Realism of sensitivity patterns

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ECMWF

Acknowledgements to Jan Barkmeijer (now KNMI) and Mike Fisher

Content of talk

- Define sensitivity patterns
- Define "Key analysis errors"
- Discuss links between the structure and realism of sensitivity patterns and data assimilation
- Show vertical and horizontal structure of sensitivity patterns
- Show links between sensitivity patterns and Eady index
- Compare sensitivity patterns and sensitivity perturbed forecasts against observations
- Conclusions

Sensitivity method

Method developed at MeteoFrance/ECMWF primarily by Florence Rabier

F. Rabier et al. "Sensitivity of forecast error to initial conditions" Q.J.R.Meteorol.Soc. (1996),122, pp. 121-150.

- **Use 48 hour forecast error as penalty term in the cost function**
- **Define a norm to enable calculation of the difference between two atmospheric states**
- **Use adjoint of tangent linear model to determine perturbation at initial time**

Sensitivity calculations

The diagnostic function to be minimised:

$$J = 0.5 < \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana}), \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana}) > \quad \textbf{or}$$

$$J = 0.5 < \mathbf{P}(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{ver.ana}), \mathbf{P}(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{ver.ana}) >$$

where $\mathbf{P}$ is the projection on the area (30ºN;90ºN)

$\mathbf{M}$ represents the non-linear model integrated for 48 hours (time t)

$\mathbf{x}_t^{ver.ana}$ represents the verifying analysis valid at 48 hour forecast time (t)

A norm is required to quantify the forecast error.

An often used definition is the square energy norm:

$$< \mathbf{x}, \mathbf{x} > = 0.5 \int_0^1 \int \int_A (u^2 + v^2 + R_d T_r (\ln p_s)^2 + T^2 C_p / T_r) dA (\partial p_r / \partial \eta) d\eta$$

Sensitivity gradients

The gradient of J at time t can be written as:

$$\nabla J_t = \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

If the tangent linear approximation is valid for 48 hours:

$$\delta \mathbf{x}_t = \mathbf{M}(\mathbf{x}_0 + \delta \mathbf{x}_0) - \mathbf{M}(\mathbf{x}_0) \approx \mathbf{R}\delta \mathbf{x}_0$$

where $\mathbf{R}$ represents the tangent linear model,
it can be shown that

$$\nabla J_0 = \mathbf{R}^* \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

where $\mathbf{R}^*$ represents the adjoint of the tangent linear model

∇J_0 is the sensitivity of the forecast error to the initial condition

Sensitivity gradient example

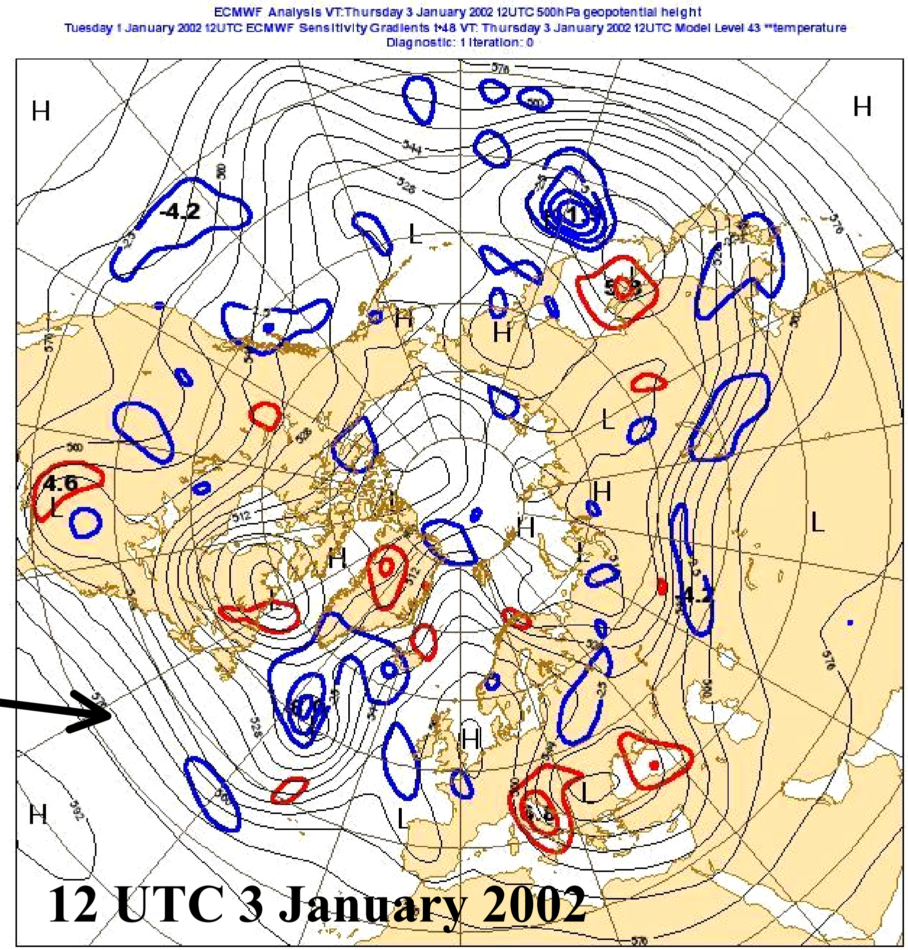
$$\nabla J_t = \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

Example of the gradient of J

$$\nabla J_t = \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

at time t=48h for temperature at level 43 (650 hPa) on 12 UTC 3 January 2003

Black contours: Z500 hPa analysis valid at 12 UTC 3 January 2003

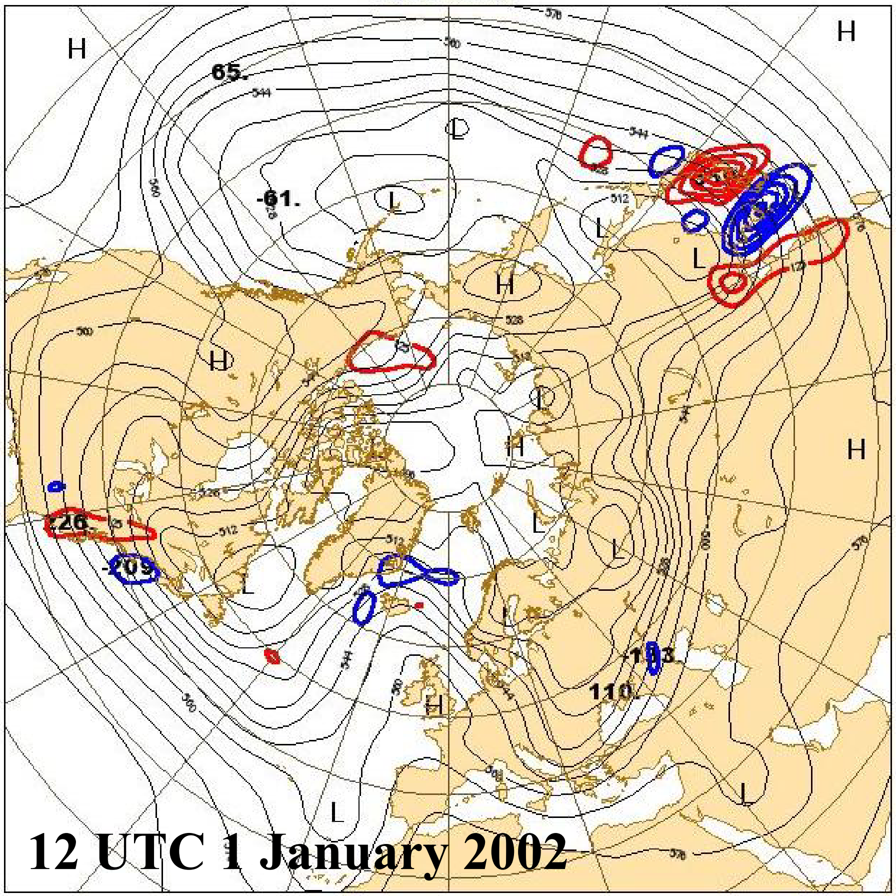


Sensitivity gradients at t=0 and t=48

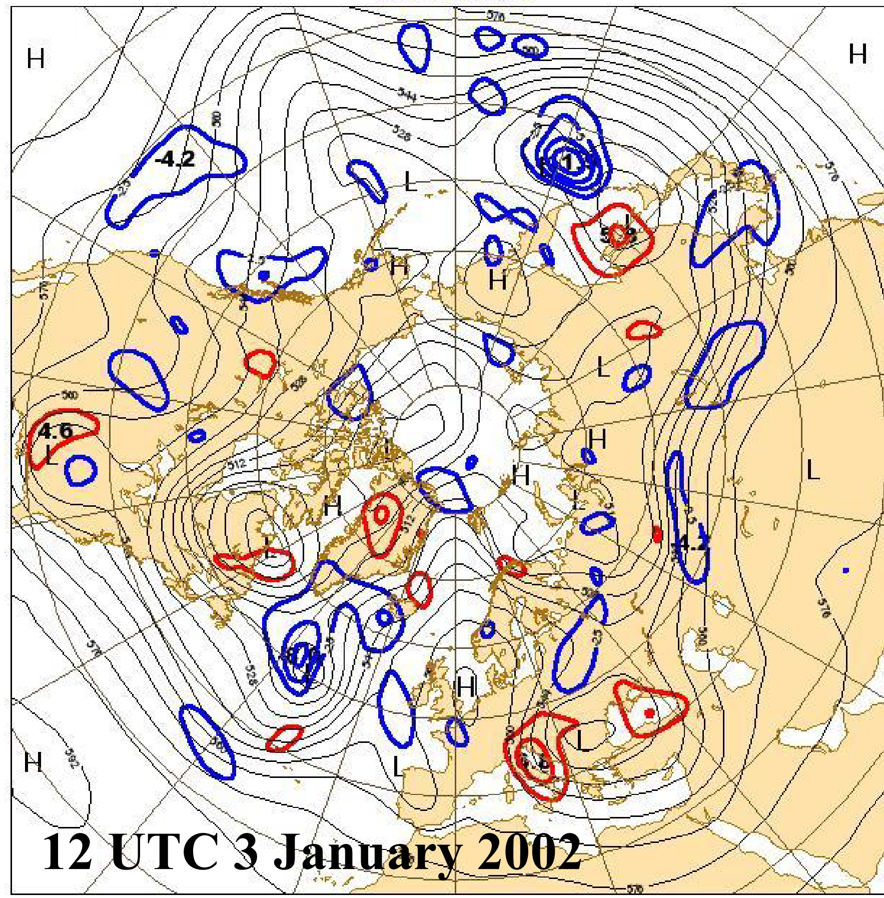
$$\nabla J_0 = \mathbf{R}^* \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

$$\nabla J_t = \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

ECMWF Analysis VT:Tuesday 1 January 2002 12UTC 500hPa geopotential height
Tuesday 1 January 2002 12UTC ECMWF Sensitivity Gradients t+0 VT: Tuesday 1 January 2002 12UTC Model Level 43 **temperature
Diagnostic: 1 Iteration: 0



ECMWF Analysis VT:Thursday 3 January 2002 12UTC 500hPa geopotential height
Tuesday 1 January 2002 12UTC ECMWF Sensitivity Gradients t+48 VT: Thursday 3 January 2002 12UTC Model Level 43 **temperature
Diagnostic: 1 Iteration: 0



From sensitivity gradient to perturbation

This method only determines the <u>gradient</u> at initial time:

$$\nabla J_0 = \mathbf{R}^* \mathbf{P}(\mathbf{x}_t^{fc} - \mathbf{x}_t^{ver.ana})$$

A perturbation is found by trial-and-error, based on typical values for fastest growing singular vectors ($\lambda = 10 - 15$ times amplification in 48 hours).

It can be shown (Rabier et al. 1996 QJRMS) that a good perturbation estimate can be expected if:

$$\delta x_0 = -\alpha \nabla J_0 \approx -\frac{1}{\lambda^2} \nabla J_0 \qquad \alpha \approx \left[\frac{1}{15^2}; \frac{1}{10^2}\right] = [0.004; 0.01]$$

Adding such a perturbation to the initial analysis field in most cases improve the 2-5 day forecast - because information from observations during the first two forecast days is included .

“Key analysis errors”

- Klinker, Rabier and Gelaro “Estimation of key analysis errors using the adjoint technique” QJRMS (1998),124, pp. 1909-1933

- **Extended the sensitivity method so it could determine the perturbation step-size**
- **Performed a number of iterations to partially minimize the objective cost function**
- **Three iterations with the energy norm gave the best fit to observations and meteorologically reasonable perturbations**
- **These perturbations were called “Key analysis errors” because they were expected to describe the most important analysis errors**

“Key analysis errors”

For the sensitivity gradient we previously defined:

$$J = 0.5 < \mathbf{P}(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{\mathrm{ver.ana}}), \mathbf{P}(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{\mathrm{ver.ana}}) >$$

where $\mathbf{P}$ is the projection on the area (30ºN;90ºN)

$\mathbf{x}_t^{\mathrm{ver.ana}}$ represents the verifying analysis valid at 48 hour forecast time (t)

This can be also be written as:

$$J = 0.5(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{\mathrm{ver.ana}})^{\mathrm{T}} \mathbf{A}(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{\mathrm{ver.ana}})$$

where $\mathbf{A}$ is the matrix defining the inner product including the projection on the area (30ºN;90ºN)

The first order approximation of cost function change with respect to increment is:

$$\delta J = (\mathbf{R}\|\delta \mathbf{x}_0\|)^{T} \mathbf{A}(\mathbf{M}(\mathbf{x}_0) - \mathbf{x}_t^{\mathrm{ver.ana}})$$

where $\mathbf{R}$ represents the tangent linear model

“Key analysis errors”

It can be shown (Klinker et al. 1998 QJRMS) that the maximum cost function change under the constraint $\left\| \delta \mathbf{x}_0 \right\|_c^2 = N$ **is:**

$$\delta x_0 = \frac{1}{2\lambda} \nabla J_c \qquad \textbf{where} \quad \lambda^2 = \frac{1}{4N} \nabla J_c{}^T \mathbf{C} \nabla J_c$$

“Key analysis errors”

It can be shown (Klinker et al. 1998 QJRMS) that the maximum cost function change under the constraint $\left\| \delta \mathbf{x}_0 \right\|_c^2 = N$ is:

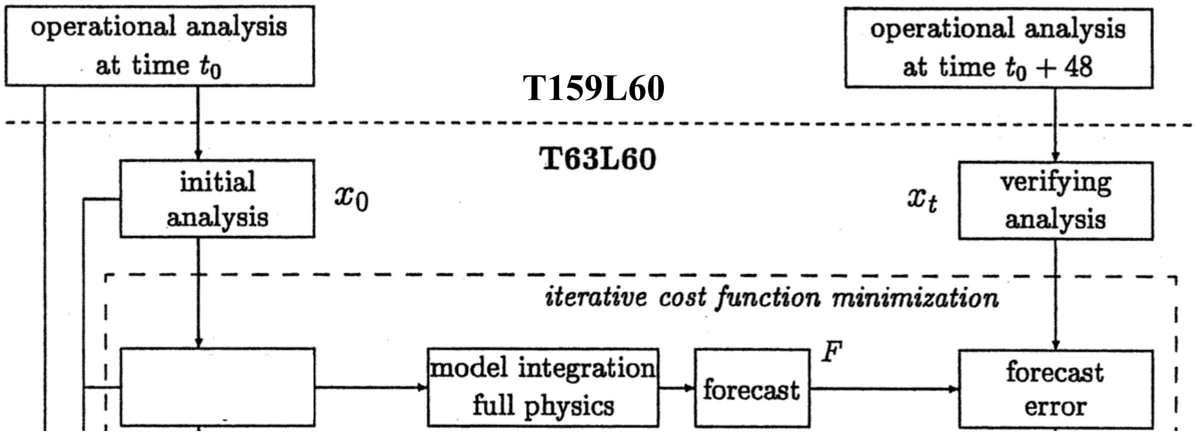
$$\delta x_0 = \frac{1}{2\lambda} \nabla J_c \quad \text{where} \quad \lambda^2 = \frac{1}{4N} \nabla J_c{}^T \mathbf{C} \nabla J_c$$

Ignore the mathematics!

The important things to note:

- An optimal step-size δx_0 can be determined
- The step-size depends on the choice of inner-product norm
- The spatial pattern depends also on the norm
- Validity of tangent linear approximation for 48 hours assumed

Layout of “key analysis error” calculations



Thanks! Francois

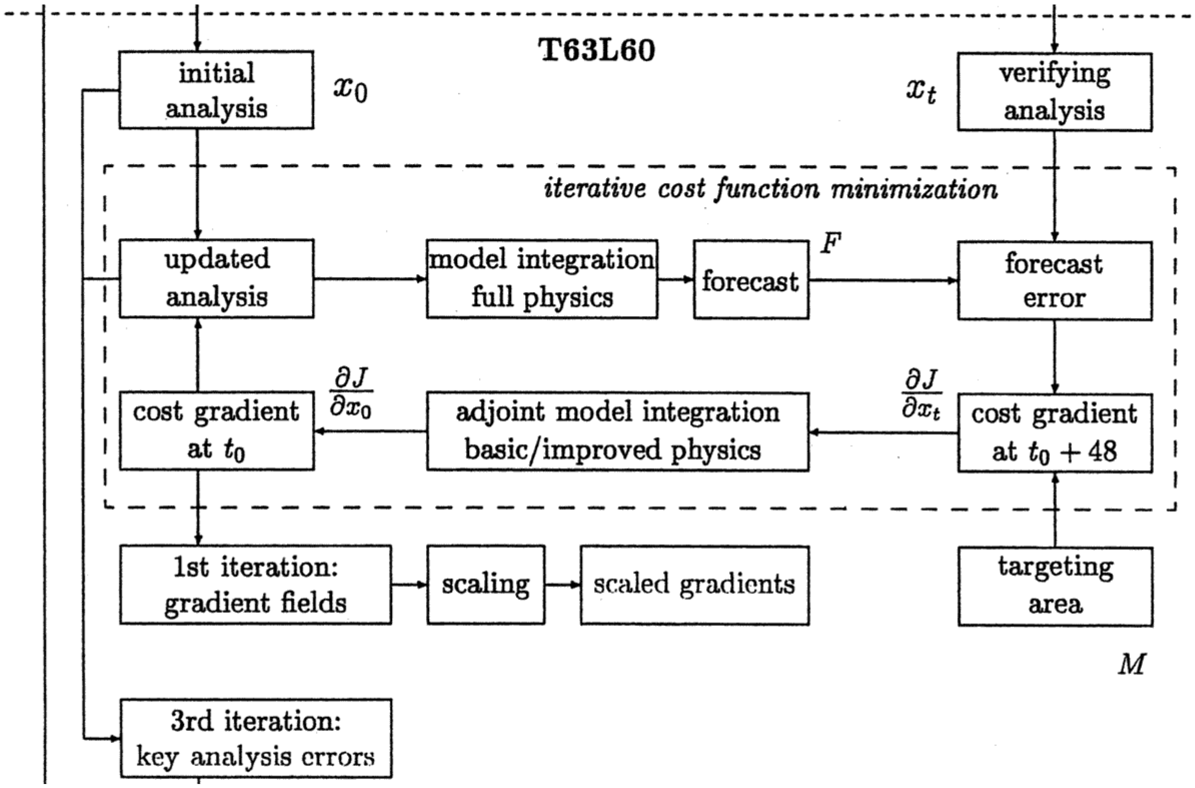
Climatologies of sensitive areas for short-term forecast errors over Europe

EUMETNET-EUCOS Study

TM 334 2001

G.J. Marseille and F. Bouttier

Layout of "key analysis error" calculations



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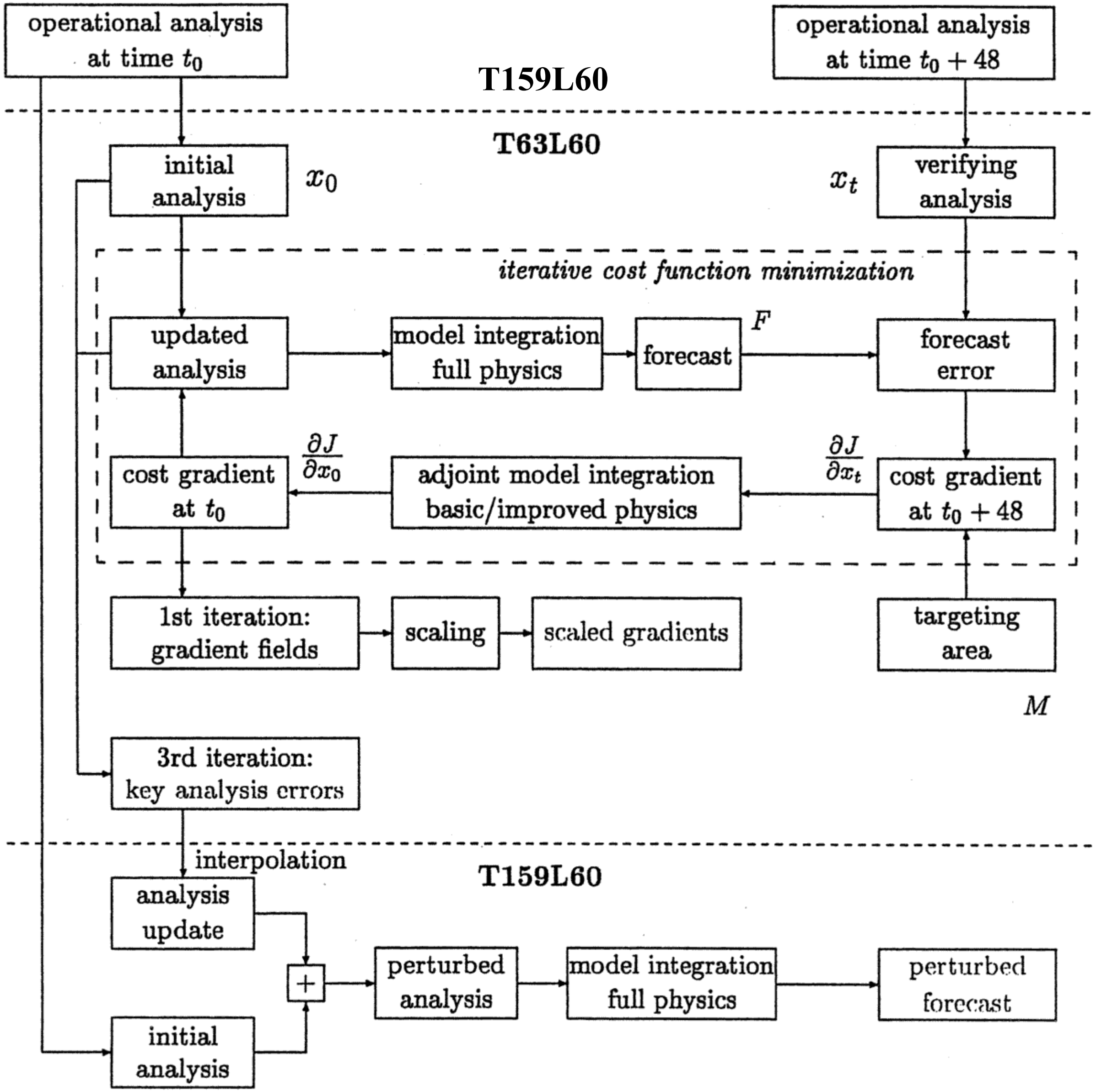

