A New Method for Directional Wave Analysis Based on Wavelets

Mark A. Donelan
Rosenstiel School for Marine and Atmospheric Science
University of Miami

1. Introduction

The traditional approaches to the analysis of in situ data to yield directional properties of waves assume stationarity and homogeneity in the wave field. Interest in conditions that clearly violate this assumption is increasing, including severe storms, wave-current interactions, wave bathymetric interactions and freak waves. Furthermore, the estimated directional spectra from the common methods, such as the Maximum Likelihood Method (MLM) or the Maximum Entropy Method (MEM), are non unique.

The development of wavelet methods of analyzing non-stationary data has opened up the possibility of deducing directonal spectra of wind waves without the restrictive conditions of stationarity and homogeneity. Donelan et al., (1996) have proposed a method of analyzing time series data from observations at three or more locations in an arbitrary array. The geometry of the array is contained in the separation vectors of pairs of points. The data are first resolved in time and in "scales" or frequency bands by applying a complex wavelet transform. The stratification of the "spectral content" of the data in time simplifies the directional estimation problem, since it is very likely that only a single packet in each scale is present at the measurement array at any one time. The *a posteriori* check of the directional spread against the ratio of crosswind to downwind mean square slopes lends credence to this assumption.

Thus the phase between the i^{th} and j^{th} wave gauges is given by:

$$\phi_{ij}(t) = k_x(t)X_{ij} + k_y(t)Y_{ij} \tag{1.1}$$

where (X_{ij}, Y_{ij}) denotes the separation vector of the pair of gauges. For 2 or more independent pairs of gauges, this system of linear equations can be solved for the magnitude and direction of the wavenumber $\mathbf{k} = (k_x, k_y)$. With more than 2 pairs the system is overdetermined and therefore the multiple estimates of $\mathbf{k}$ yield an estimate of the random error in the estimate of $\mathbf{k}$. The energy of the wave having that wavenumber at that time is given by the magnitude squared of the corresponding wavelet coefficient. Finally, the directional spectrum is obtained by weighting the energy by the number of arrivals in each (2-dimensional) wavenumber bin. For further details see Donelan et al., (1996).

2. Measurements

The data used here for illustration were obtained from the research tower in Lake Ontario during the Water-Air Vertical Exchange Studies (WAVES) experiment n 1987 – the third and final year (see Figure 1). The tower is fixed to the bottom in 12 m of water at the western end of Lake Ontario, 1.1 km from the shore. It is exposed to fetches that vary from 1.1 km for westerly winds to 300 km for ENE winds. Wave directional measurements were made with an array of six capacitance wave gauges arranged in a centered pentagon of radius 25 cm.

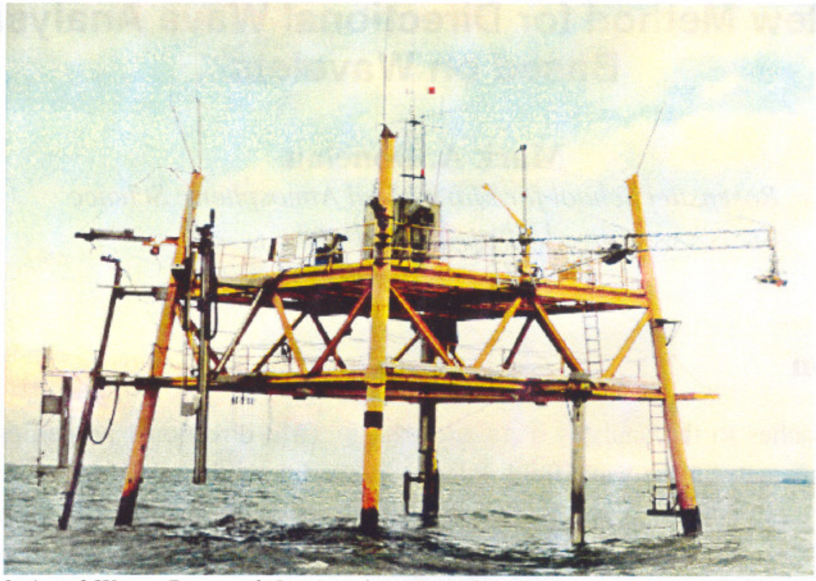

Figure 1. The National Water Research Institute's research tower in Lake Ontario. The array of six wave gauges are mounted on the east face of the tower. This view is from the north-west. The top deck measures 10 m by 10 m.

