
0.1 E -01	0.99212325
0.1 E +00	0.92123255
0.1 E +01	0.21232550
0.1 E +02	-0.68767518

4.2 Use of the observed wind in the form dd, ff

The observed winds can have the form dd,ff, i.e. a direction and a force. The interest of this representation compared to the form u,v lies in the possibility to use only the direction or the speed in the analysis if one of these two variables is too poor.

The optimum interpolation analysis, contrary to variational methods, does not have this possibility, because there is no linear link between the direction or the speed of the wind and the model variables.

4.2.1 Calculation of a cost function and its gradient with respect to the variables u, v and Ps

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The cost function is: $J_o = J_{od(direction)} + J_{of(speed)}$

$$J_{od} = \left(\frac{d_o - d_p}{s_d}\right)^2$$

$$J_{of} = \left(\frac{f_o - f_p}{s_f}\right)^2$$

The following notations are used:

- d_o: direction of the observed wind (measured with the positive sign in the trigonometric sense) from the Eastern direction;
- f_o: speed of the observed wind;
- d_p: direction of the "postprocessed" model wind, vertically interpolated to the observation point;
- f_p: speed of the model wind;
- u_p: zonal wind interpolated to the observation point;
- v_p: meridional wind ....;
- s_d: observation error standard derivation on the direction;
- s_f: observation error standard derivation on the speed.

The direction and speed are linked to u_p and v_p by:

$$f_p = \sqrt{u_p^2 + v_p^2}$$

$$d_p = \arcsin\left(\frac{v_p}{f_p}\right) \quad \text{if} \quad u_p \geq 0$$

$$d_p = \pi - \arcsin\frac{v_p}{f_p} \quad \text{if} \quad u_p < 0$$

Gradient with respect to variables u_p and v_p:

$$J_o = J_d + J_f \quad \text{so} \quad \frac{\partial J_o}{\partial u_p} = \frac{\partial J_{od}}{\partial u_p} + \frac{\partial J_{of}}{\partial u_p}$$

$$\frac{\partial J_{of}}{\partial u_p} = -2\,\frac{f_o - f_p}{s_f^2}\,\frac{u_p}{f_p}$$

$$\frac{\partial J_{of}}{\partial v_p} = -2 \frac{(f_o - f_p)}{s_d^2} \frac{v_p}{f_p}$$

If $u_p \geq 0$ then

$$\frac{\partial J_{od}}{\partial u_p} = 2 \left(\frac{d_o - d_p}{s_d^2}\right) \frac{v_p}{f_p^2}$$

and $$\frac{\partial J_{od}}{\partial v_p} = 2 \left(\frac{d_o - d_p}{s_d^2}\right) \frac{u_p}{f_p^2}$$

If $u_p < 0$ then

$$\frac{\partial J_{od}}{\partial u_p} = -2 \left(\frac{d_o - d_p}{s_d^2}\right) \frac{v_p}{f_p^2}$$

and $$\frac{\partial J_{od}}{\partial v_p} = 2 \left(\frac{d_o - d_p}{s_d^2}\right) \frac{u_p}{f_p^2}$$

At this point a similar computation to the one performed in 2.1.1 can be made. This is true also for the gradient of the cost function with respect to p_s.

4.2.2 Test of the calculation

Compared to 4.1, the only extra operator to consicer is the one performing the computation of (d_p, f_p) from (u_p, v_p). In terms of code development it is only a few lines of Fortran to add at the appropriate place in the direct and adjoint routines. The simplicity of the analytical computation gives the possibility to make the cross-check of the results. The standard test of the gradient described in 4.1 has also been performed successfully when working with the direction and speed of the wind.

5. MINIMIZATION TESTS OF THE COST FUNCTION

The computations described before have been put in the form of a "SATOB" subroutine fully consistent with the development of the ECMWF variational analysis. Several versions of the subroutine SATOB are actually available: one of them is using the wind components, another one is using direction and speed. Before including and testing the SATOB part in the real size variational analysis, it is worth testing it in a very simple 1D problem reduced to one simulated SATOB observation, and one simulated model profile at the observation point. Such a variational problem is strongly under-determined (because one observation is much too little to determine the whole model profile), however it is worth running such a simplified 1D variational analysis for at least two reasons:

i) to validate this part of the variational analysis;

ii) to check how the minimization is working (code M1GC2 described before).