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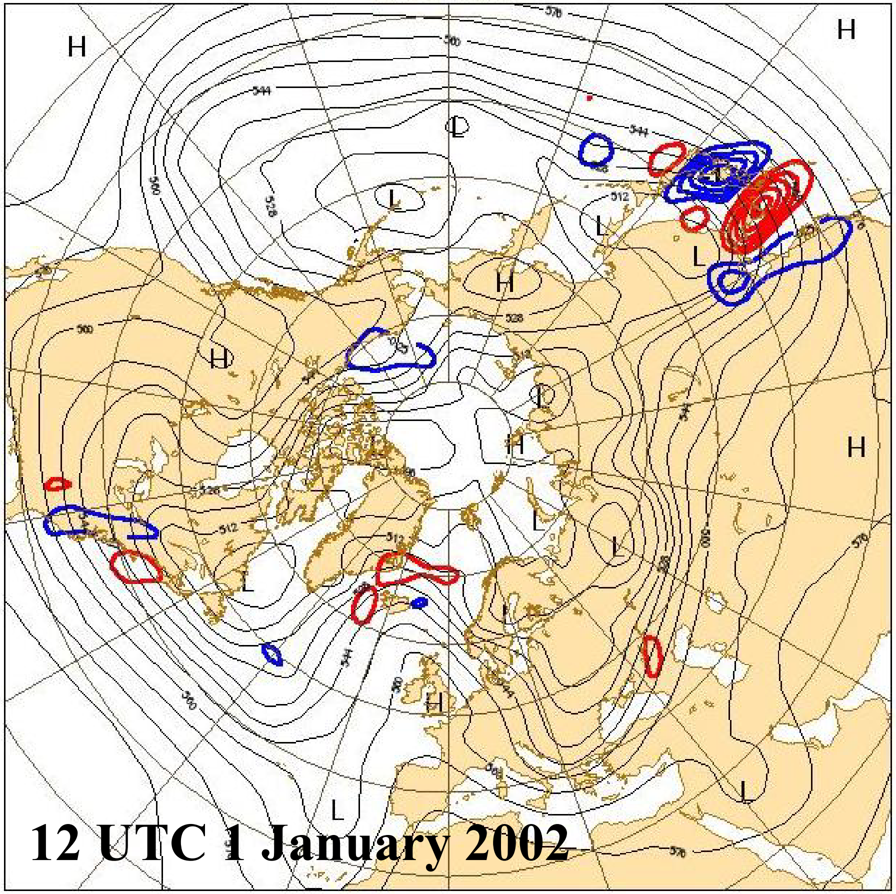
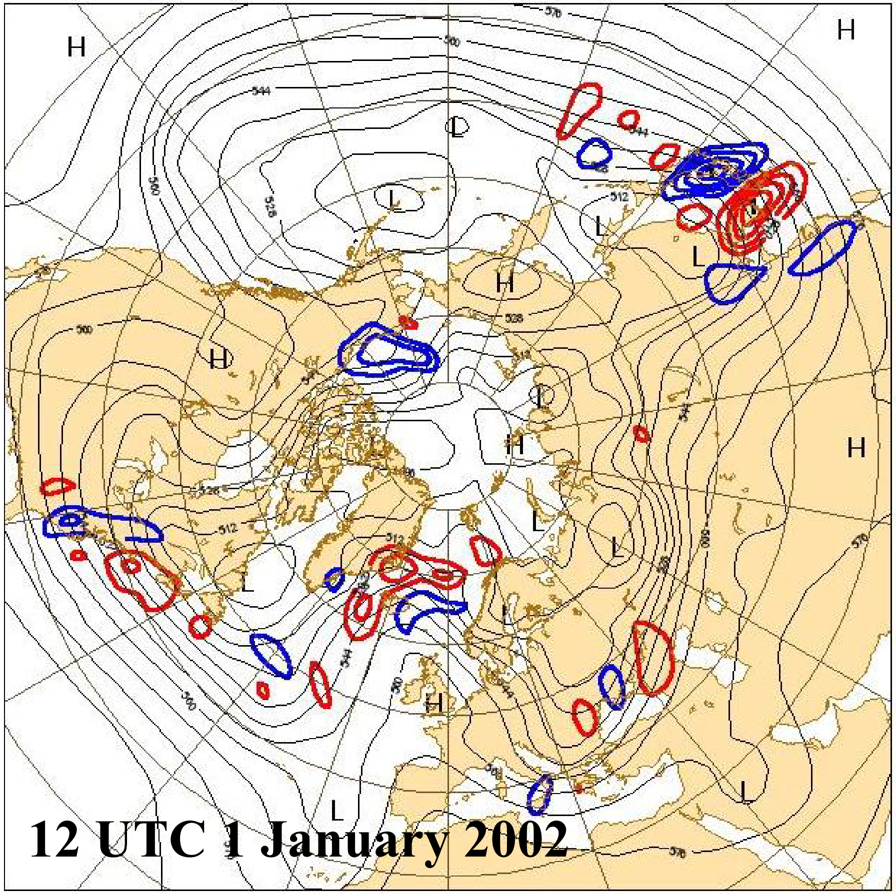
Key analysis errors – an example

Temperature perturbation at 650 hPa after 1 iteration (0.3 K contouring)

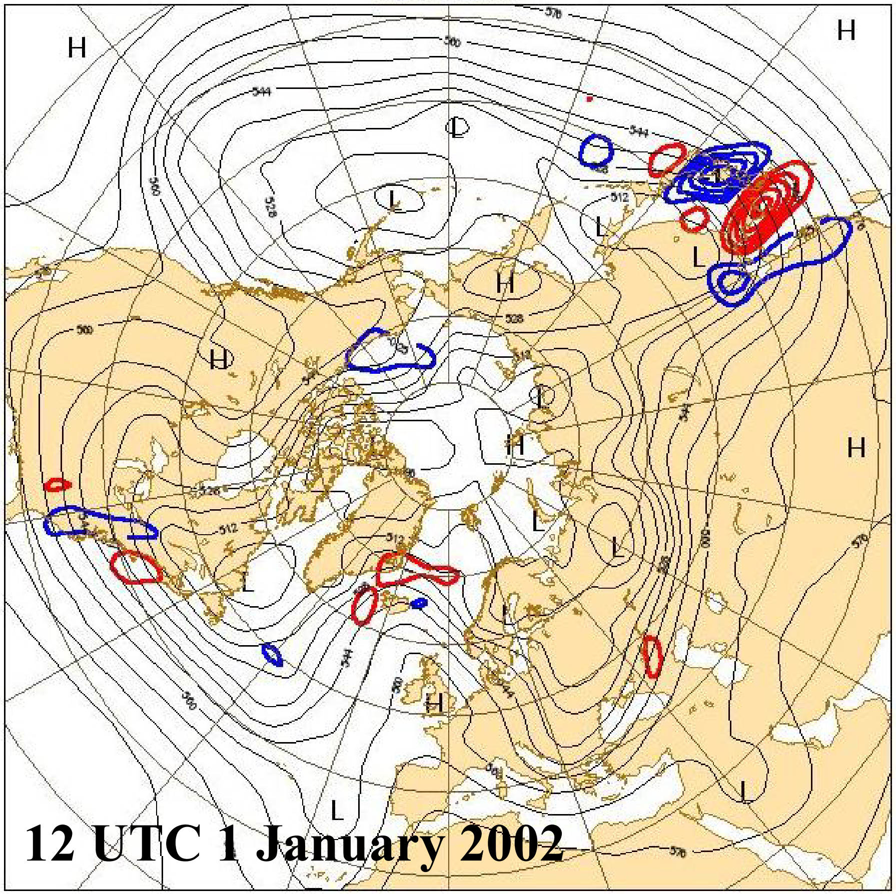


Temperature perturbation at 650 hPa after 2 iterations (0.3 K contouring)

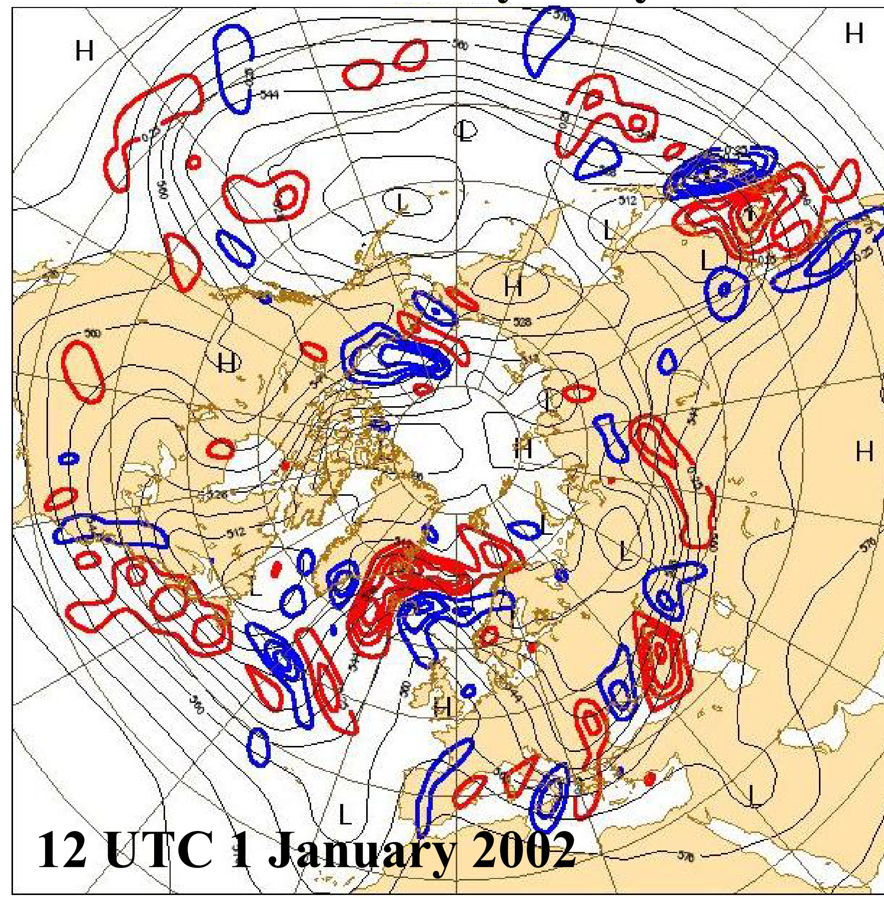


Key analysis errors – an example

Temperature perturbation at 650 hPa after 1 iteration (0.3 K contouring)



Temperature perturbation at 650 hPa after 3 iterations = Key analysis errors



Hessian norm and Jb norm

The energy norm and two other norms have been used in my study: the "approximate Hessian norm" and the J_b norm.

The Hessian approximation used in the assimilation system is

$$\mathbf{H} = \mathbf{B}^{-1/2}(\mathbf{I} + \sum_{i=1}^{L}(\mu_i - 1)\mathbf{w}_i\mathbf{w}_i^T)\mathbf{B}^{-1/2}$$

where $\mathbf{w}_i$ are the L=100 leading eigenvectors of the Hessian and B is the background error covariance matrix

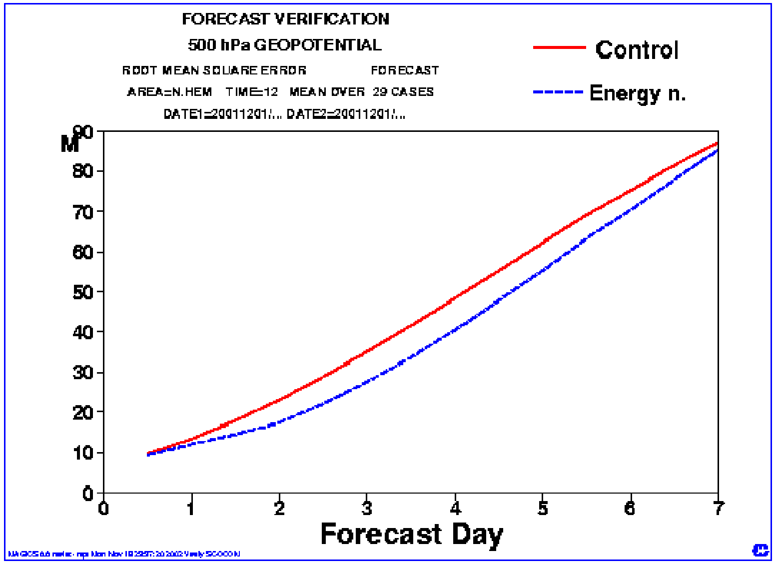
The J_b norm does not include any Hessian information, i.e. L=0 above, so:

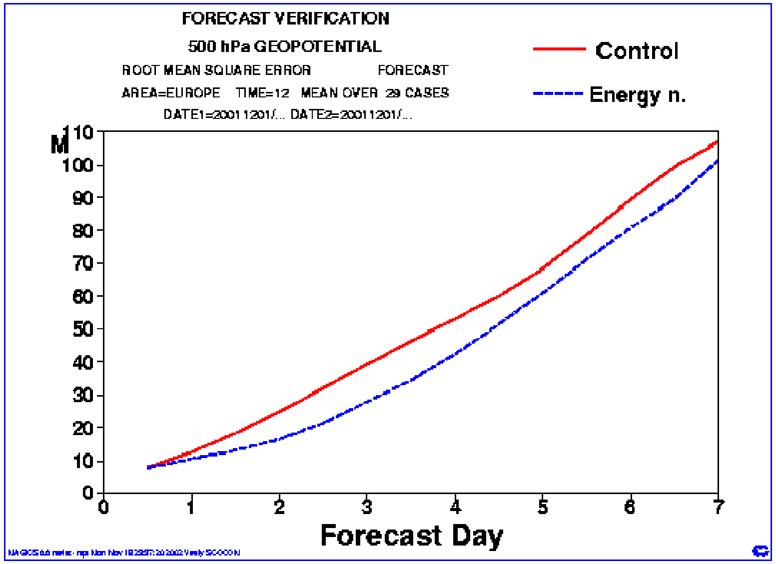
$$\mathbf{H} = \mathbf{B}^{-1/2}(\mathbf{I})\mathbf{B}^{-1/2} = \mathbf{B}^{-1}$$

My sensitivity experiments

- **T159/T159 4D-Var assimilations were performed for December 2001 and January 2002**
- **For the assimilation experiments "key analysis errors" were calculated daily based on respectively:**
- **Energy norm sensitivities at 1200 UTC + 48 hours**
- **Jb norm sensitivities at 0300 UTC + 48 hours**
- **Hessian norm sensitivities at 0300 UTC + 48 hours**
- **The structure of the different sensitivity patterns were explored**
- **Short range (24 hour) forecasts which included comparison against good observations at proper time and location were run**
- **Observation statistics from these runs were used to investigate the realism of sensitivity patterns**

Scores for control and sensitivity forecasts December 2001/January 2002

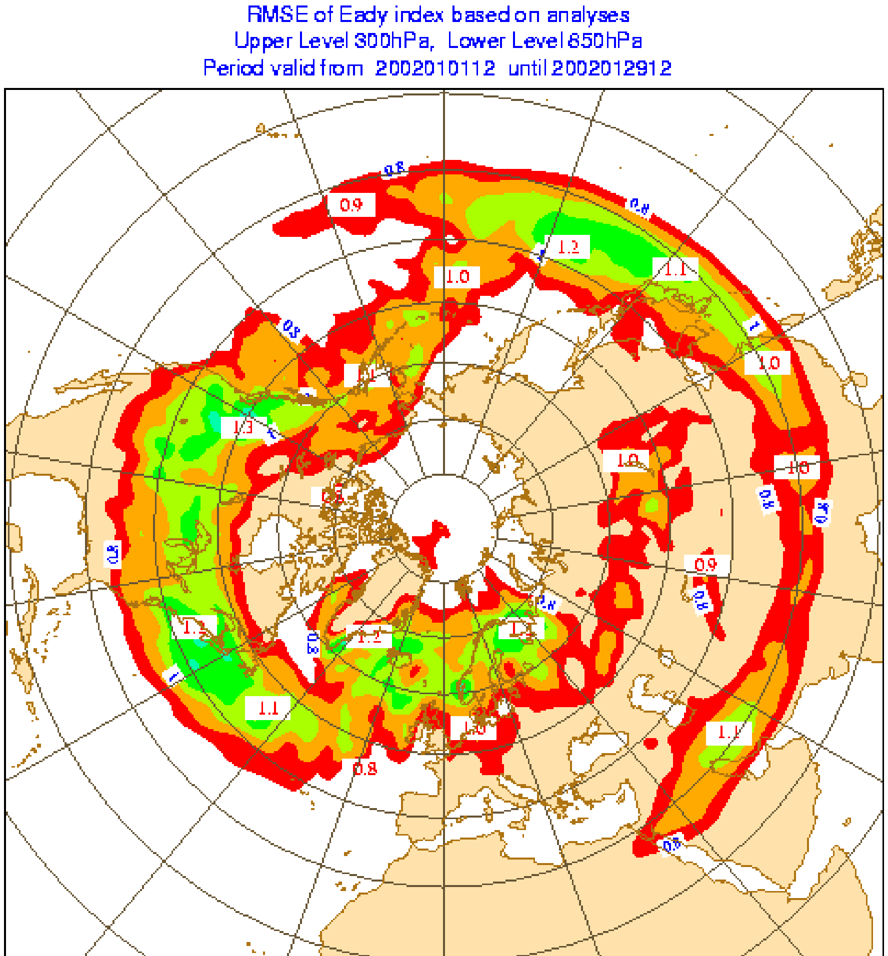




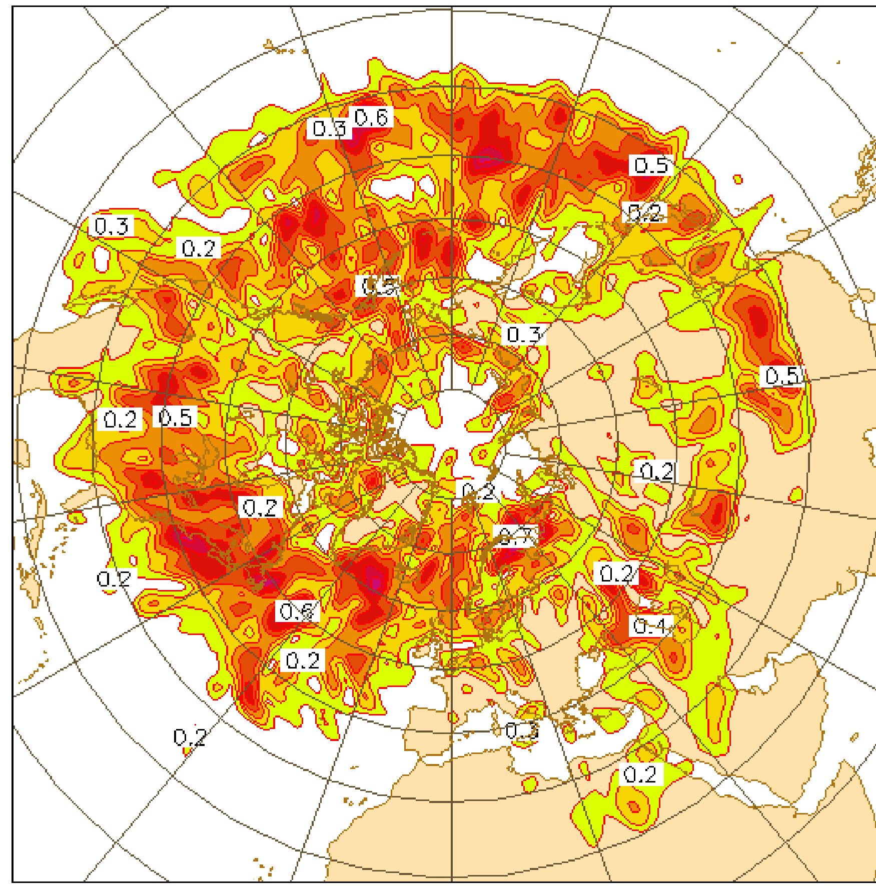
As expected:
The "key analysis error" modified analyses results in improved 2-7 day forecasts