3. Results

The first step of the WDM is the calculation of the wavelet transform for the individual wave gauges. The "mother wavelet" chosen is the Morlet wavelet (Grossmann and Morlet, 1984). Figure 2 (a, b & c) shows the periodogram or time dependence of energy in various frequency bands or "scales". The panels (a), (b) and (c) are for three wave gauges separated by 10 m and 11 m in the downwind direction. The most energetic waves (spectral waves) have a frequency of 0.13 Hz and a wavelength of 72 m in this depth of water (12 m). The imprint of the passage of groups from one location to the other is evident and the time lag, though small, may be discerned. The phase difference between pairs of wave gauges in various frequency bands yields the vectorial wavenumber information as above; while the amplitude of the wavelet transform squared yields the energy in each component. This process therefore yields the variance of surface elevation assigned to various vectorial wavenumbers; i.e. the wavenumber directional spectrum. Since the original data are in the form of time series, the resulting frequency distribution of energy can be compared to the wavenumber directional information to yield information on the dispersion relation.

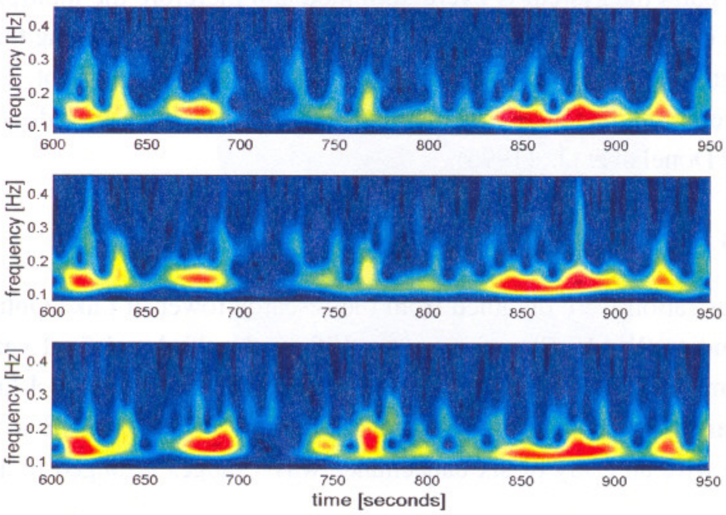

Figure 2. Time-frequency plots of the wavelet amplitude from three gauges separated by 10 m and 11 m in the downwind direction. The data are from the same tower in an earlier configuration (see Donelan et al., 1985).

The histogram of observed wavenumber magnitude for a case of long fetch (mature) waves is shown in Figure 3. Each panel of the figure is for a different frequency band. At and near the peak frequency the observed wavenumbers fall within the expected linear dispersion shell corresponding to the center (solid) and limits (dashed) of the frequency band. Figure 4 shows the corresponding histogram of direction. We note that the waves near the peak have a much narrower spreading than those above the peak.