5.1 Wind Observation in the form u,v

Fig.2 summarizes one of these simple simulated 1D experiments. It shows that the minimization is working towards a profile which is consistent with the (u,v) observation: one solution among an infinity of possible solutions as a problem is undetermined. The model profile is shown for each wind component by the solid line, the observation is marked by a cross (+). The left-hand diagrams in the figure show the profile before the minimization starts. The convergence is obtained in 1 iteration , i.e. in the most efficient way, as expected due to the spherical property of the cost function.

5.2 Observed wind in the form (direction dd, speed ff)

Starting from a simulated initial state and a SATOB observation given in the form (dd,ff), we minimize the cost function which uses:

i) direction and speed (figure 3)

ii) speed only (figure 4)

iii) direction only (figure 5)

Figures 3, 4 and 5 show the behaviour of the minimization in the horizontal plan of the wind observation. The full line arrow represents the wind observation, and the dashed one the model wind (control variable) interpolated to the observation level.

Using only the speed, or only the direction, is not usual in current operational analysis schemes. As explained before it is actually impossible in the current ECMWF OI formulation which works with the wind components and the geopotential height. It is still interesting for SATOB winds, as it gives possibility to use the observed direction only when the observed speed is known to be too poor (like in the jet streams where the cloud winds are famous for being underestimated). Such a feature is also interesting for preparing the use of future observations, such as those that will be produced by a scatterometer flown on future satellites like ERS 1: a scatterometer is able to get a measure of the wind speed near the surface, over the oceans, but cannot determine the direction with certitude.

Figures 3, 4 and 5 show that in all three cases i), ii) and iii), the minimization converges towards the expected minimum. The number of iterations in the minimization has been found between 2 and 6 (instead of 1 iteration when we used u and v). This is likely to be due to the shape of the cost function:

the shape is not spherical as it was in the (u,v) case, due to the operators "Arcsin" and " SQRT " which are introduced for computing (dd,ff) from (u,v).