Eady index and rms of Energy norm sensitivity temperatures. January 2002

Eady index



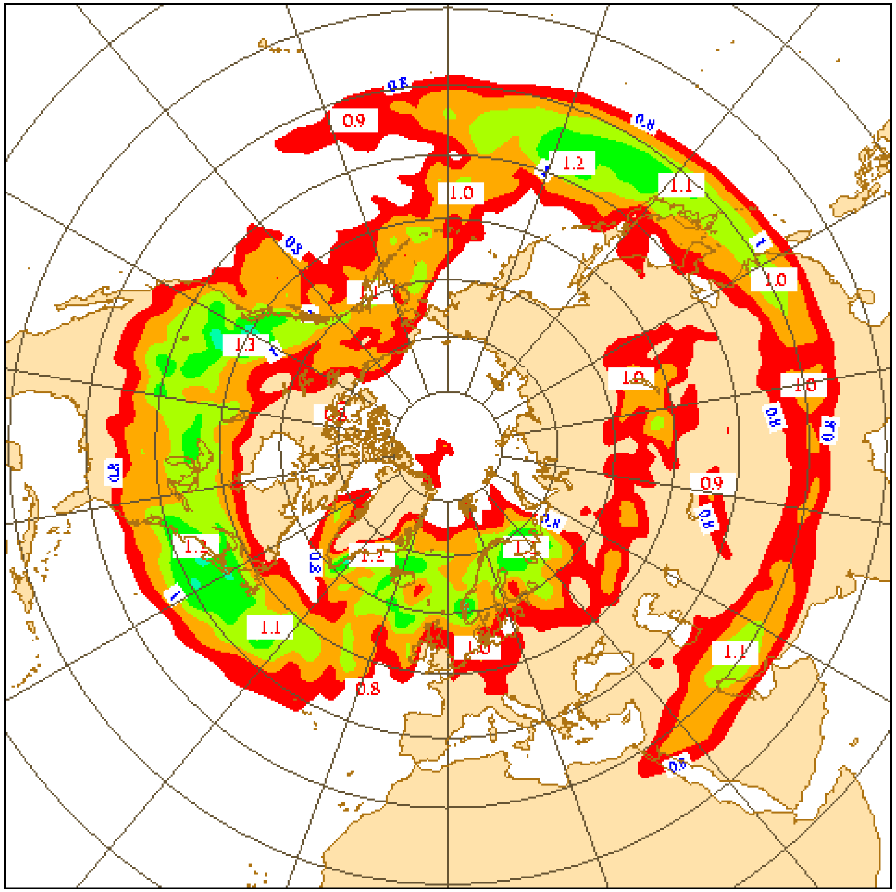
Rms of energy norm sensitivity temperatures level 42 January 2002



Eady index and rms of Hessian norm sensitivity temperatures. January 2002

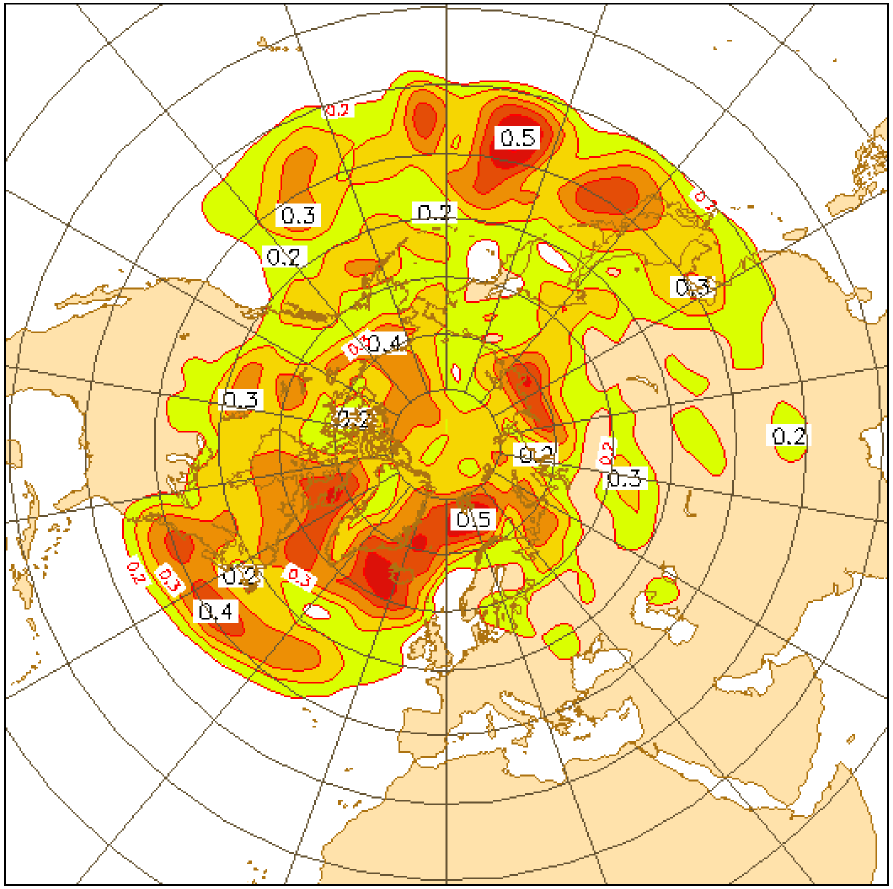
Eady index

RMSE of Eady index based on analyses
Upper Level 300hPa, Lower Level 850hPa
Period valid from 2002010112 until 2002012912



Hessian norm sensitivities

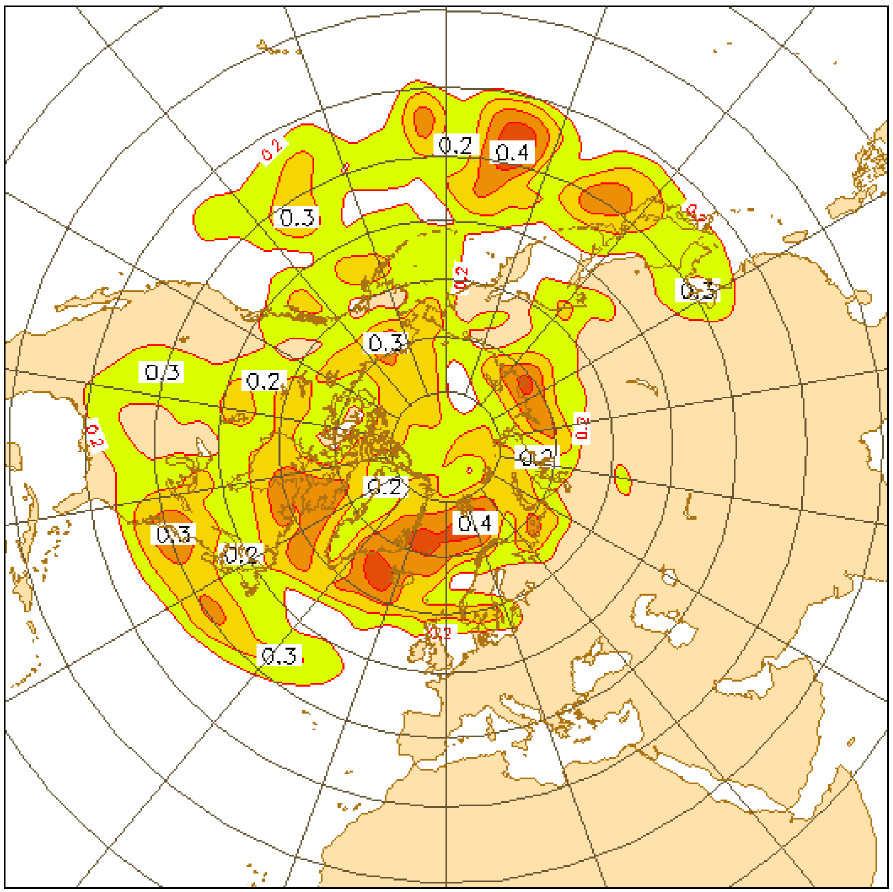
ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 **temperature



rms of Jb and Hessian norm sensitivity temperatures. January 2002

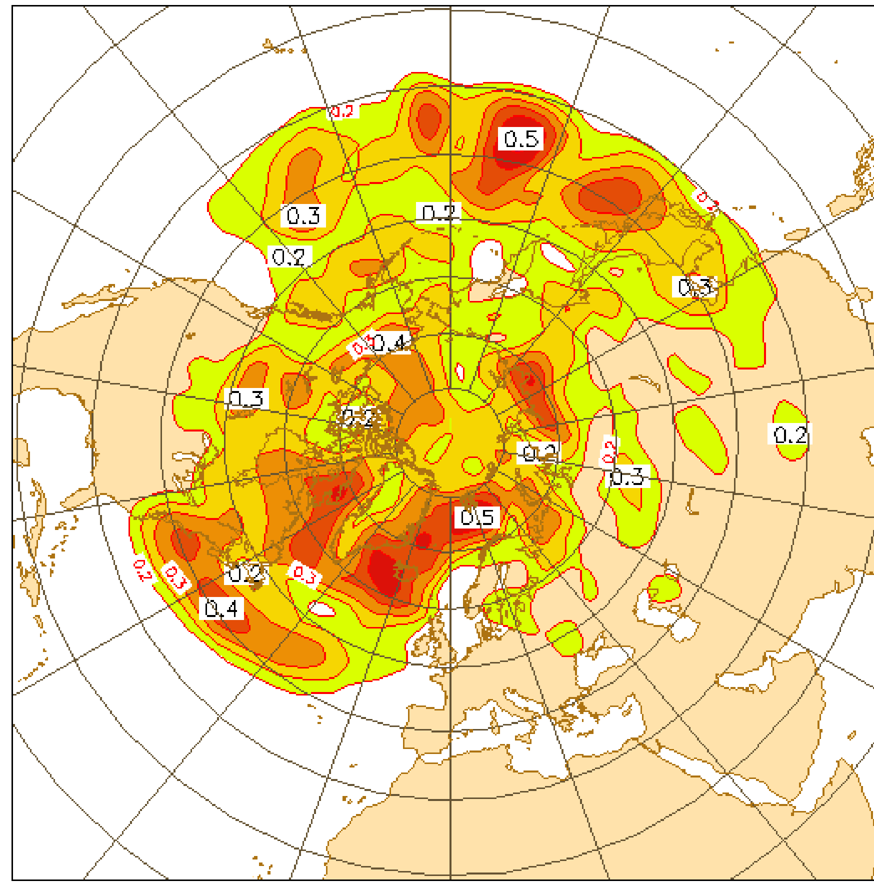
Jb norm sensitivity

ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 **temperature



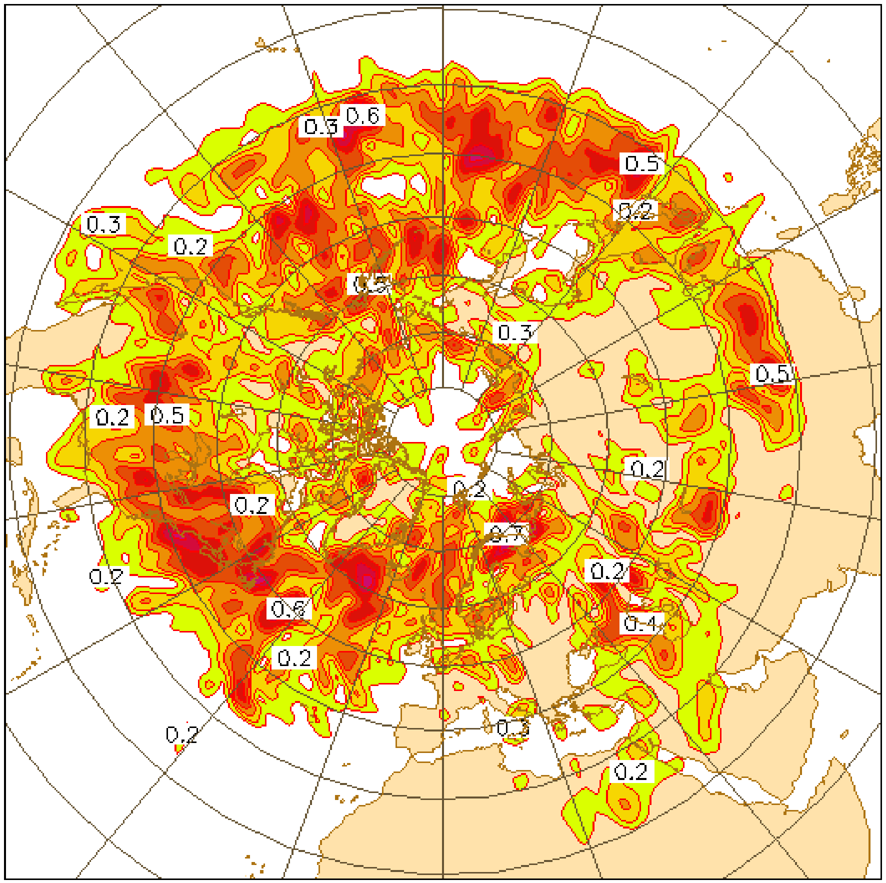
Hessian norm sensitivity

ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 **temperature

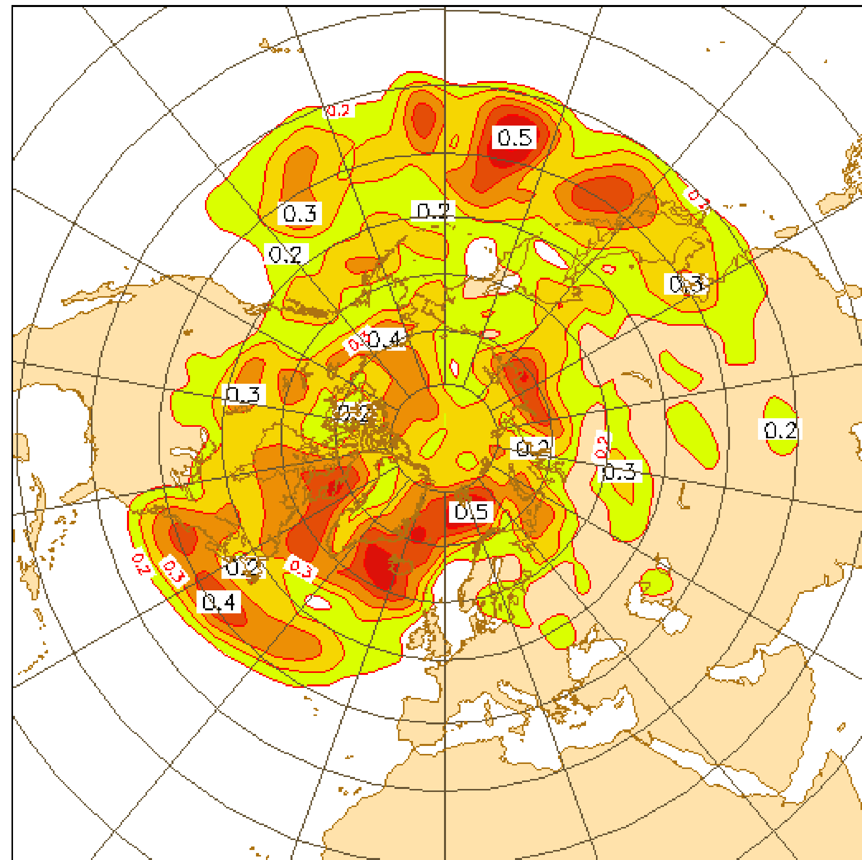


rms of energy norm and Hessian norm sensitivity temperatures. January 2002

- Energy norm sensitivity   Hessian norm sensitivity



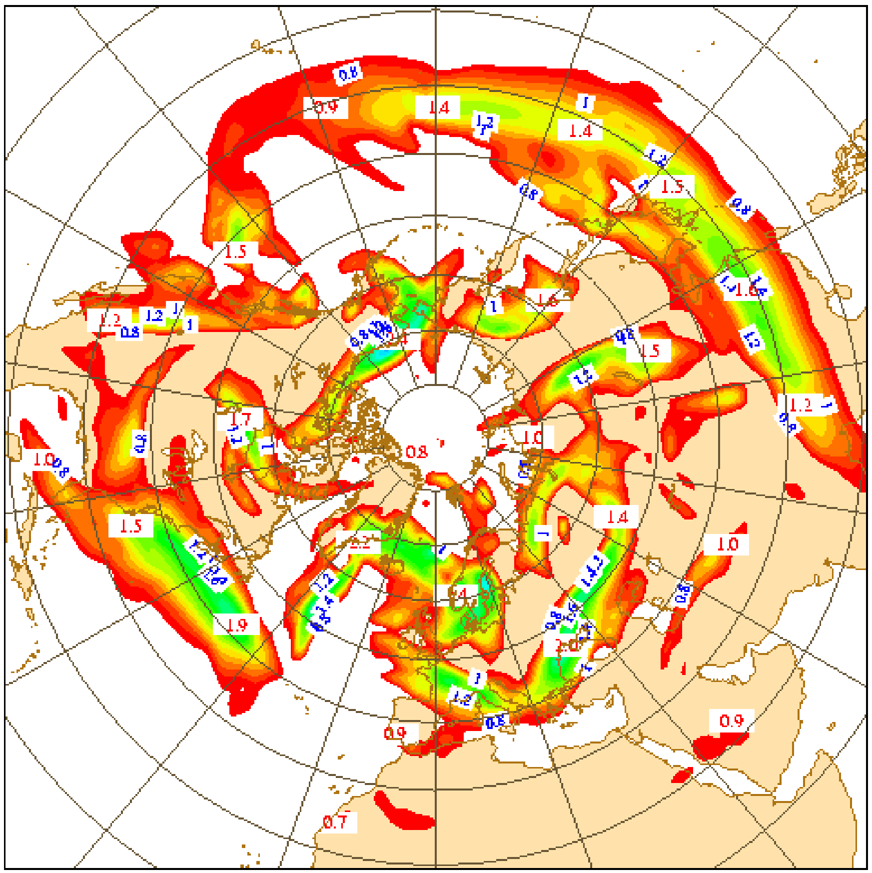
ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 **temperature