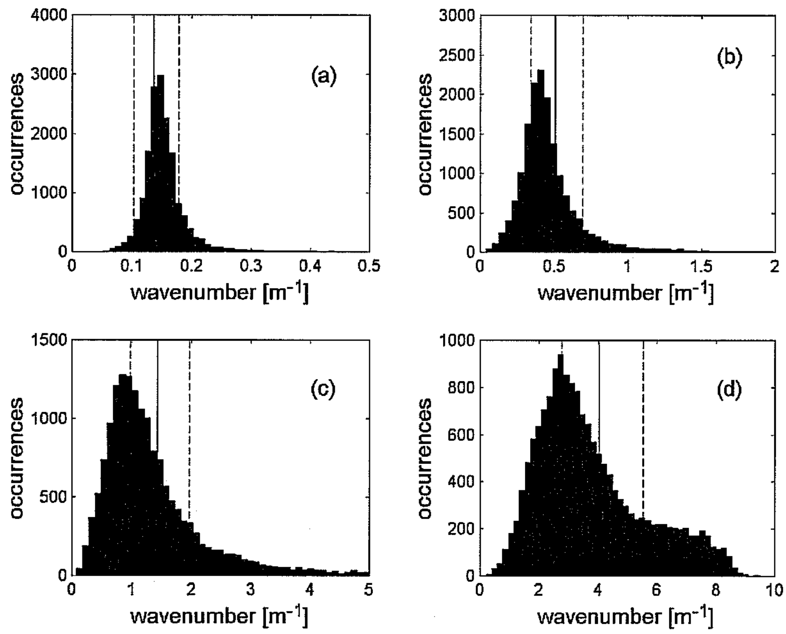

Figure 3. Histograms of the observed wavenumbers in particular frequency bands. The solid line is at the location of the wavenumber corresponding to the linear dispersion relation at the center frequency of the band. The dashed lines are the locations corresponding to the edges of the band. The center frequencies are 0.177, 0.354, 0.595 and 1.000 Hz. The corresponding wavenumbers and wave lengths are respectively: 0.136, 0.504, 1.42 and 4.03 m^{-1} and 46.2, 12.5, 4.41 and 1.56 m.

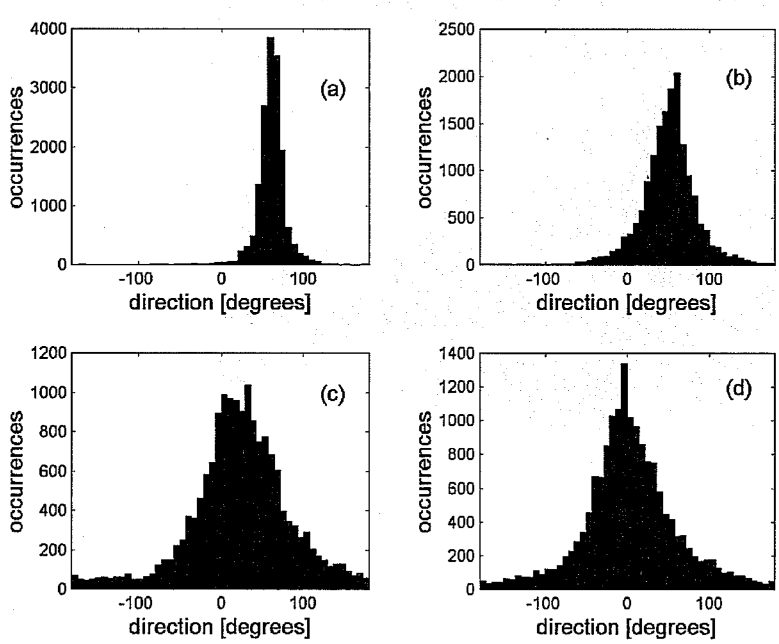

Figure 4. Histograms of the observed directions in particular frequency bands. The center frequencies are 0.177, 0.354, 0.595 and 1.000 Hz. The corresponding wavenumbers and wave lengths are respectively: 0.136, 0.504, 1.42 and 4.03 m^{-1} and 46.2, 12.5, 4.41 and 1.56 m.

Each wavenumber magnitude/direction pair is associated with a surface elevation amplitude and these are the basic wavenumber resolved parameters produced by the Wavelet Directional Method (WDM). The statistical sum of such time series data properly combined yields 2-D spectra such as the wavenumber-direction spectrum (Figure 5) or the frequency-direction spectrum (Figure 6) or the complete 3-D wavenumber-frequency spectrum. Examples of these are given below.