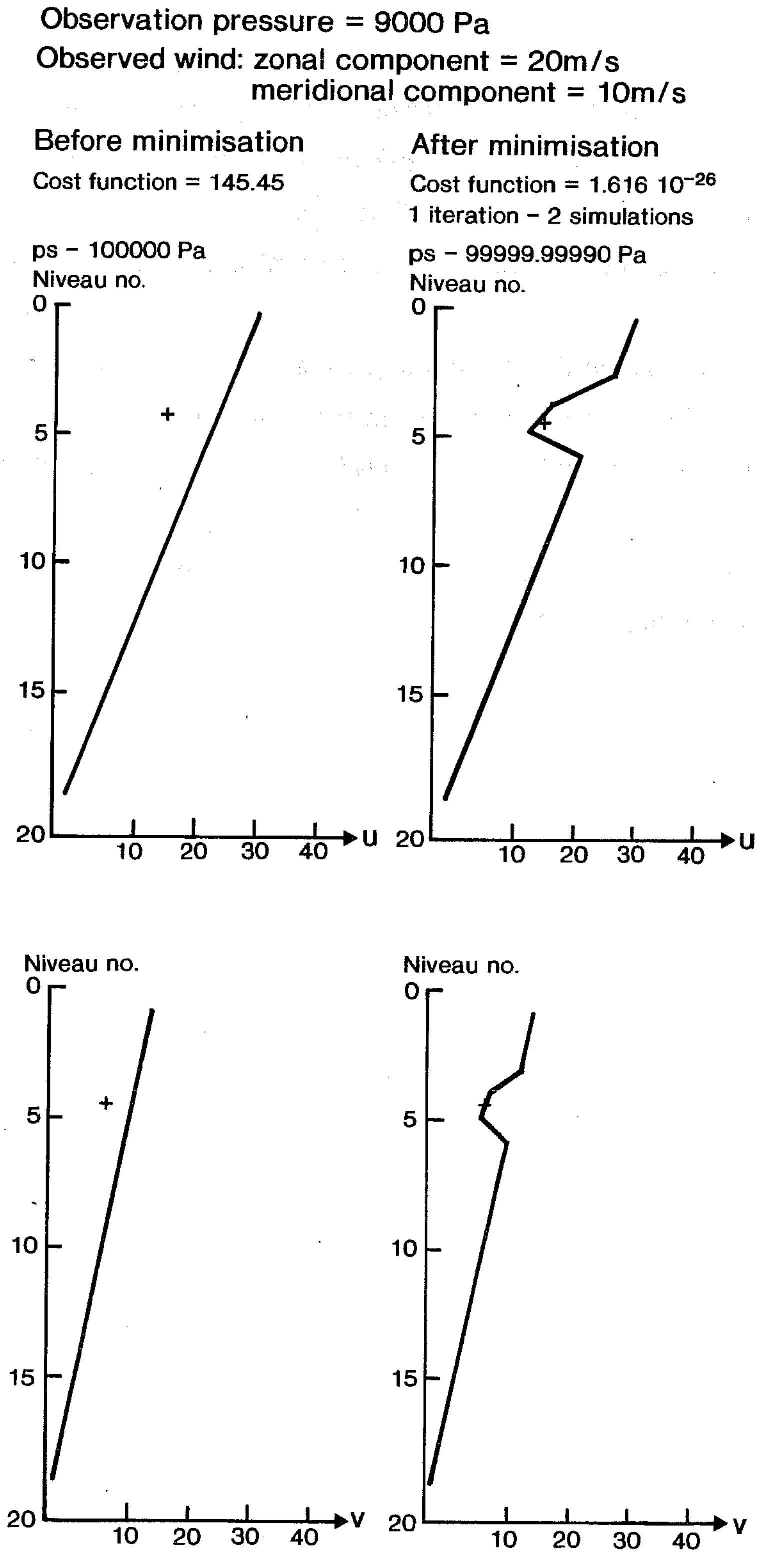

Fig 2 Minimization of a SATOB cost function written with u and v.