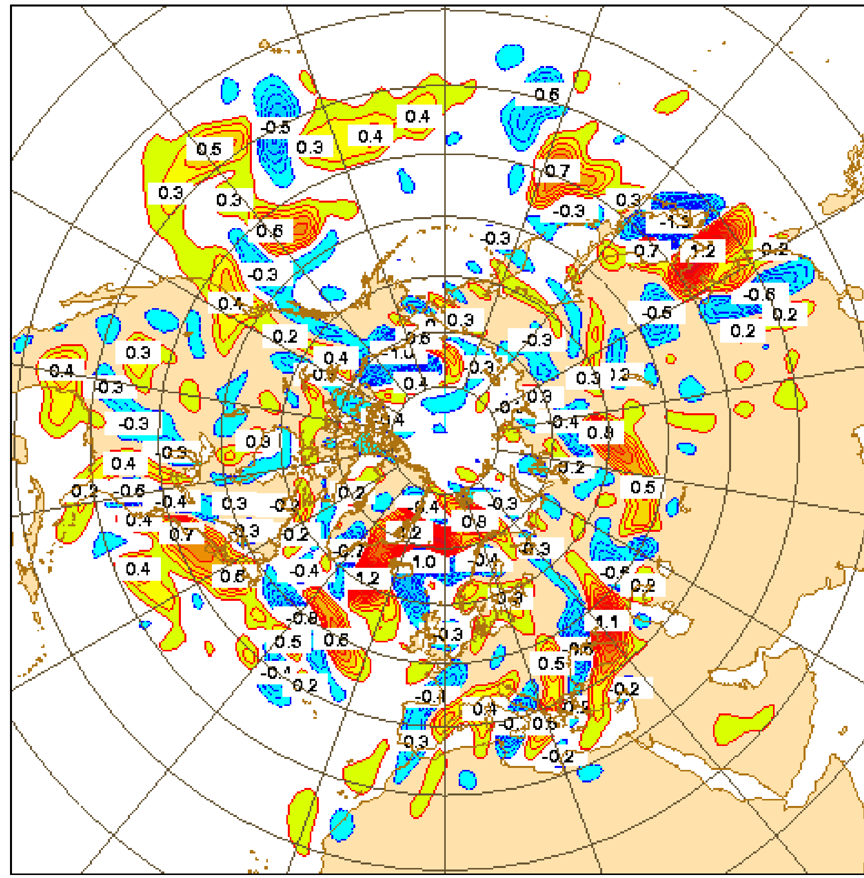
1 January 2002 case study

Eady index

ECMWF Analysis VT:Tuesday 1 January 2002 06UTC 300hPa **geopotential height

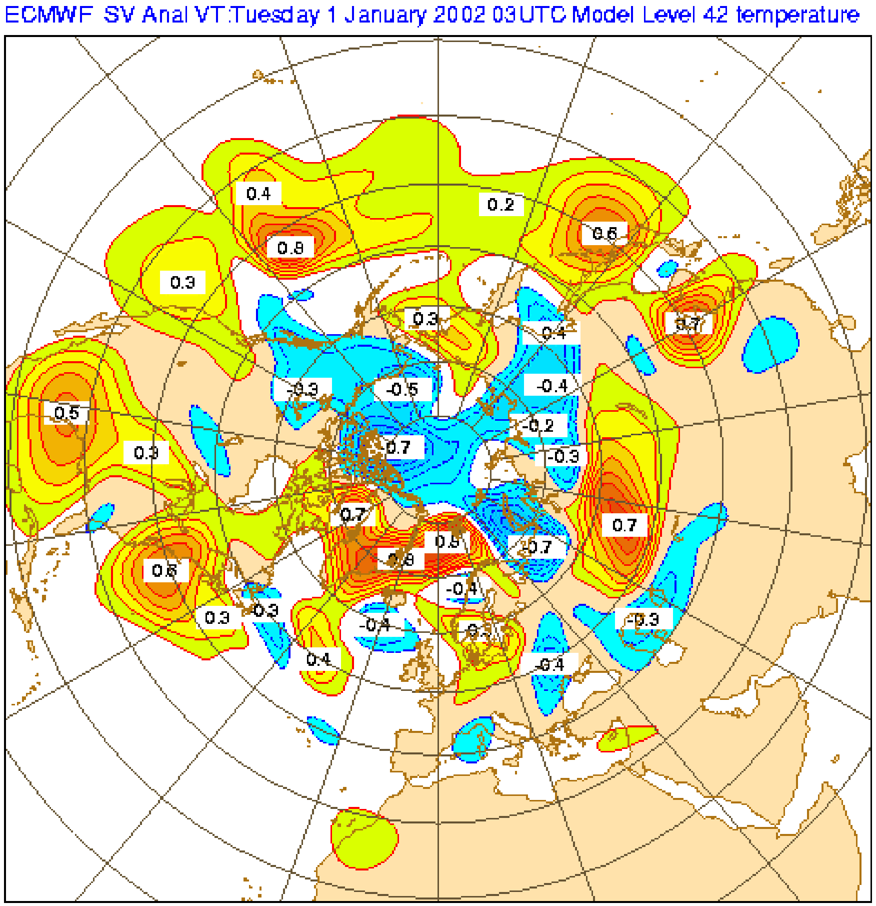


Energy norm sensitivity Temperature level 42

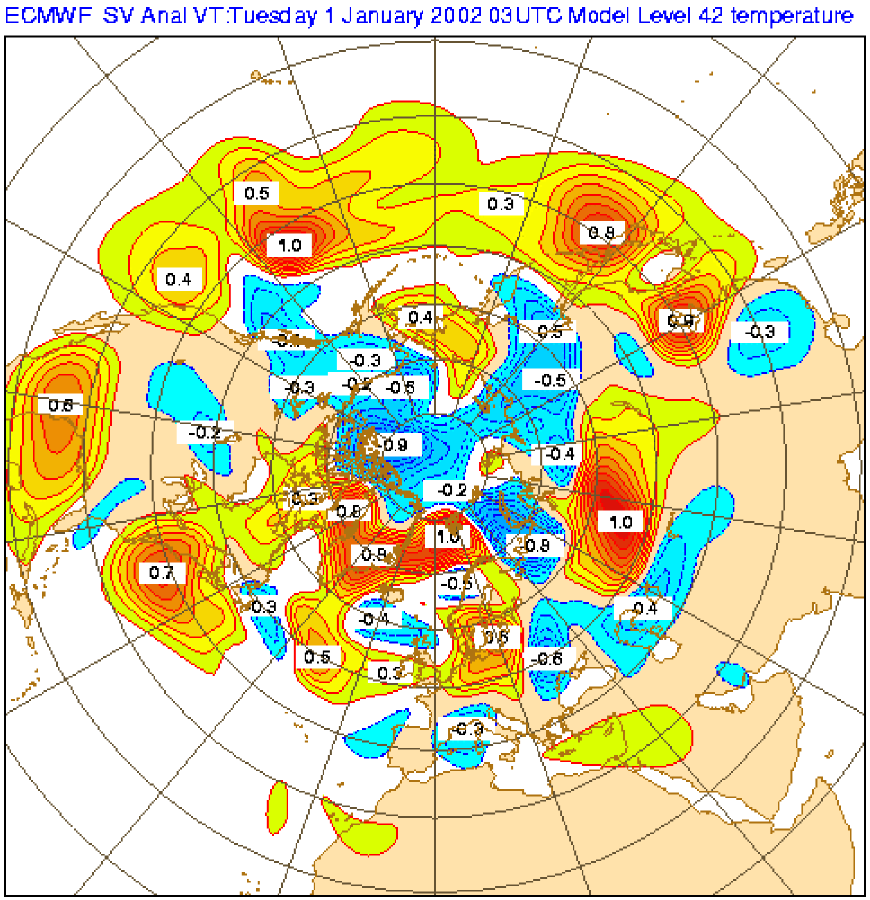


1 January 2002 case study

Jb norm sensitivity

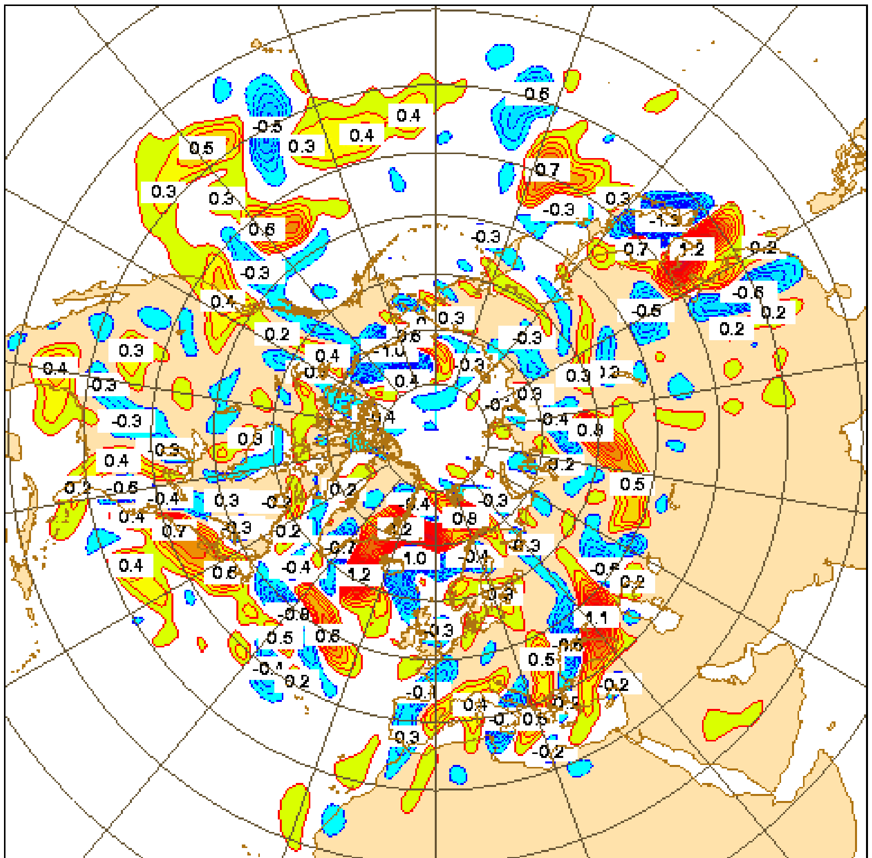


Hessian norm sensitivity



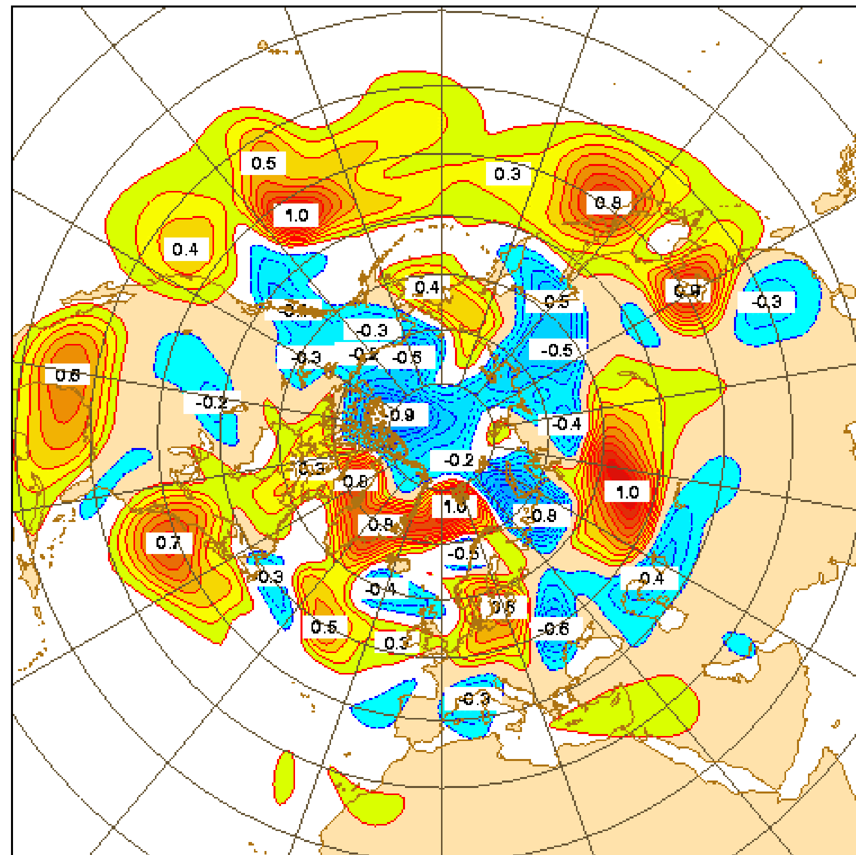
1 January 2002 case study

Energy norm sensitivity



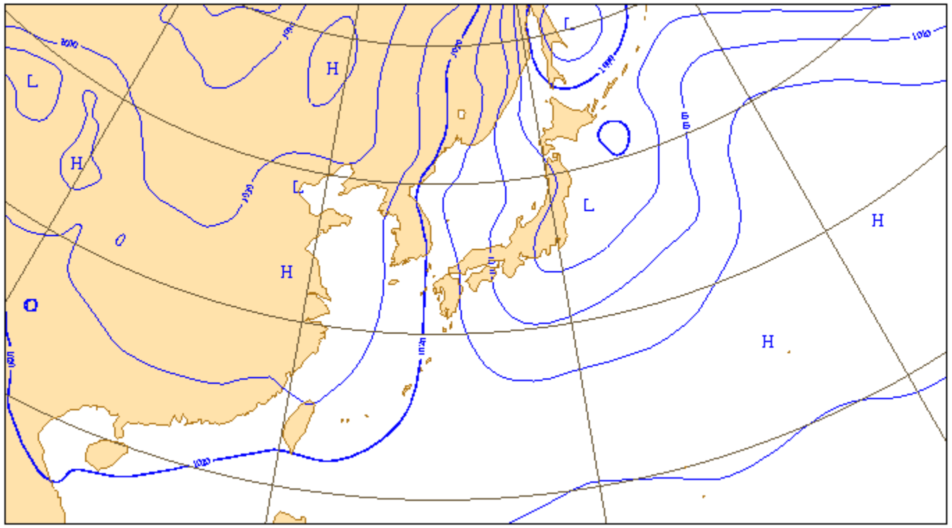
Hessian norm sensitivity

ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 temperature



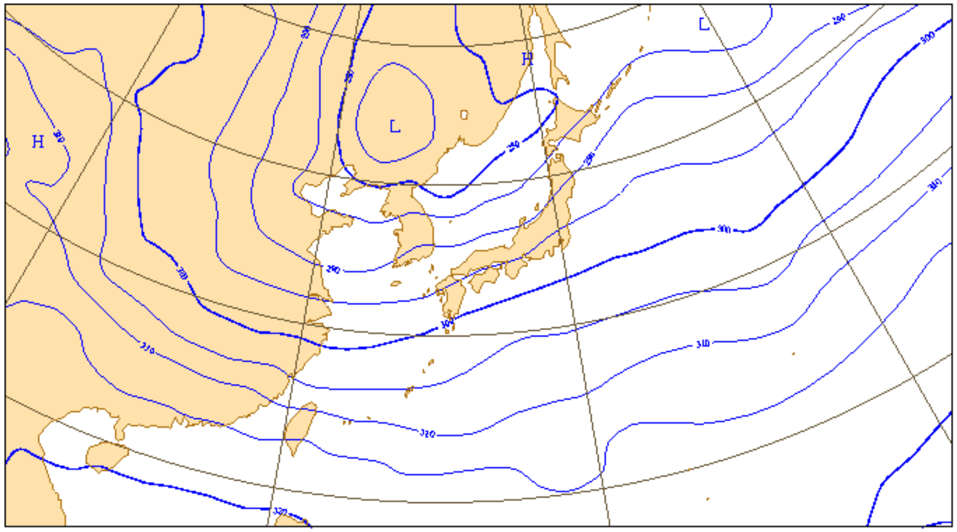
Analysis fields valid 1 January 2002

ECMWF Analysis VT:Tuesday 1 January 2002 12UTC Surface: mean sea level pressure



MSL pressure

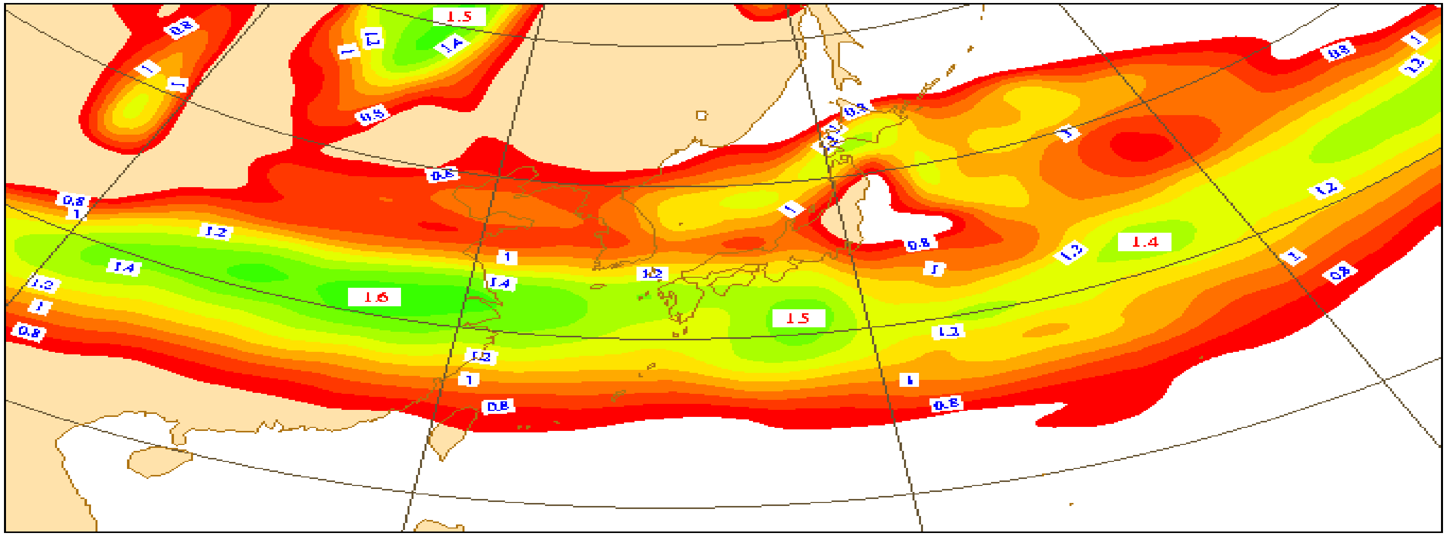
ECMWF Analysis VT:Tuesday 1 January 2002 12UTC Model Level 42 **potential temperature



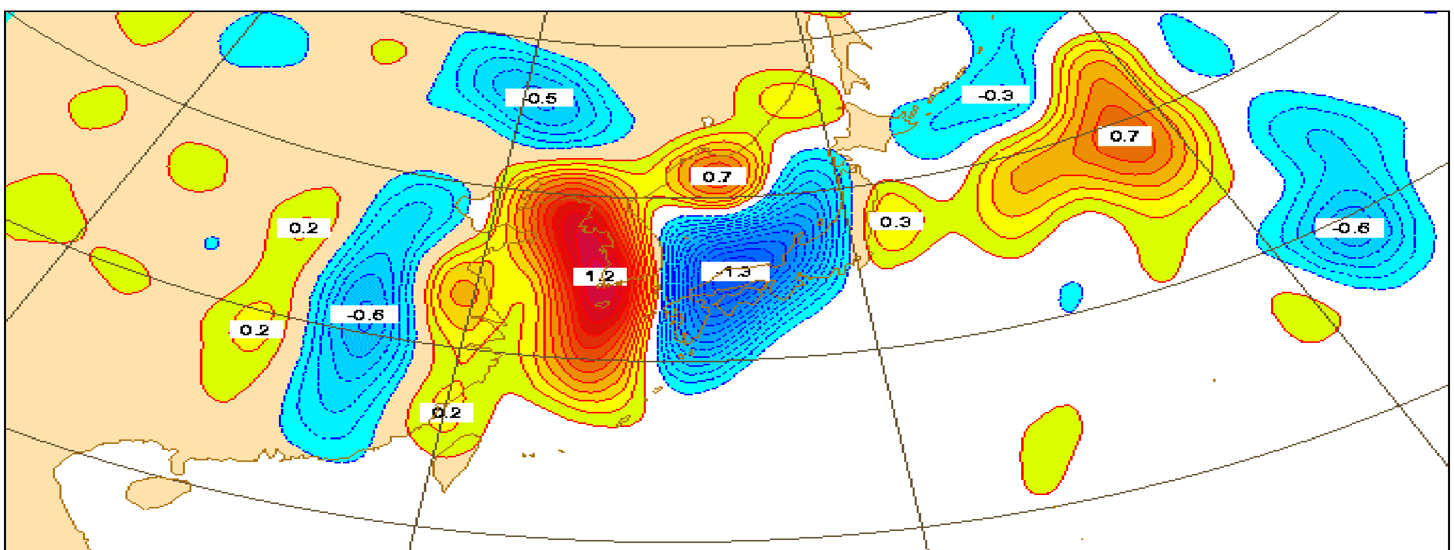
Potential temperature
Model level 42

1 January 2002 Japan case study

ECMWF Analysis VT:Tuesday 1 January 2002 06UTC 300hPa **geopotential height



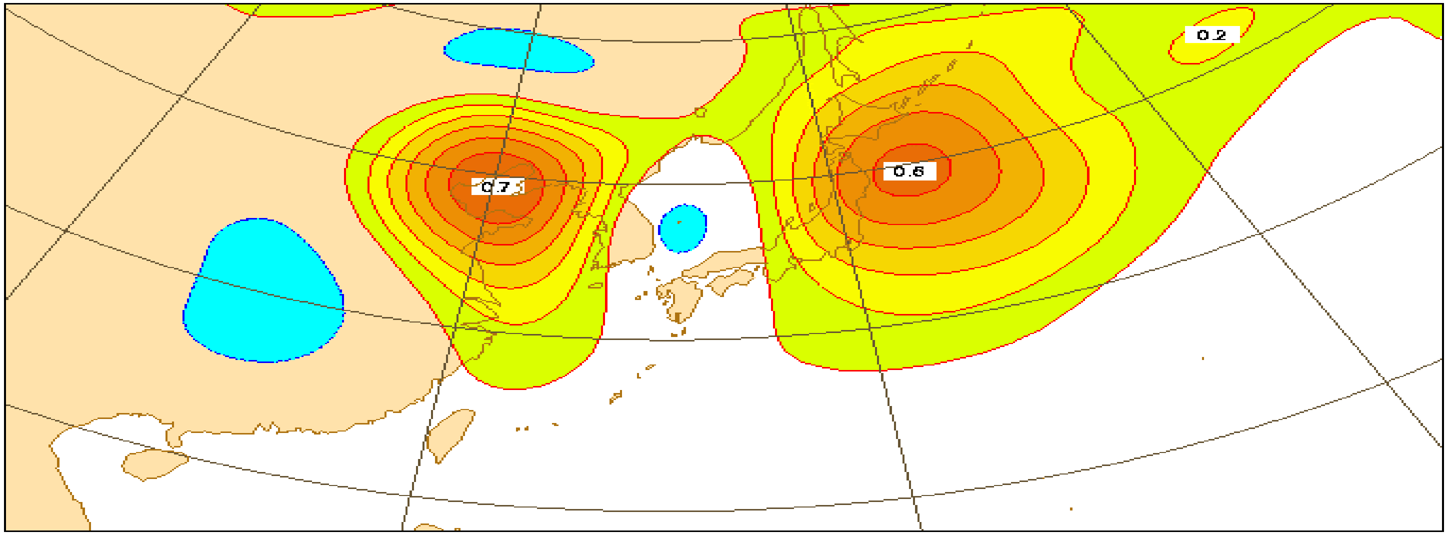
Eady index



Energy norm Sensitivity Temperature Level 42

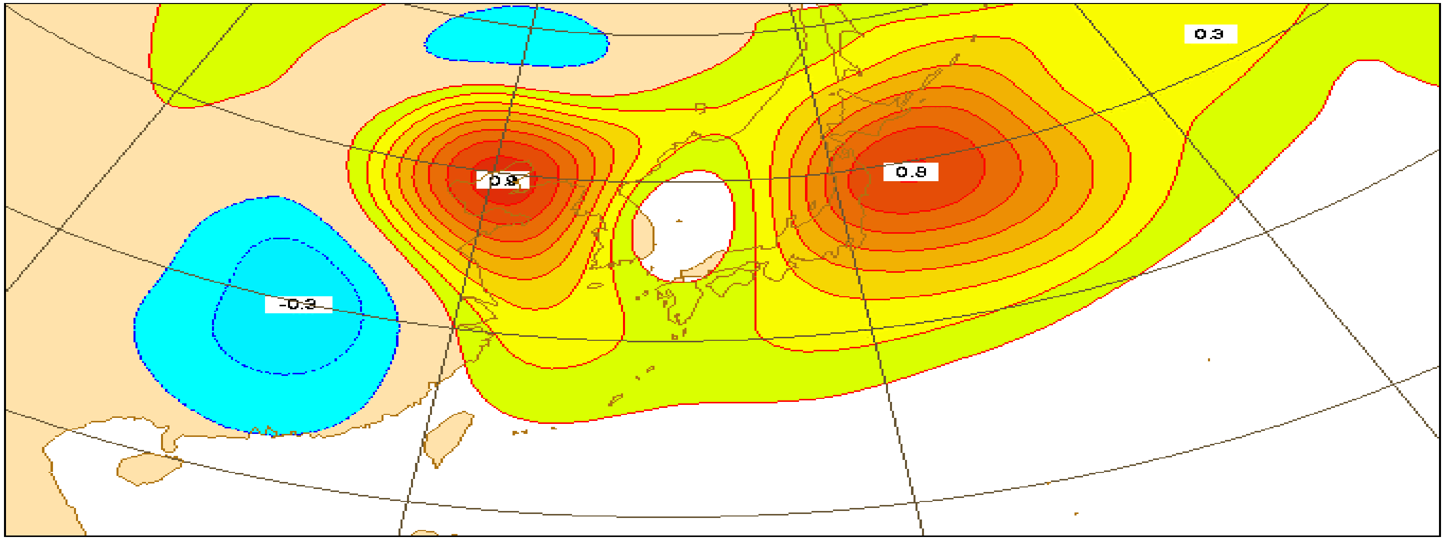
1 January 2002 Japan case study

ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 temperature



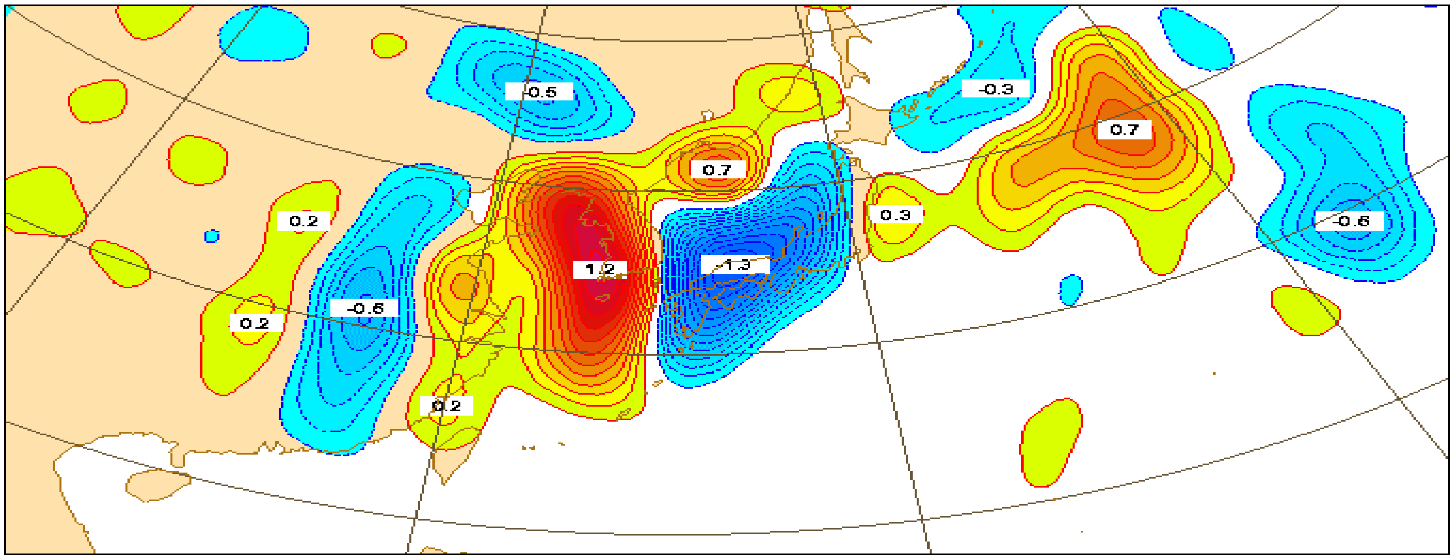
Jb norm Sensitivity Temperature Level 42

ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 temperature



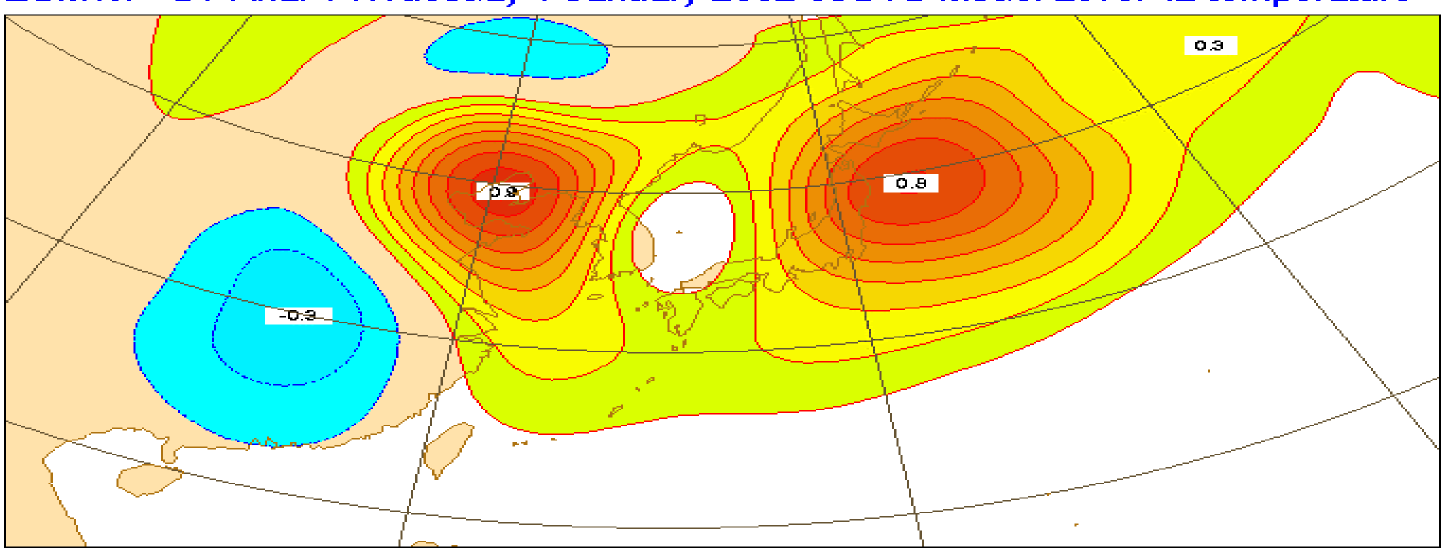
Hessian norm Sensitivity Temperature Level 42