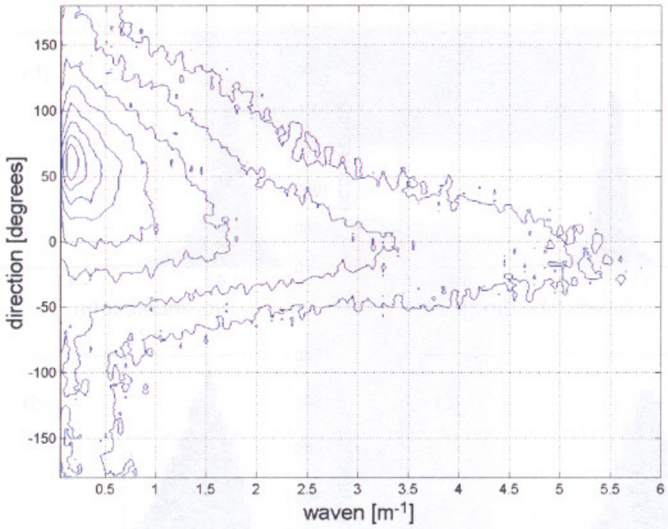

Figure 5a. The wavenumber directional spectrum. The eight contour lines are drawn at values decreasing by 4^{-n} so that the largest contour is drawn at ¼ of the peak spectral value and the smallest at 1/65536 of the peak.

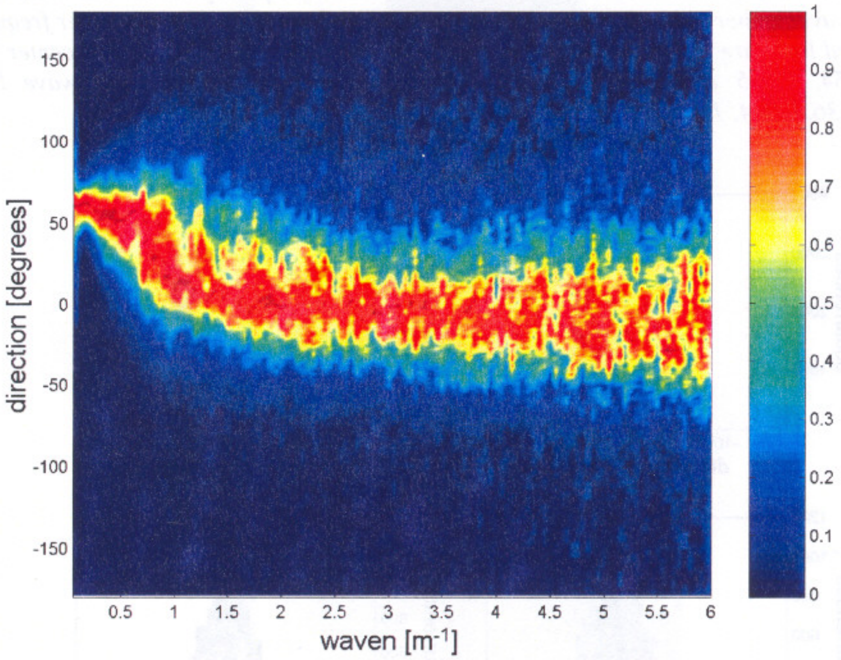

Figure 5b. The wavenumber spreading function displayed by normalizing each wavenumber slice of the directional spectrum by its maximum value.