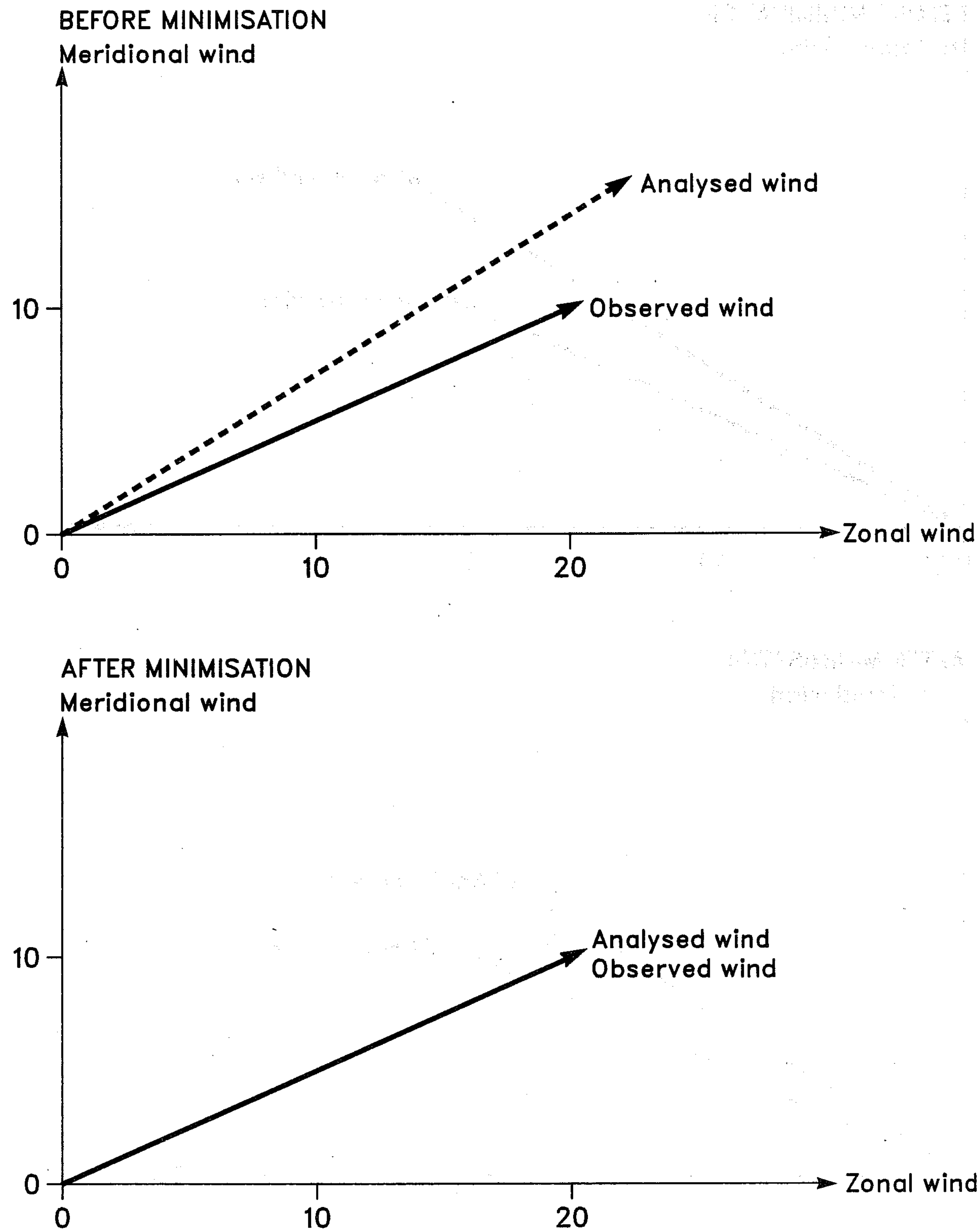

Fig 3 Use of direction and speed of the observed wind.

Before minimization : Meridional wind
Zonal wind
Analysed wind
Observed wind.