1 January 2002 Japan case study



Energy norm
Sensitivity
Temperature
Level 42

ECMWF SV Anal VT:Tuesday 1 January 2002 03UTC Model Level 42 temperature

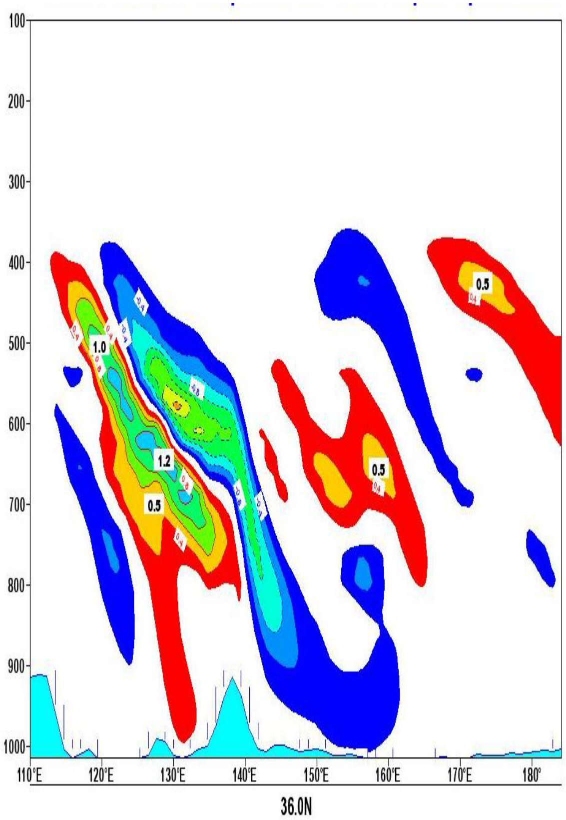


Hessian norm
Sensitivity
Temperature
Level 42

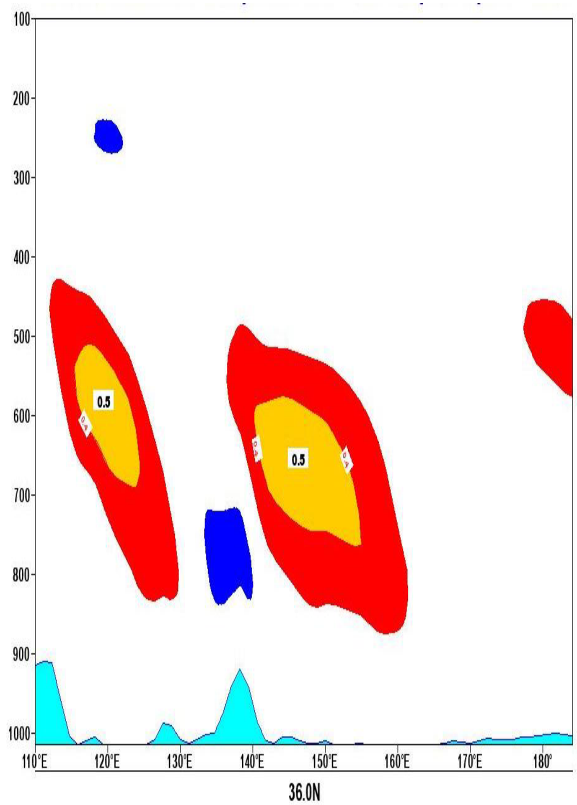
1 January 2002 Japan case study

Cross-sections for temperature sensitivity patterns

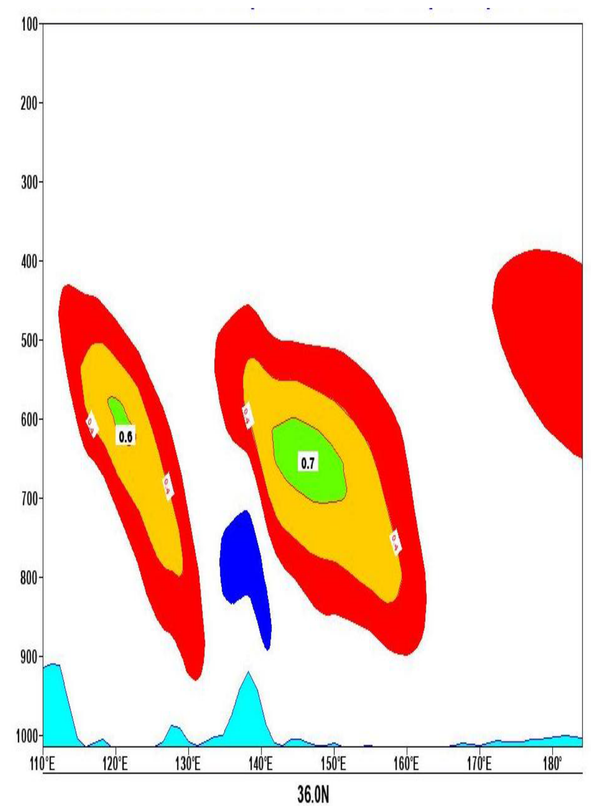
Energy norm



Jb norm



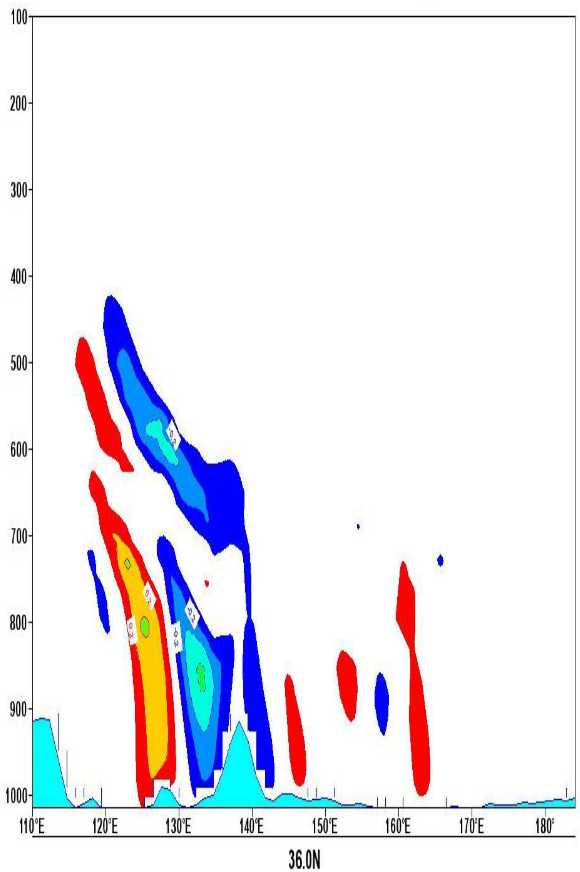
Hessian norm



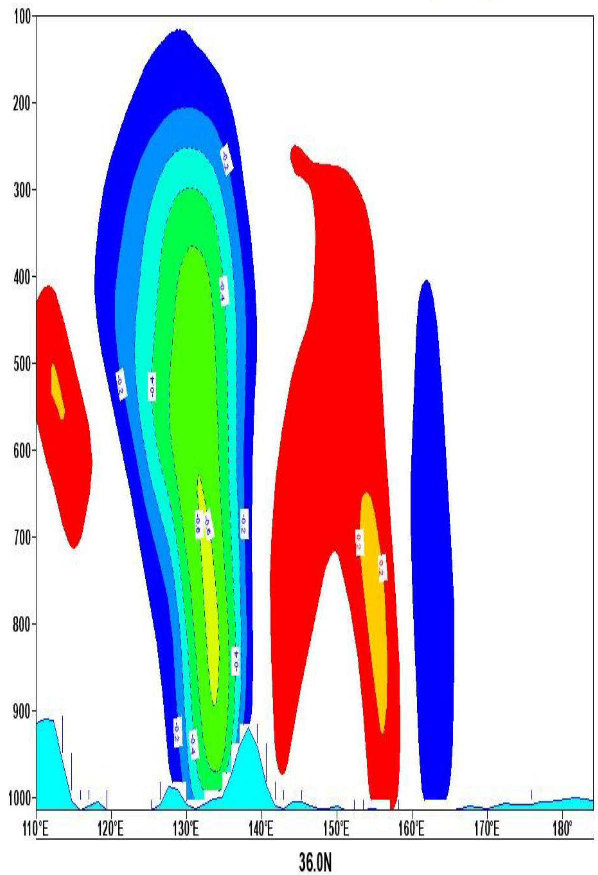
1 January 2002 Japan case study

Cross-sections for vorticity sensitivity patterns

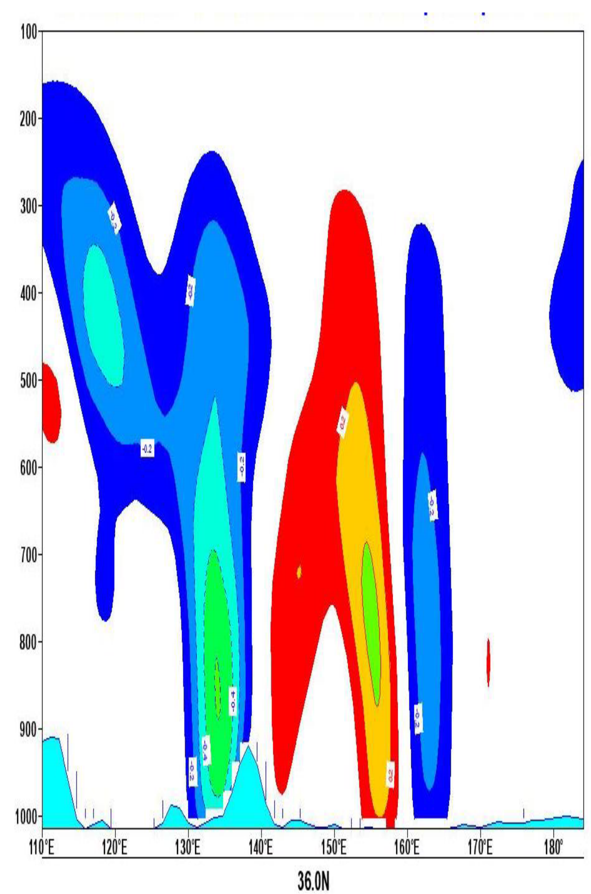
Energy norm



Jb norm

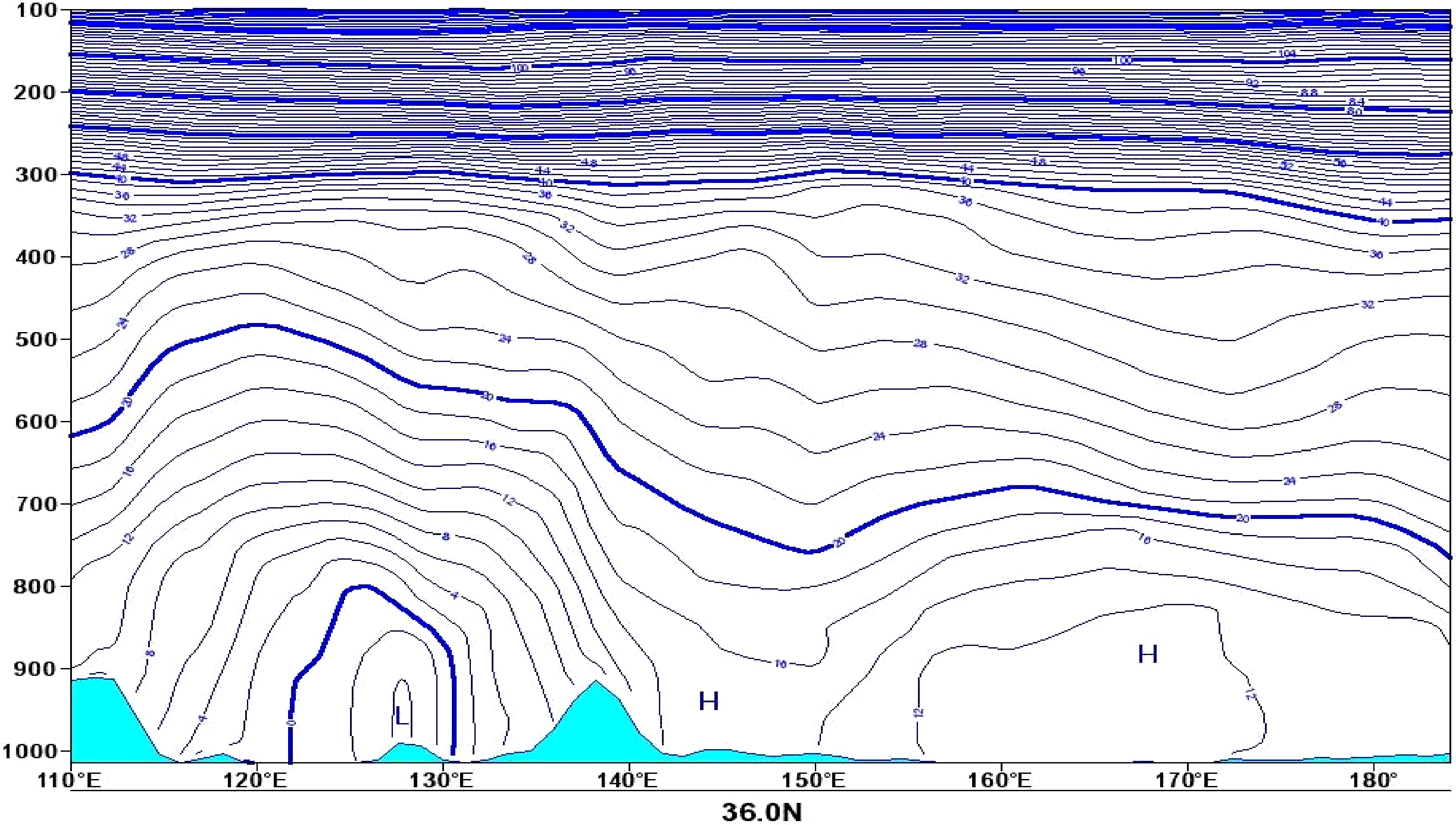


Hessian norm

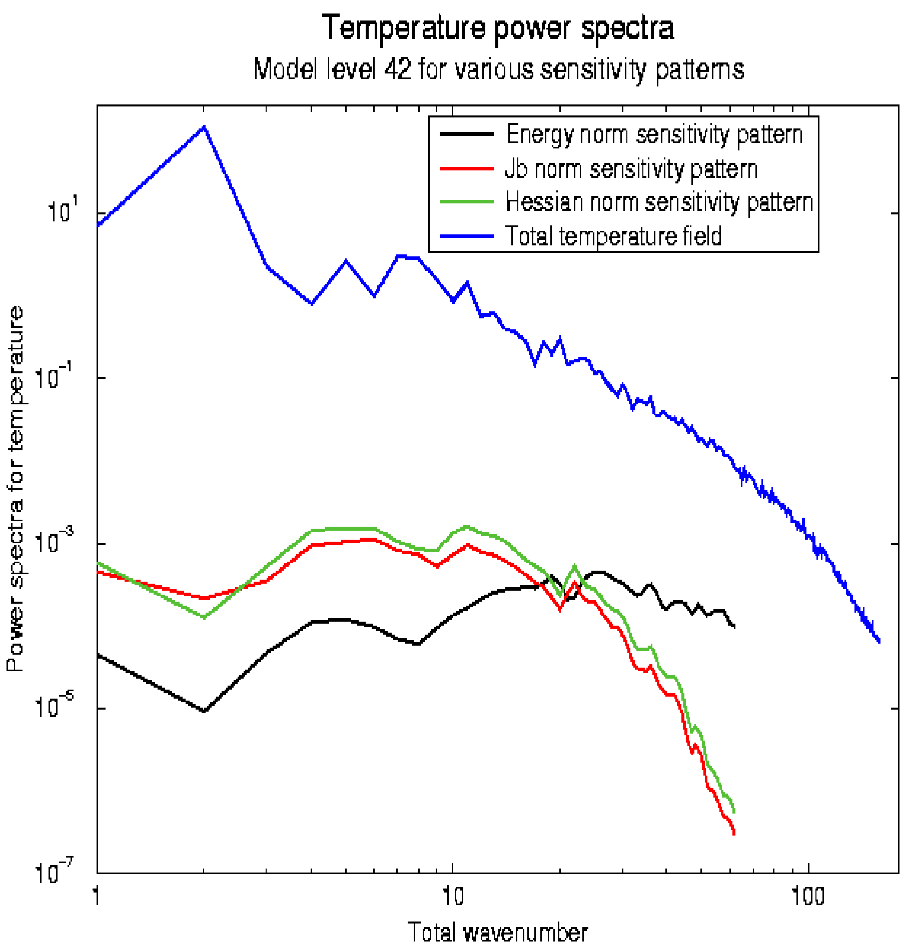


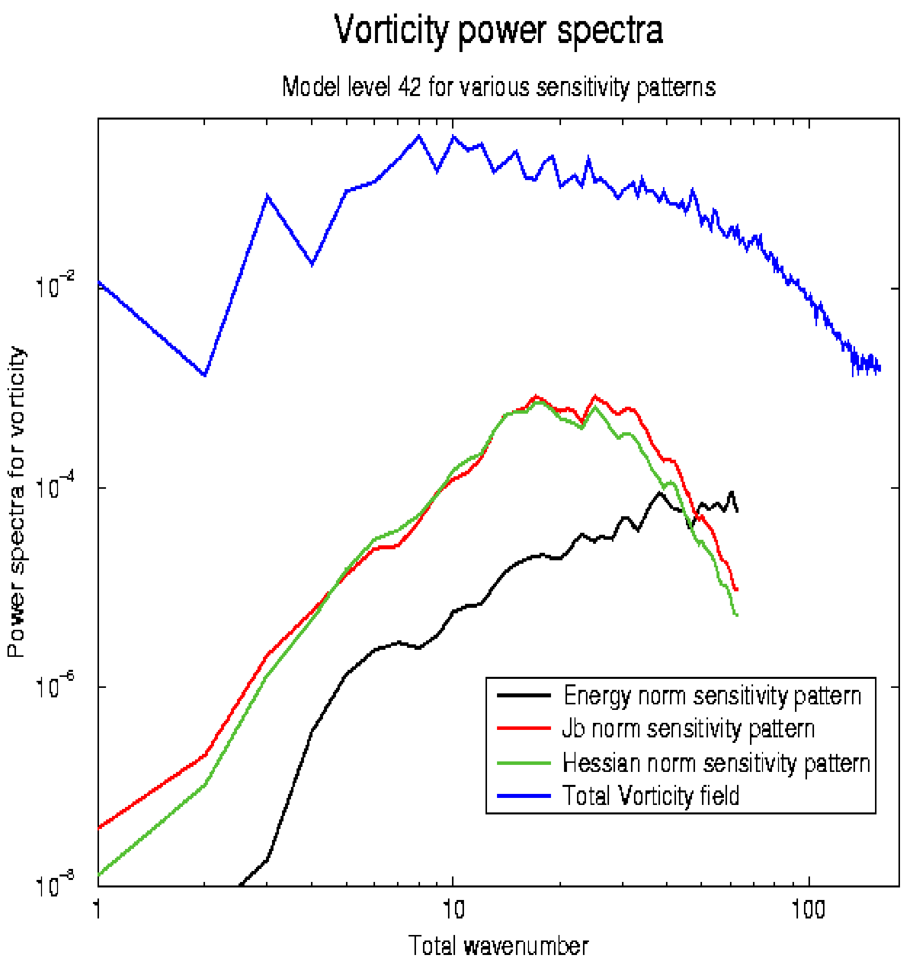
Analysis field valid 1 January 2002

Potential temperature east-west cross section

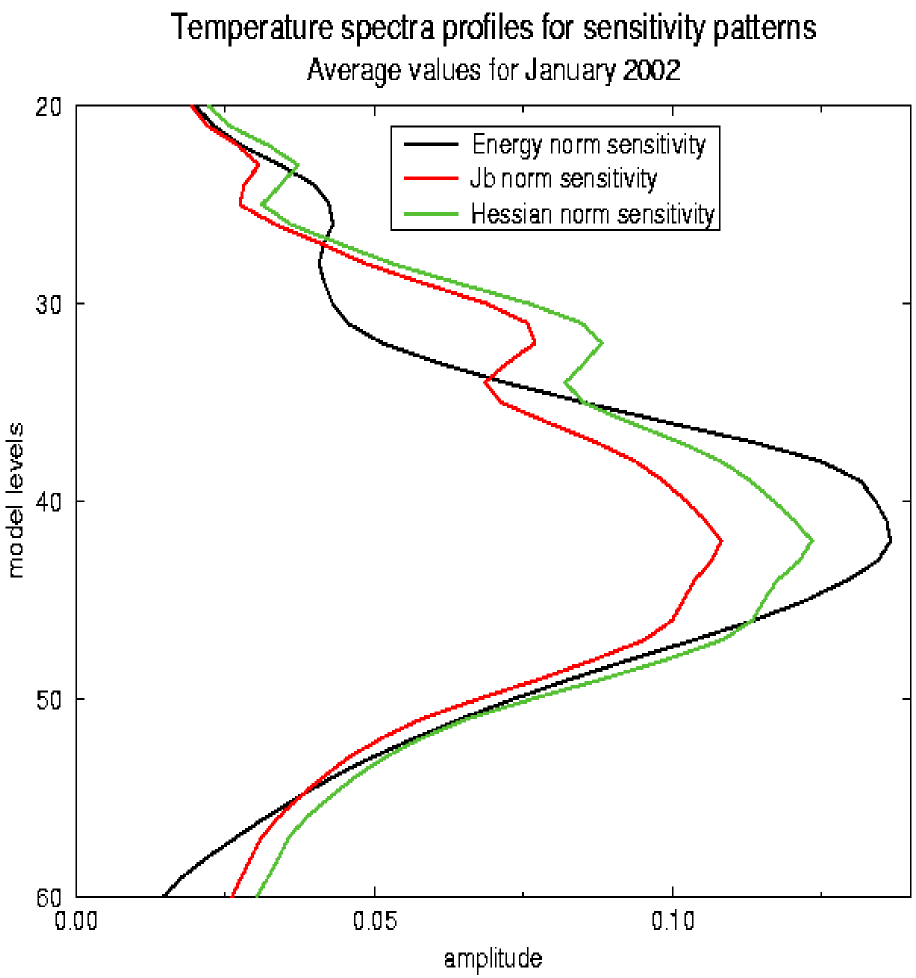


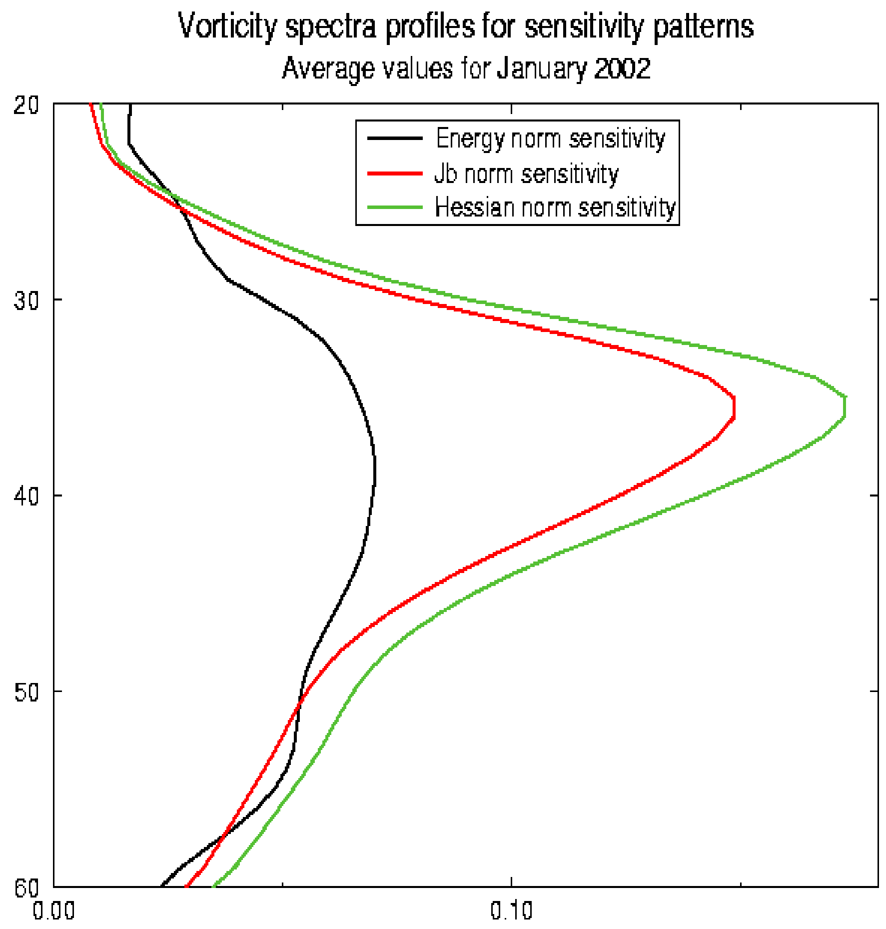
Temperature and vorticity spectra





Temperature and vorticity spectra profiles

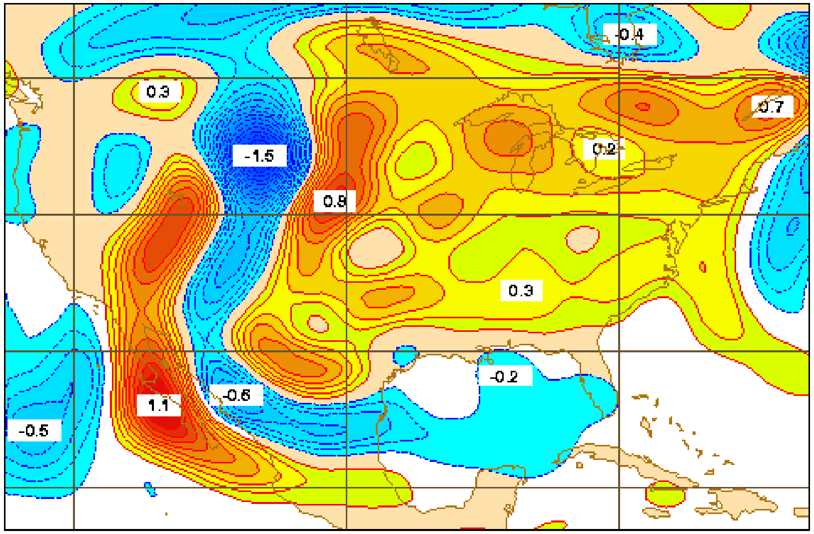


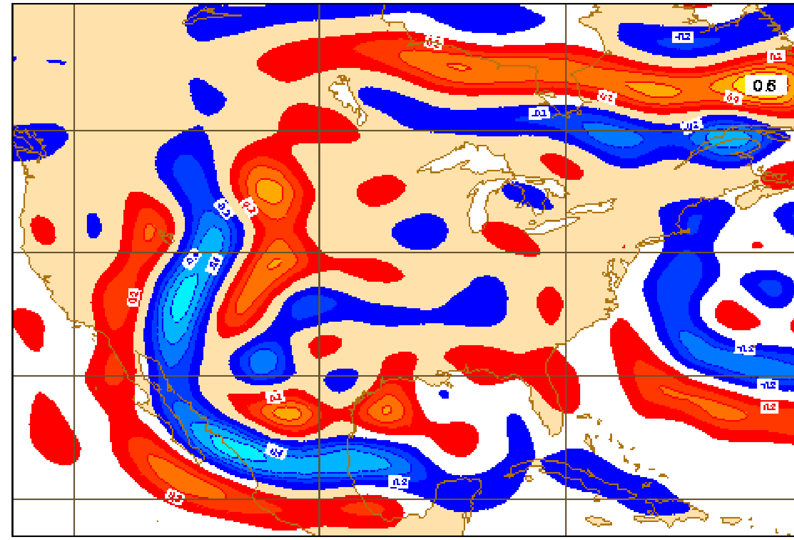


Temperature and vorticity sensitivity

Energy and Hessian patterns often differ a lot

Energy norm

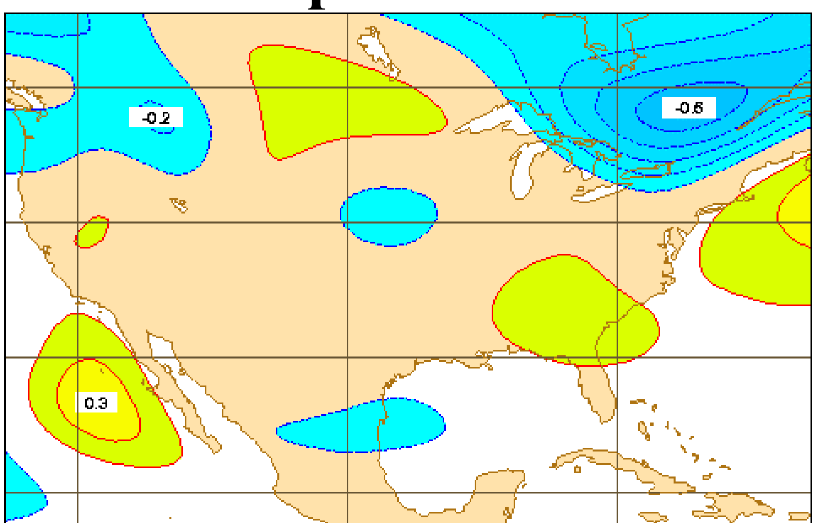


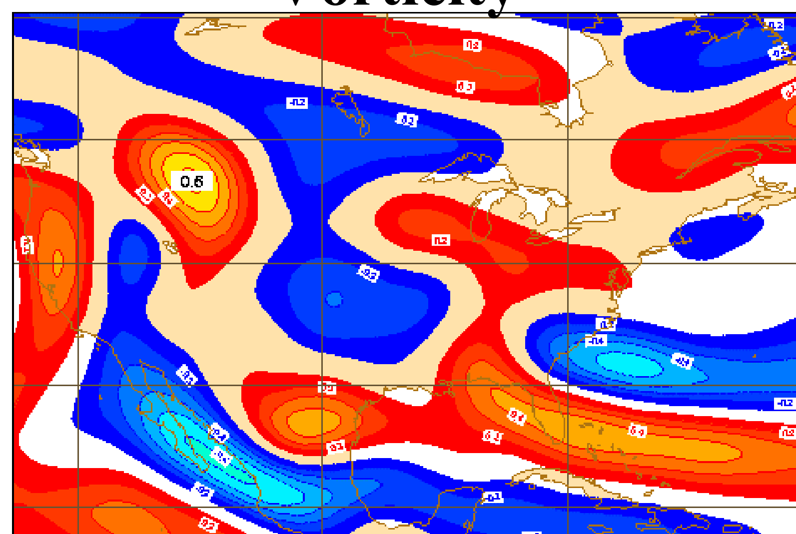


Temperature

Vorticity

Hessian norm

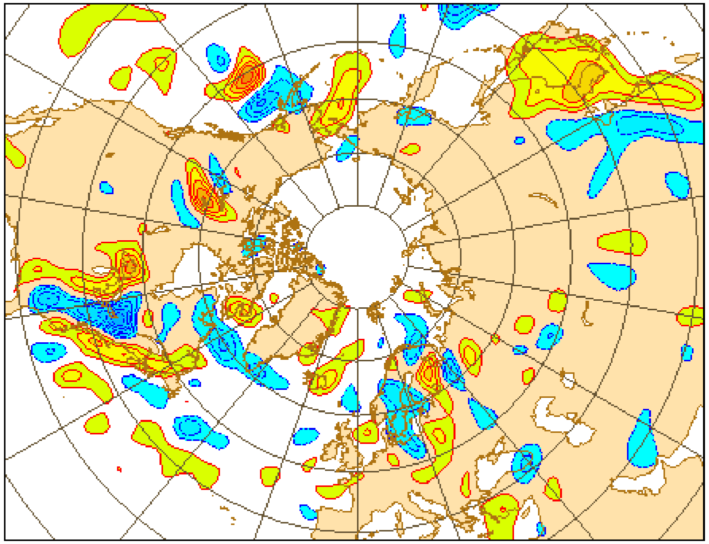


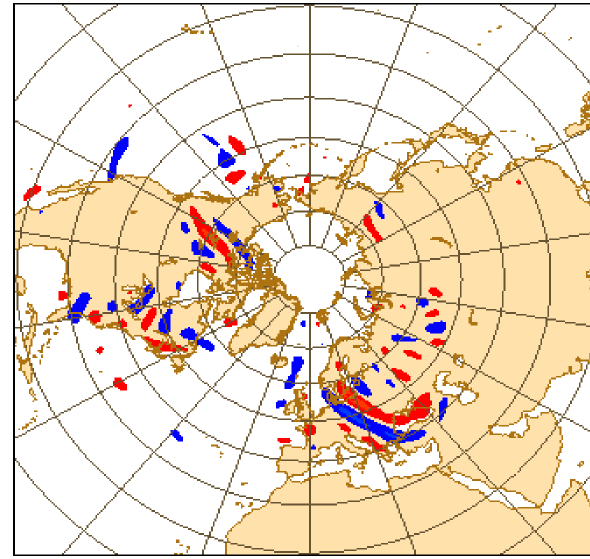


Temperature and vorticity sensitivity

Energy and Hessian amplitudes often differ a lot

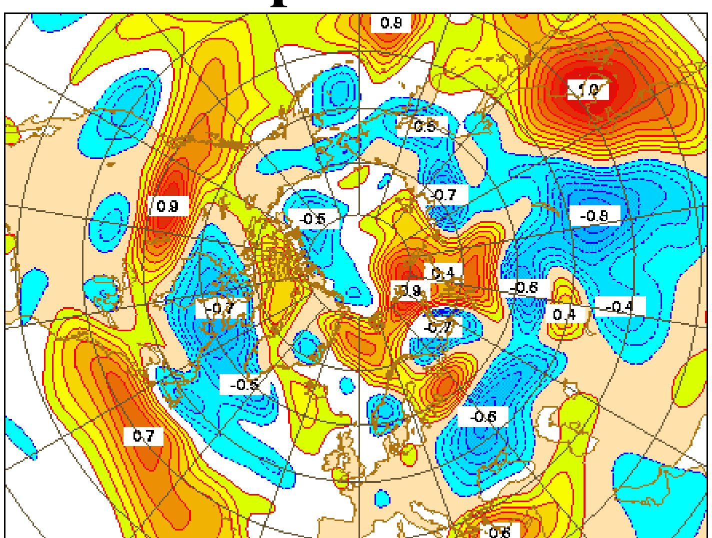
Energy norm

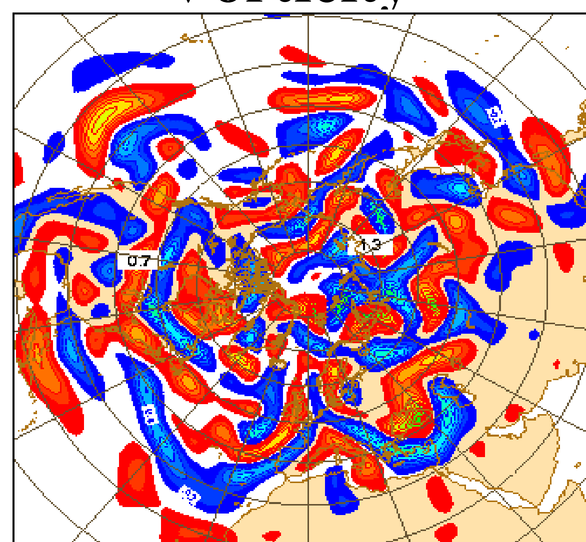




Temperature | **Vorticity**

Hessian norm

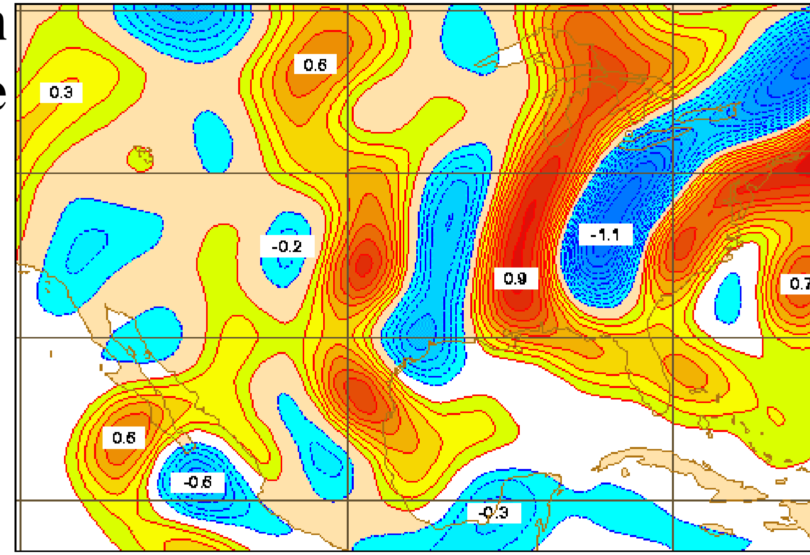


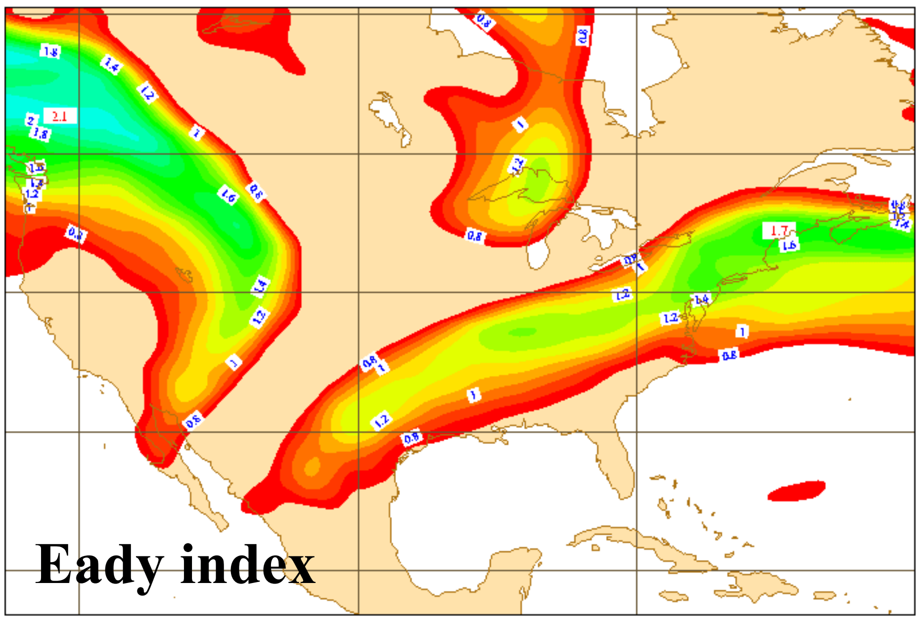


Temperature sensitivity

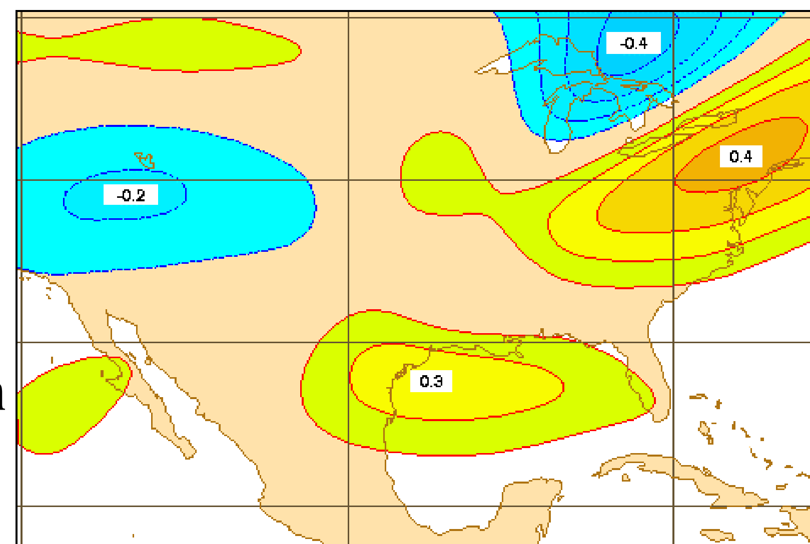
Energy norm more linked to unstable regions