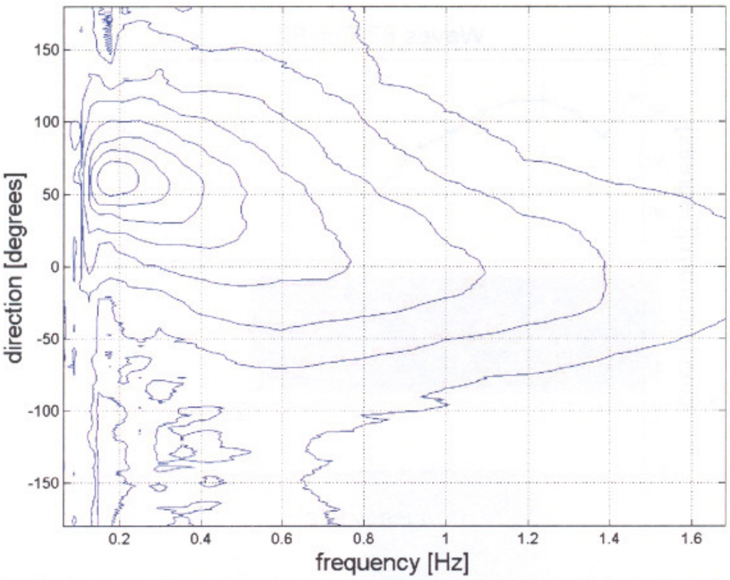

Figure 6a. The frequency directional spectrum. The eight contour lines are drawn at values decreasing by 4^{-n} so that the largest contour is drawn at ¼ of the peak spectral value and the smallest at 1/65536 of the peak.

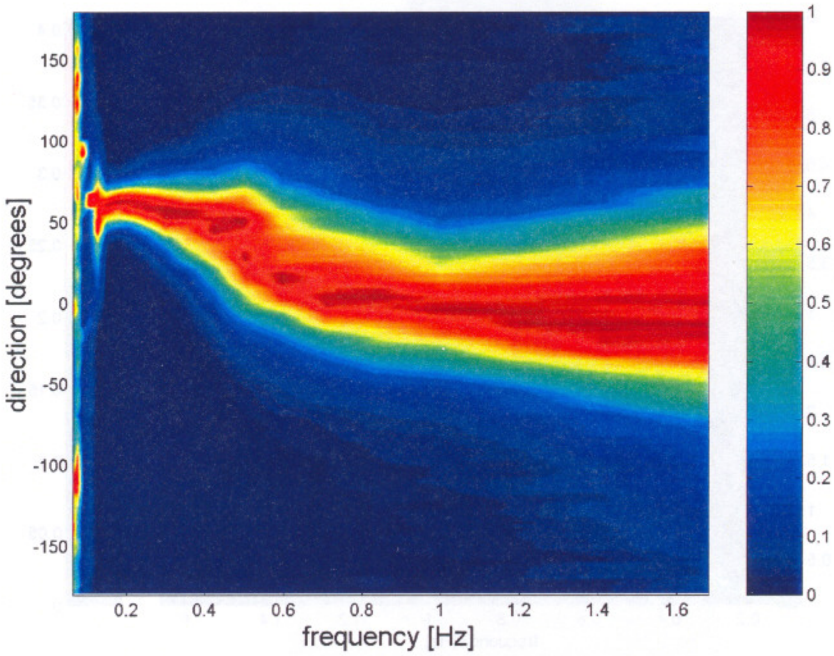

Figure 6b. The frequency spreading function displayed by normalizing each frequency slice of the directional spectrum by its maximum value.

Figures 5 and 6 illustrate the power of the method in delivering directional spectra. It is important to realize that the method yields the vector wavenumber spectrum directly as well as the frequency-direction spectrum. The above figures illustrate the spreading of the spectra away from the peak (0.177 Hz and 0.136 m^{-1}) as well as the turning of the waves towards the wind direction for the shorter components and towards the long fetch direction for the longer components (see also figure 7 and Donelan et al., 1985). Fetch-limited, steady-state cases like this, in which the shoreline geometry causes differential turning of the wave components, may well provide the most sensitive test-bed for examining the physics and numerics of advanced wave models.