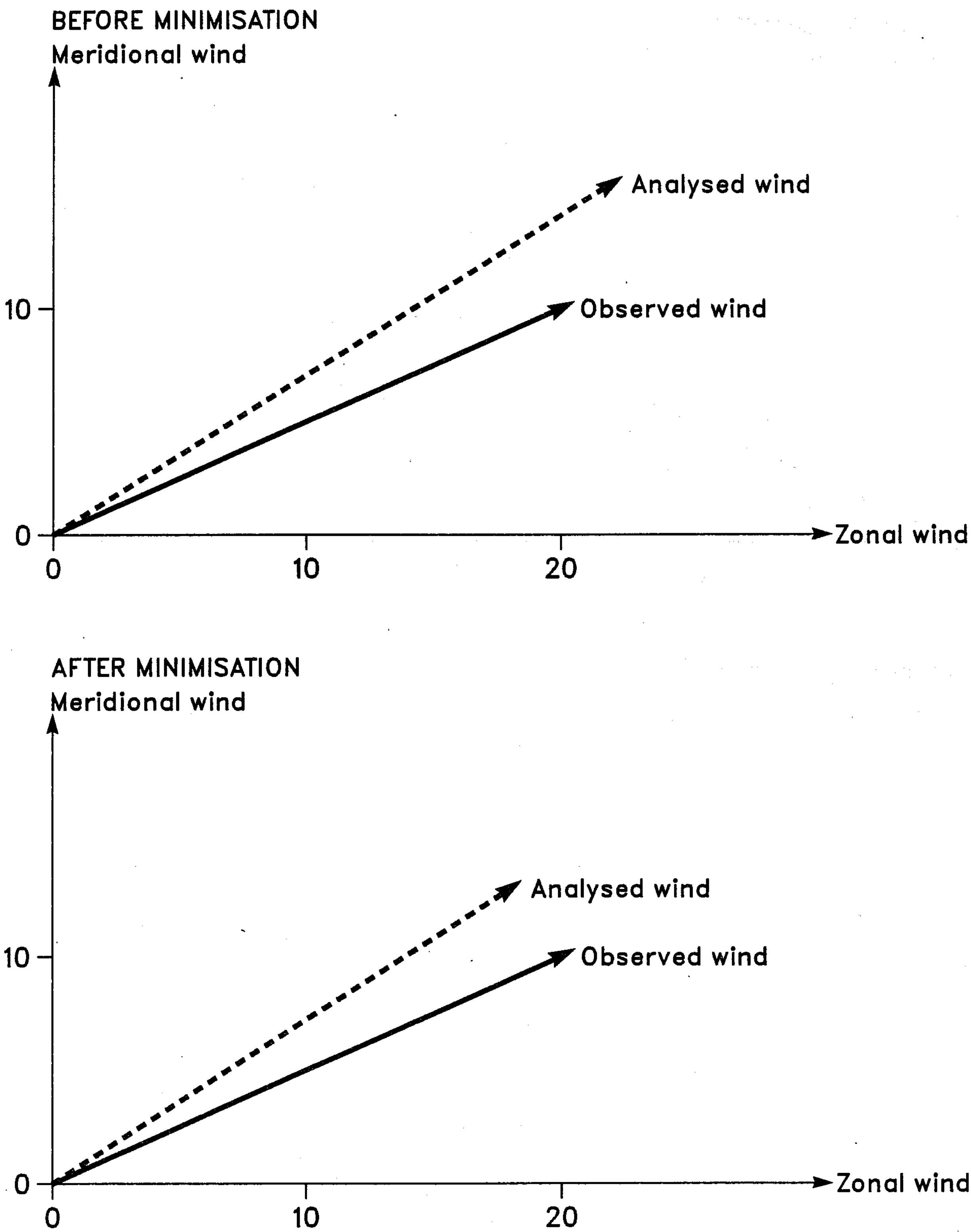

Fig 4 Same as fig 3 but using observed speed only.