Energy norm
Temperature
sensitivities



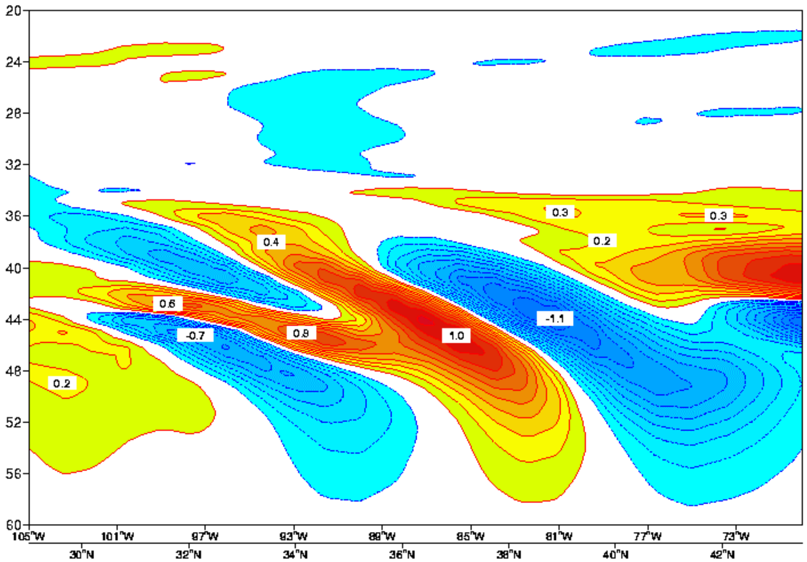


Hessian norm
temperature
sensitivities

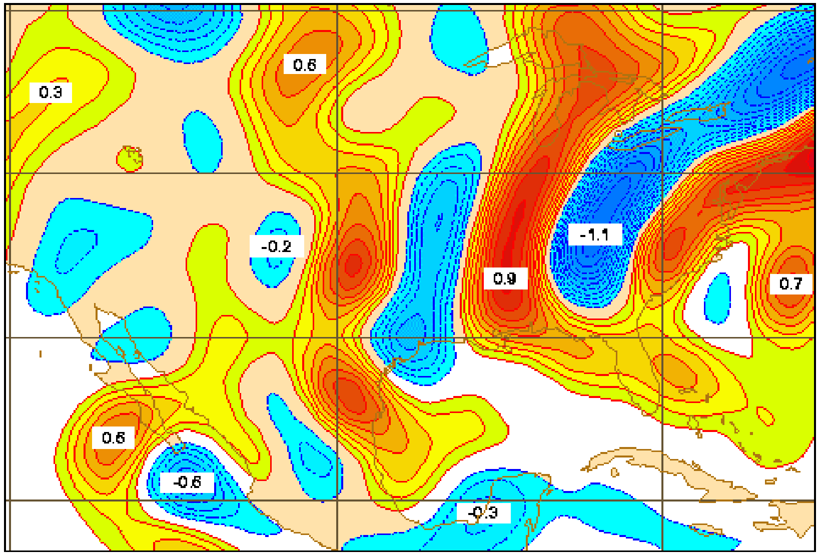


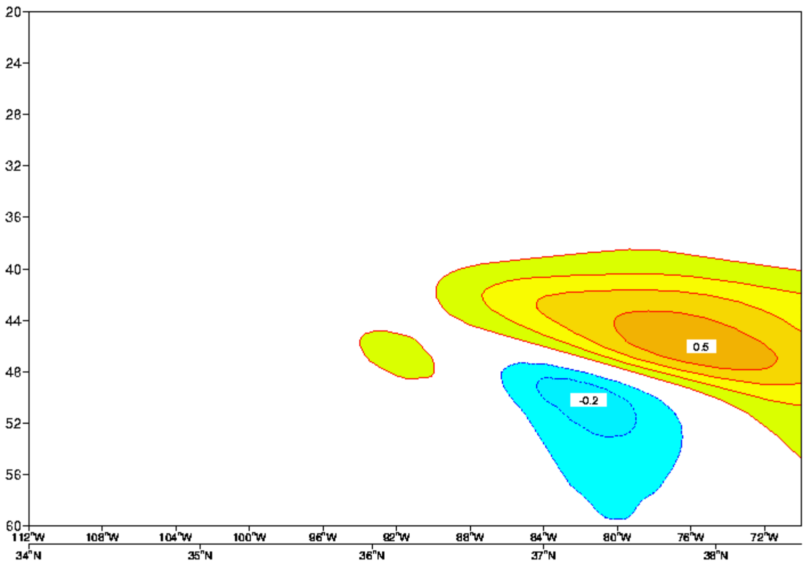
Temperature sensitivity

Energy norm more linked to unstable regions

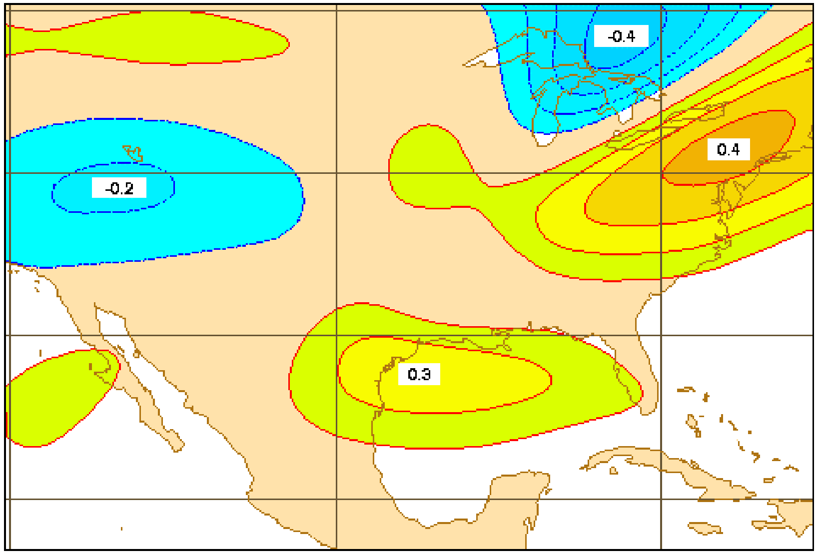


Energy norm sensi





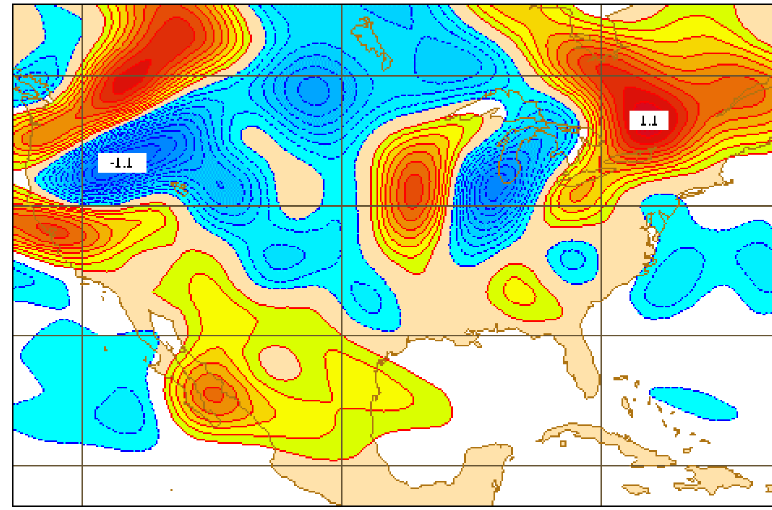
Hessian norm sensi

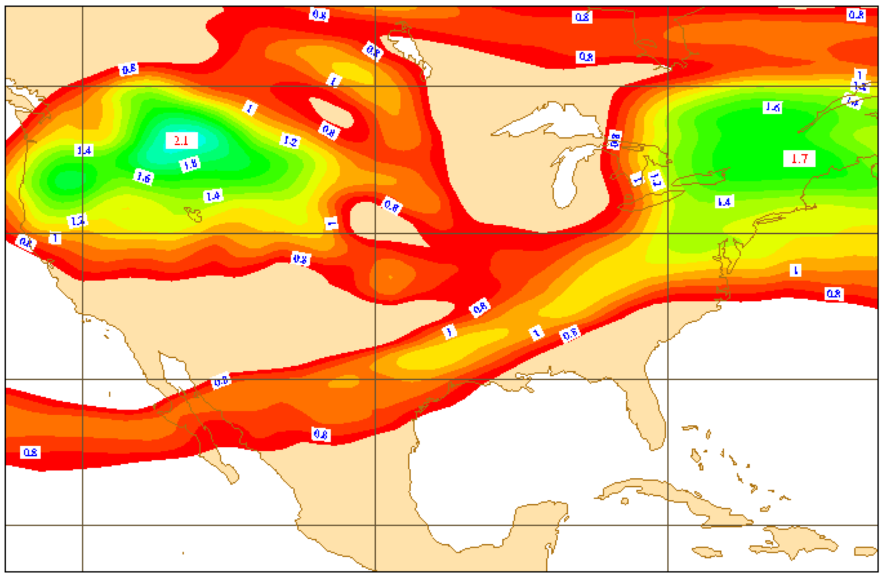


Temperature sensitivity

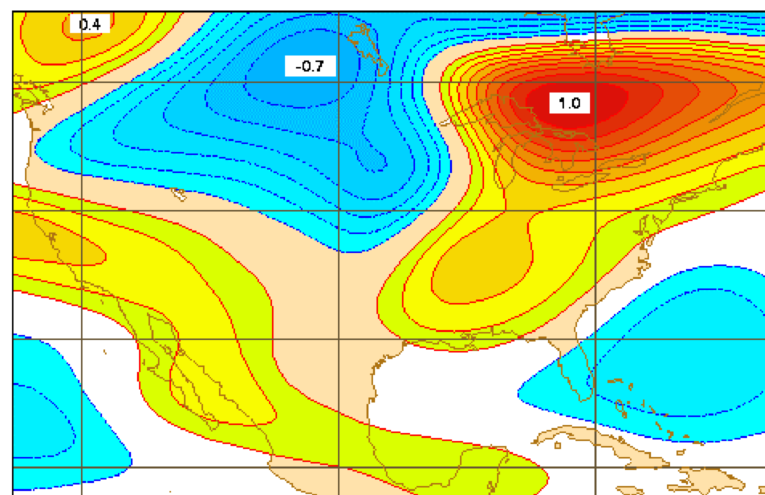
Energy and Hessian norm are sometimes very similar