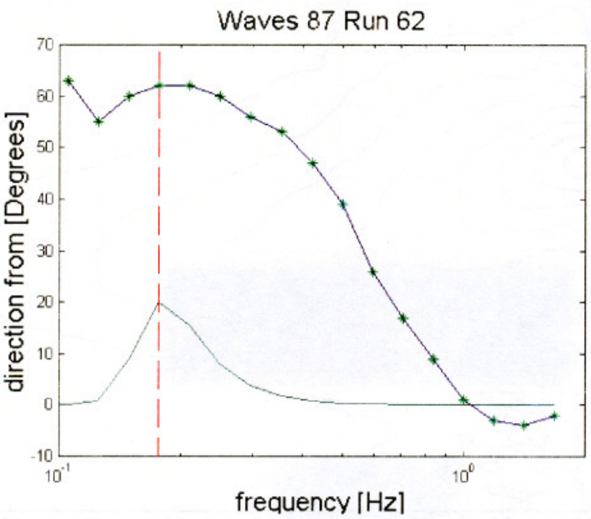

Figure 7. The frequency spectrum (solid line) and the peak direction at each frequency (line with asterisks) showing relaxation from the long fetch direction (east-north-east) to the wind direction (north).

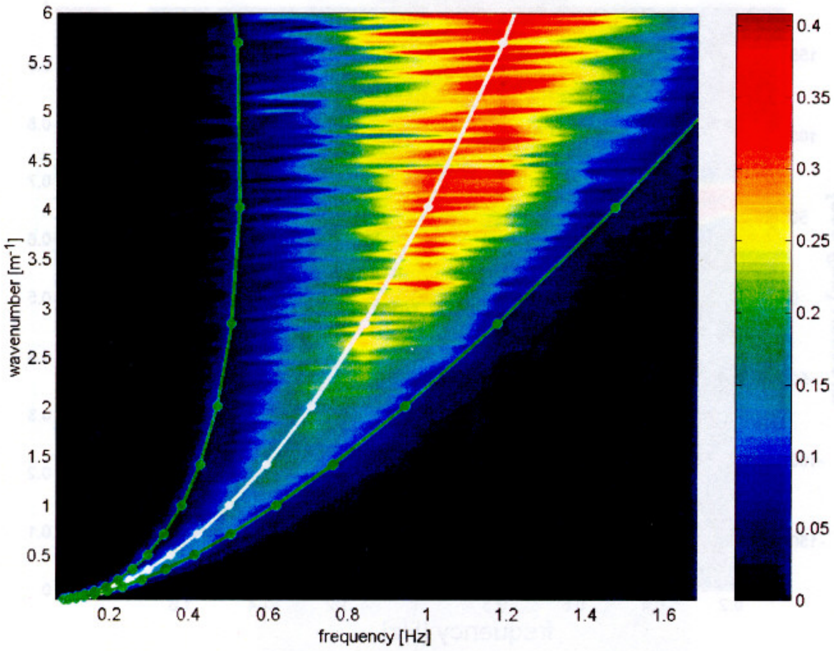

Figure 8. Curvature spectra (k^4 times the wavenumber spectrum) plotted on frequency and wavenumber magnitude. The ridge of the spectrum falls along the linear dispersion curve (white line) and the energy is bounded on both sides by the linear dispersion relation accounting for advection by the standard deviation of the orbital velocities (green lines) i.e. Doppler shifting. These mature waves are closely linear.

Figure 8 examines the dispersion characteristics of mature wind waves. The local wind speed is somewhat lower than the phase speed of the longest waves and these approach the tower from the east-north-east, some 60° away from the wind direction. These are "overdeveloped" waves in terms of the local wind and consequently are closely linear. The shorter waves travel slower than the wind and conform to its direction, but are not strongly forced enough to show much evidence of nonlinearity. The dispersion plot (figure 8) is consistent with a nearly linear wave field in which the shorter waves are Doppler shifted by the orbital velocities of the longer waves.

4. Summary

The methodology of the Wavelet Directional Method (WDM) has been outlined and illustrated by an example from Lake Ontario. The method was developed for array of wave gauges but may be applied to any of the traditional systems for gathering wave directional information such as: pitch-roll-heave buoys and pressure-current meter systems. The ability to derive wavenumber related information directly and to deal with non-steady situations are the principal advantages of the method and make it well suited to application in the verification of wave models.

Acknowledgments

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References

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