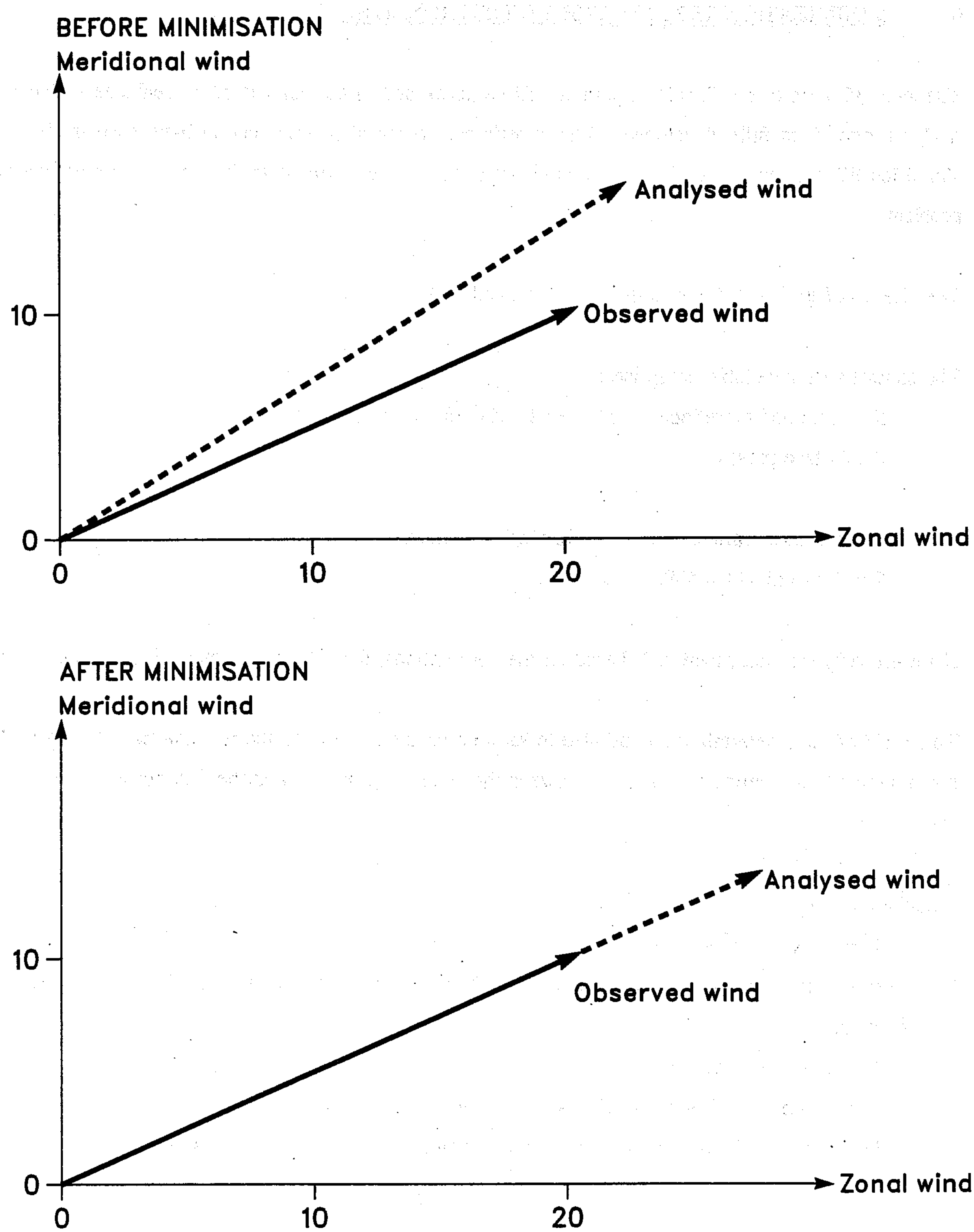

Fig 5 Same as fig 3 but using observed direction only.

6. MINIMIZATION TESTS IN THE 3 DIMENSIONAL SPACE

The ideal 3D tests of the SATOB operator will be performed in the context of the real size variational analysis, once it is fully developed. The experiments reported in this section have been made in a simplified 3D context (which is still much closer to the real variational analysis than the previous 1D problem).

We take a 7x7 grid with 5 levels in the vertical and 100 km mesh.

The control vector variable comprises:

- the zonal and meridional wind at each grid point at every level;
- the surface pressure.

Therefore the control variable has the following dimension:

$$7 \times 7 \times (5 \times 2 + 1) = 539.$$

If we use only the cost function J_o based on the observations, the problem is under-determined.

To get rid of the indetermination and also to set up a problem closer to the real analysis one, we add a cost function J_g measuring the distance between the control variable X and the first-guess X_g. Then we try to minimize

$$J = J_o + J_g.$$

Calculation of J_g:

$$J_g = (X-X_g)^t \bullet P^{-1} \bullet (X-X_g)$$

P is the matrix of the variances/covariances of forecast errors and has dimension 539 x 539. We construct it as follows:

- We assume that in 3D:
  correlation = vertical correlation x horizontal correlation,
  i.e. $r = r_v \times r_h$ (usual separability assumption made in most of the OI system)

- For the vertical correlation we take the following statistical model:

$$r_v = \frac{1_n}{1 + K\left(\ln\frac{p_1}{p_2}\right)^2}$$

with K = 1.5

- For the horizontal correlation we take between two points A and B distant by d metres:

$$r_h = e^{\frac{-d^2}{2a^2}} \qquad a = 500\text{km}$$

- The standard deviations of the forecast error are equal to 1 m/s for all winds and 100 Pa for the surface pressure.

We use one single observation assumed to be at grid point 4 x 4 and 100 hPa.

Observation: Zonal wind is 26 m/s
Meridional wind is 32 m/s

The standard deviation of the observation error is 1 m/s for both zonal and meridional wind. The first guess is a uniform wind field. The starting point of the minimization is taken as equal to the first guess; we use the observed wind in the form u,v.

Figure 6 shows the results of such a 3D test at 100 hPa (the observation level). The test is successful and encouraging in the sense that both the J_o and J_g terms seem to play the rôle we expect from them in the variational analysis: the J_o term is forcing the wind field to draw to the observation at point (4,4), and J_g is "spreading" smoothly the observation information in the horizontal. By looking at the vertical profile of the wind, we also checked that the information is correctly spread in the vertical (it was not the case in the 1D tests because of the lack of a J_g term).

However, some numerical problems have been noted in the convergence of the minimization in some cases, which were solved by increasing the diagonal of the P matrix (i.e. assuming that the horizontal correlation function for forecast errors has a discontinuity at the origin). The exact explanation of these numerical problems could not be clarified. This will have to be reconsidered, probably in the context of the real size variational analysis with a 3D discretization closer to the operational sizes and resolutions. Some technical aspects related to the J_g computation are described in Moll et al. (1988).

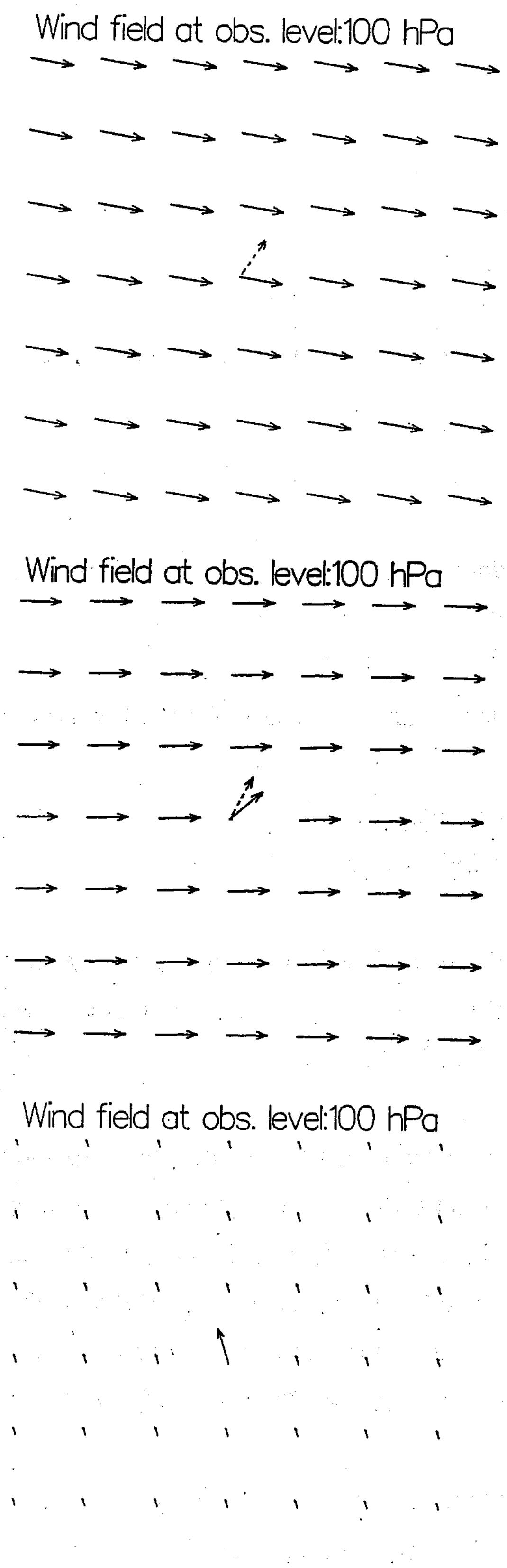

Fig 6 Impact of a single wind observation at 100 hPa on a variational anaysis (observation = dashed arrow). The first guess field is a uniform wind field (top). The variational analysis is shown in the middle together with the observed wind. The differences "Analysis - First Guess" (increments) are the bottom diagram.

7. CONCLUSIONS

The SATOB observation operators have been developed with the aim to incorporate them in the variational analysis code currently developed at ECMWF. The traditional use of a wind observation through its components (u,v) has been considered first; then the direct use of observed directions and speeds has been treated as it opens interesting possibilities not only for using cloud winds, but also other observing systems.

The gradient computations have been developed following the adjoint technique and following the general rules for the development of the variational analysis. The gradient computations have been validated by using the standard tools (Taylor formula). Also, because of the simplicity of the operators in the SATOB case, it was possible to carry out the full analytical computations of the gradient.

The gradient of the cost function (for a wind observation) with respect to the surface pressure p_s has not been neglected although it is extremely small. This non-zero gradient component means that the p_s value of the model can be modified (slightly) by a wind observation through the position of the model levels (which are dependent on p_s).

Simple 1D and 3D experiments have been run to test the SATOB operators. They show that the minimization is working satisfactorily and according to what we know of the minimization algorithm M1GC2.

The real size variational analysis is not ready yet to have the SATOB operators tested in their final environment. The more interesting tests are expected to be made in the real size variational code together with other observation operators.

Let us finally note that the SATOB observation operator is one of the simplest. Much more work has to be done for other observations. On the other hand the SATOB operator can be used for any observation type reporting a single level wind datum in the free atmosphere.

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