Energy norm Temperature sensitivities



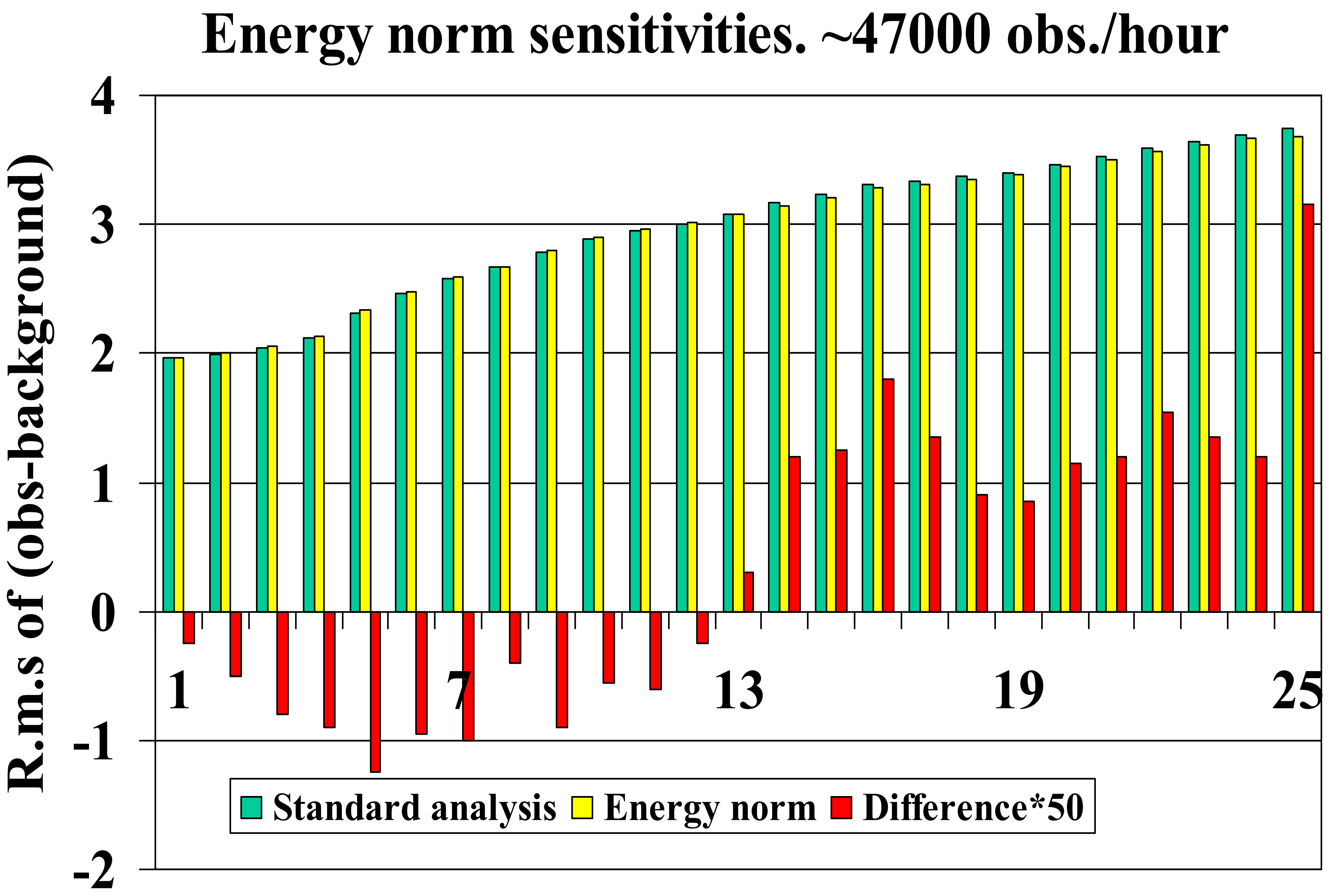


Eady index

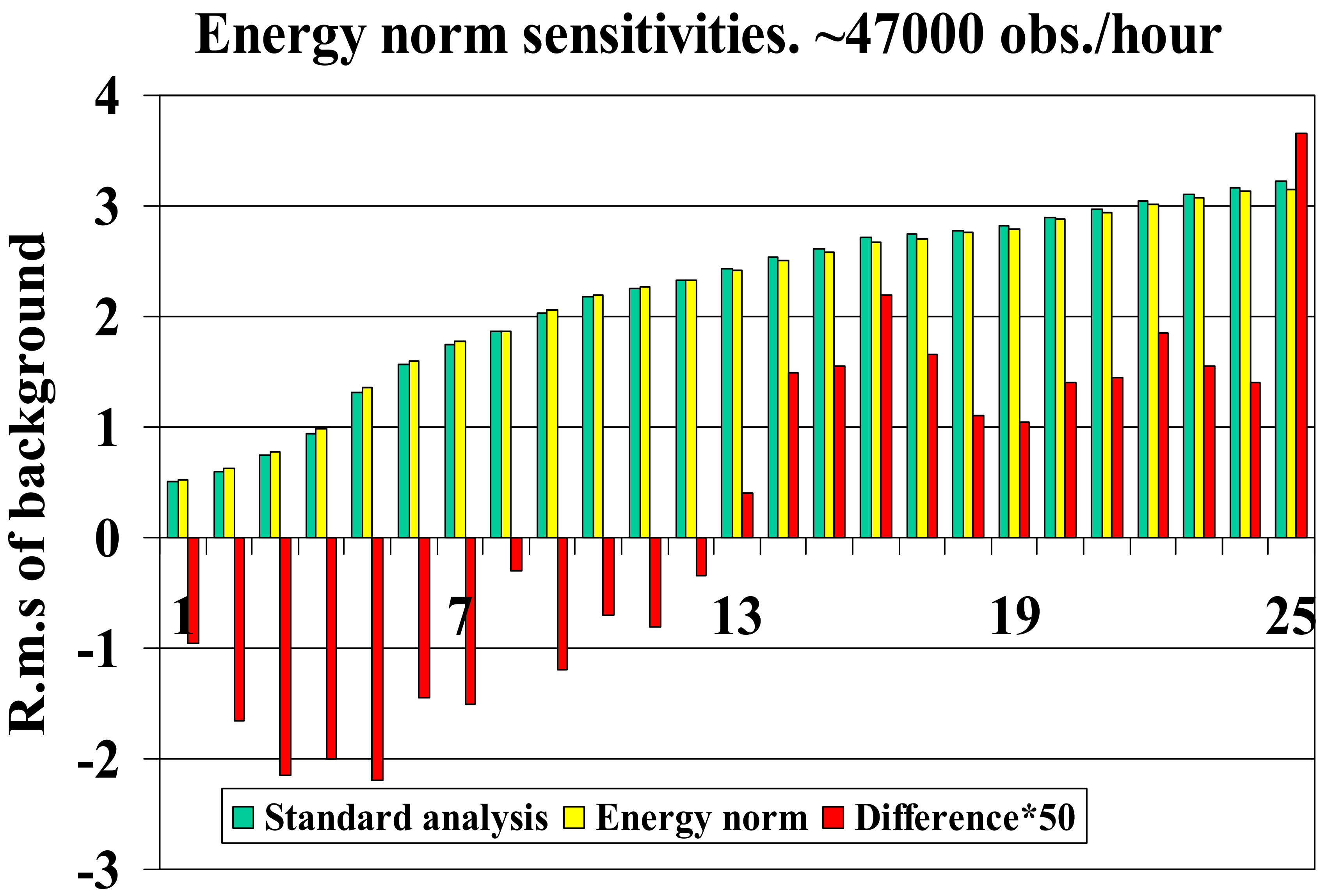


Hessian norm temperature sensitivities

American profilers zonal wind component

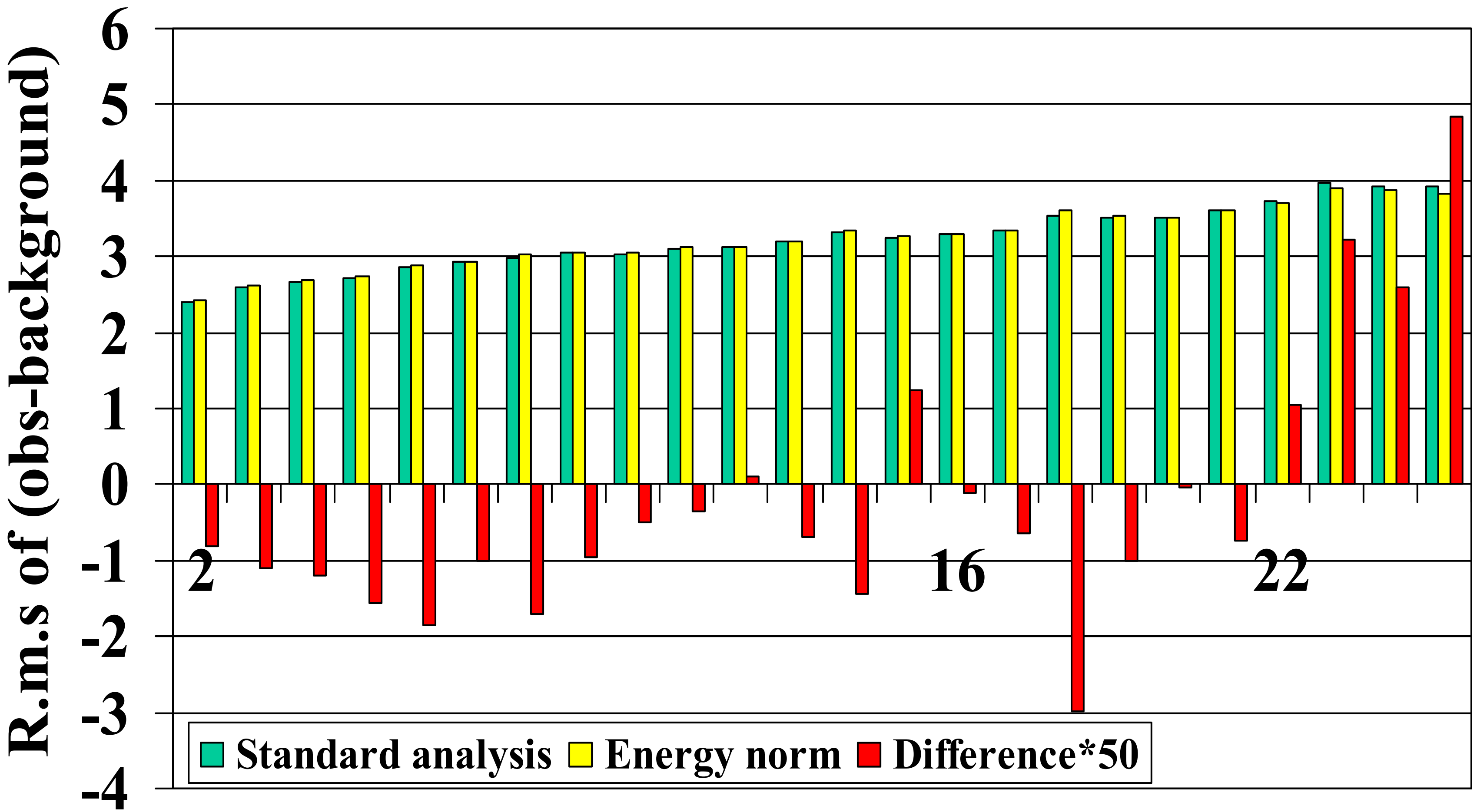


American profilers zonal wind component



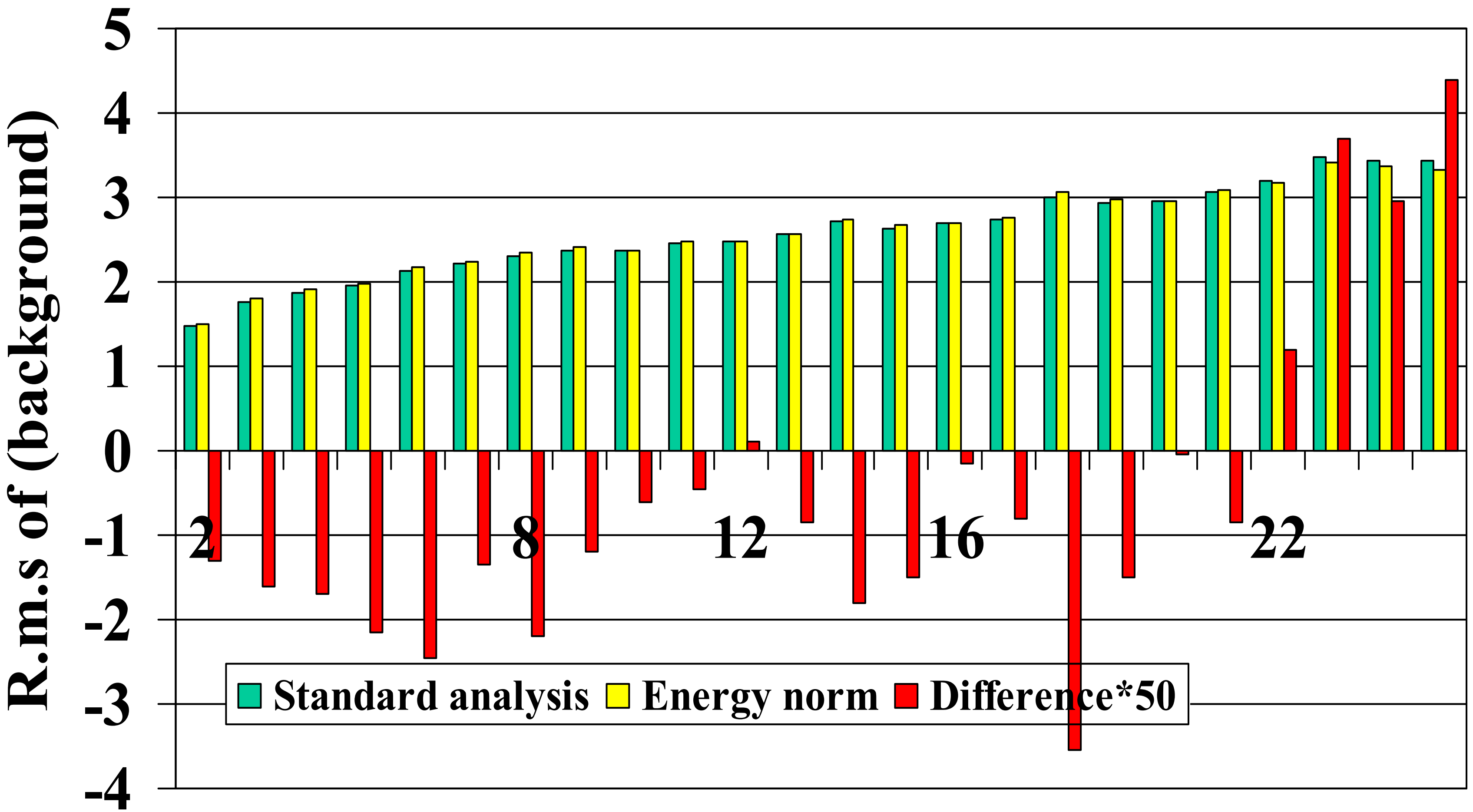
American profilers zonal wind component

Hessian norm sensitivities ~47000 obs/hour



American profilers zonal wind component

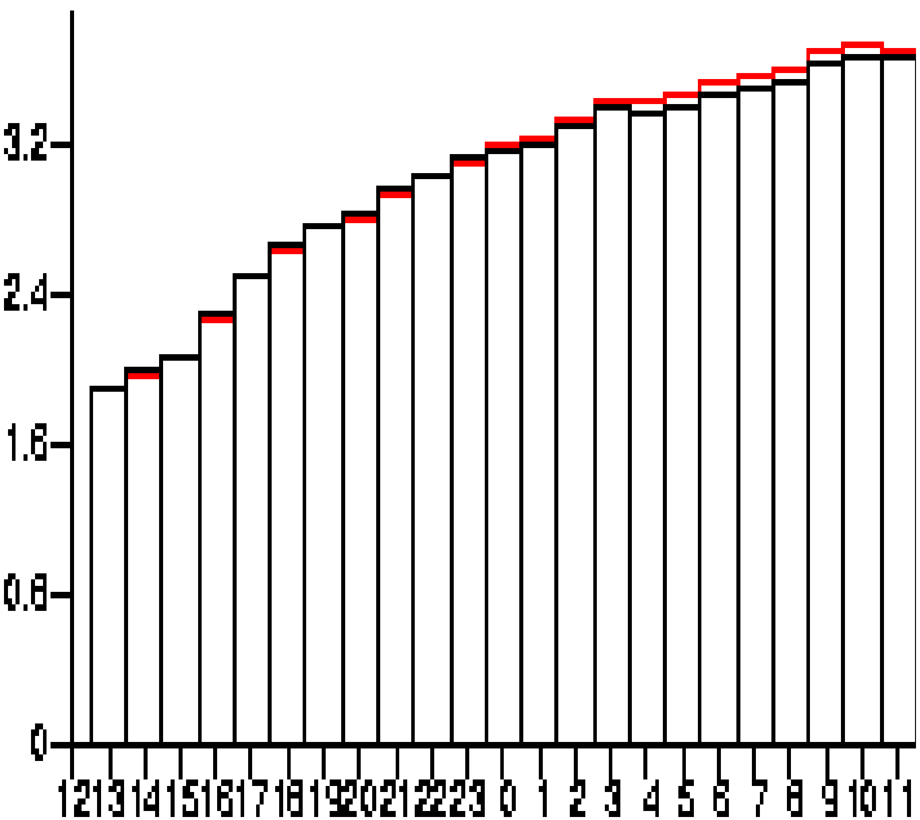
Hessian norm sensitivities ~47000 obs/hour



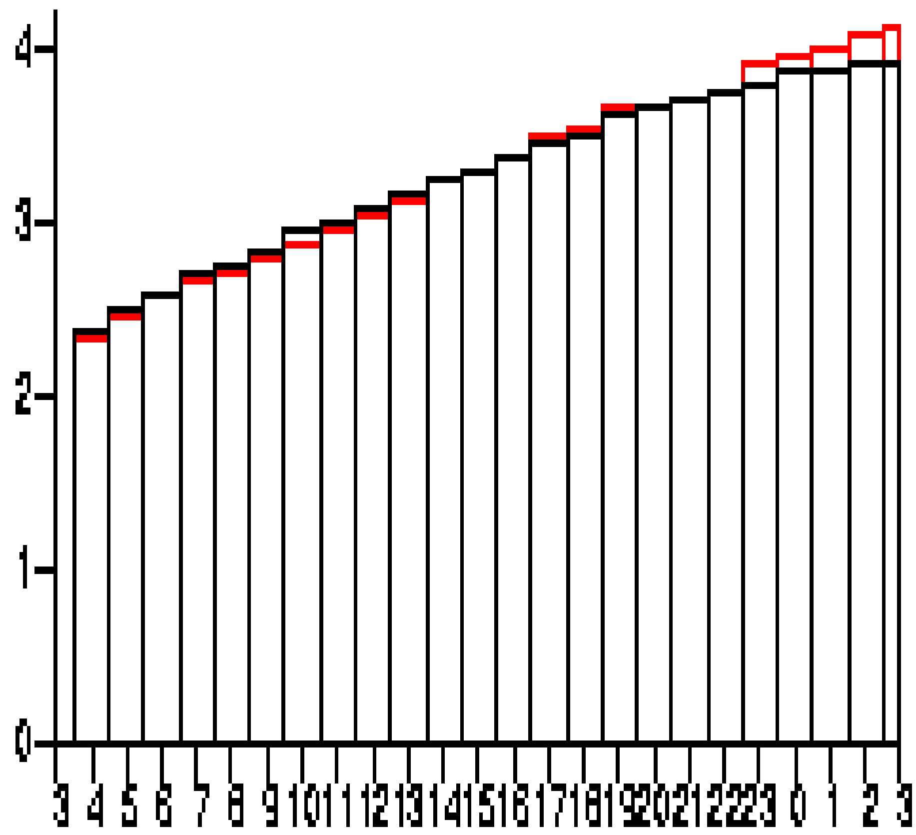
American profilers meridional wind component

Red is control forecast **Black is sensitivity run**

22000-24000 observations/hour



Energy norm sensitivity

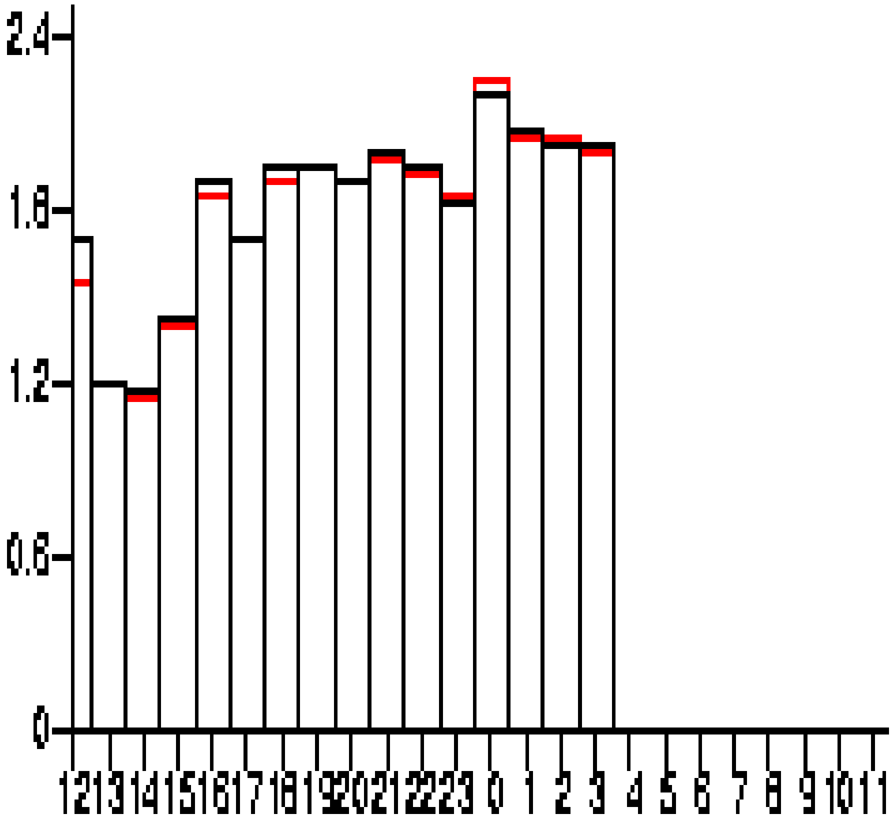


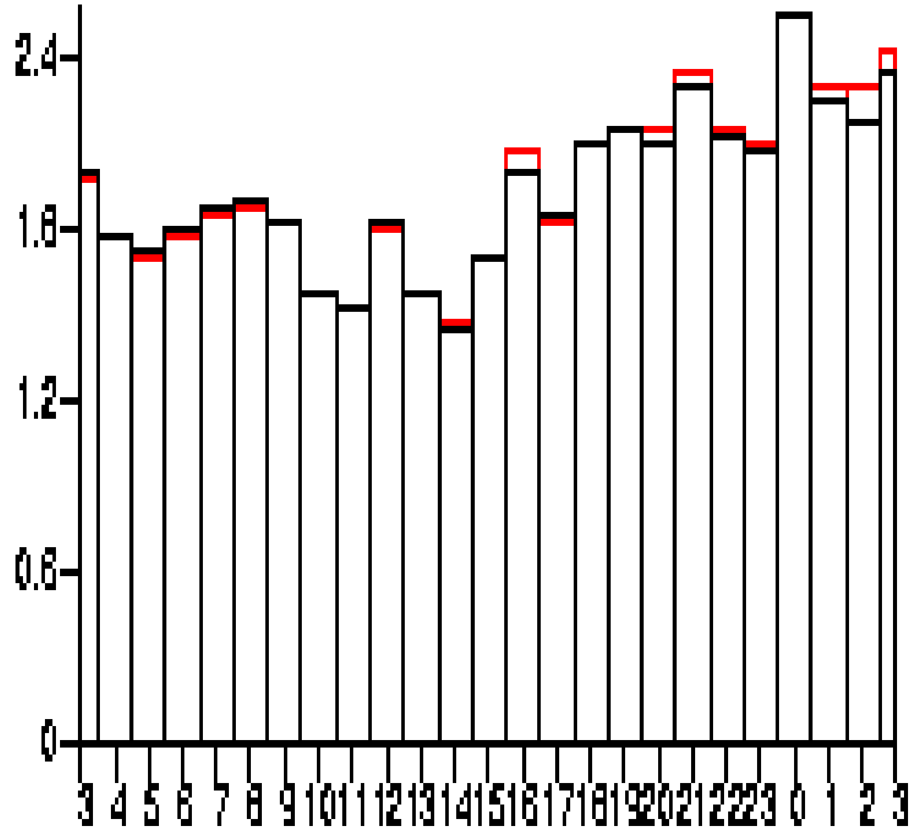
Hessian norm sensitivity

QUIKscat wind speed

Red is control forecast **Black is sensitivity run**

Up to 60000 observations/hour





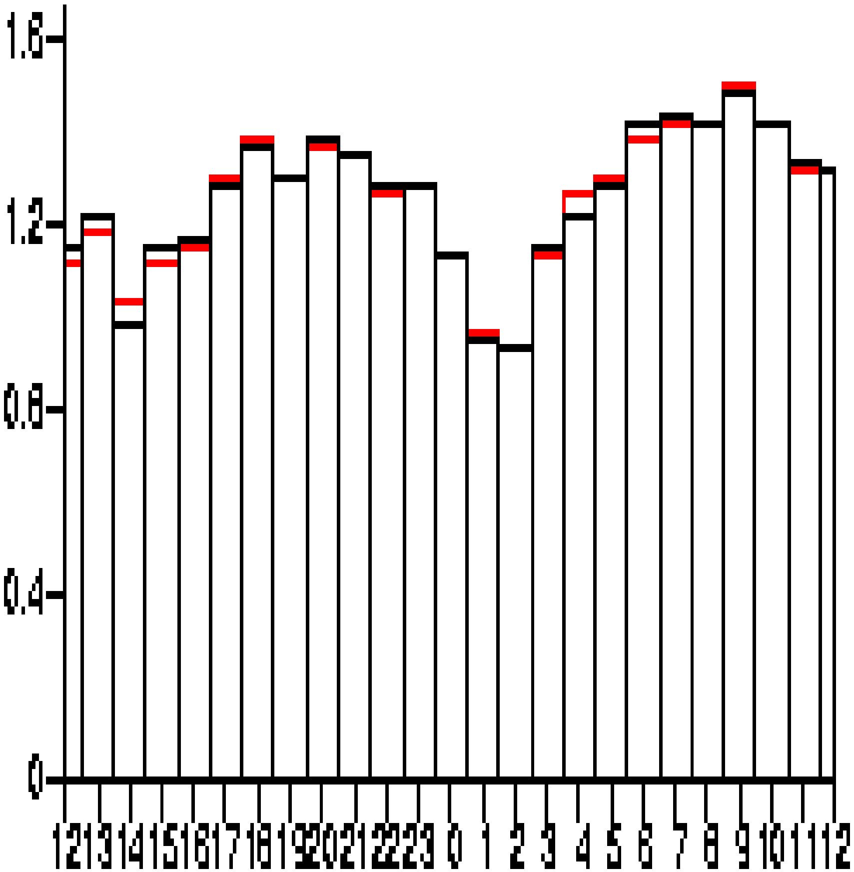
Energy norm sensitivity **Hessian norm sensitivity**

SSM/I wind speed

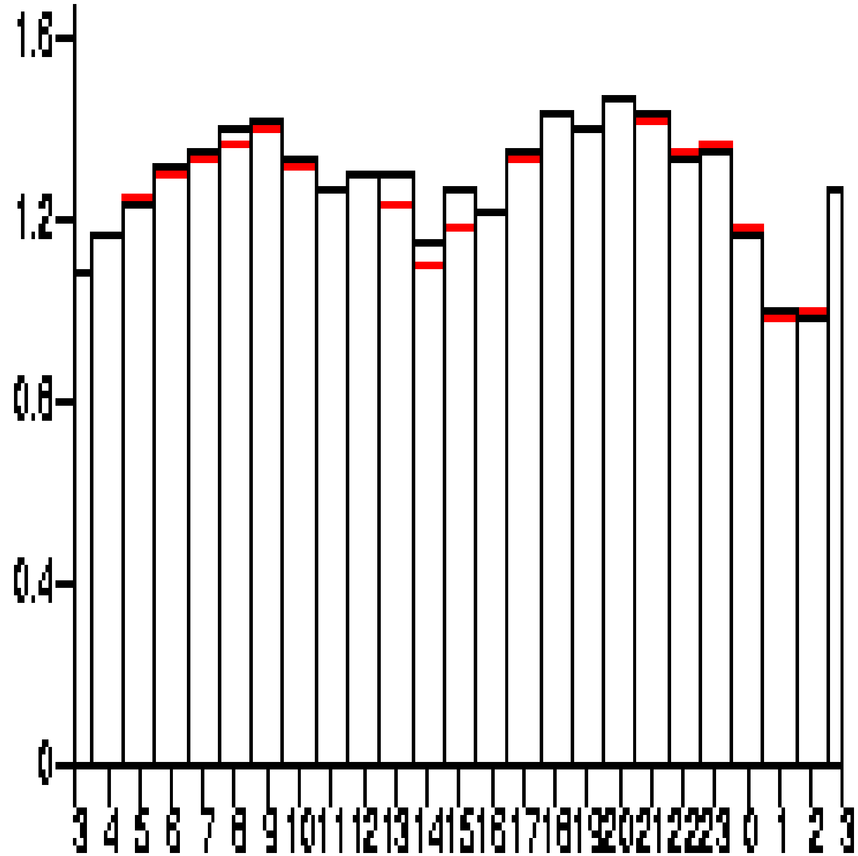
Red is control forecast

Black is sensitivity run

Up to 5000 observations/hour



Energy norm sensitivity



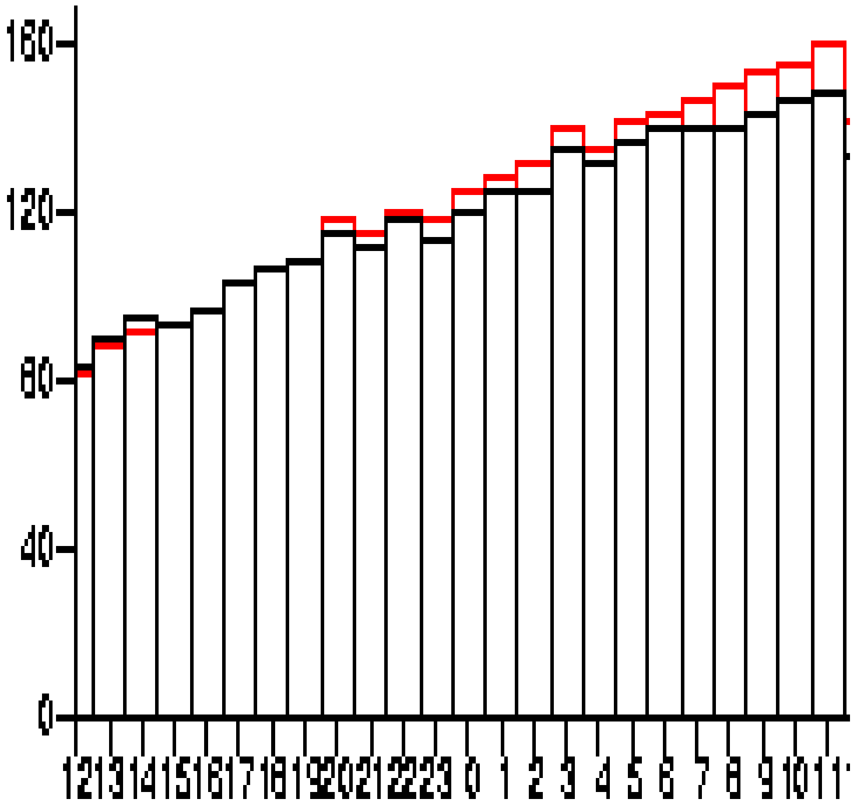
Hessian norm sensitivity

DRIBU surface pressure

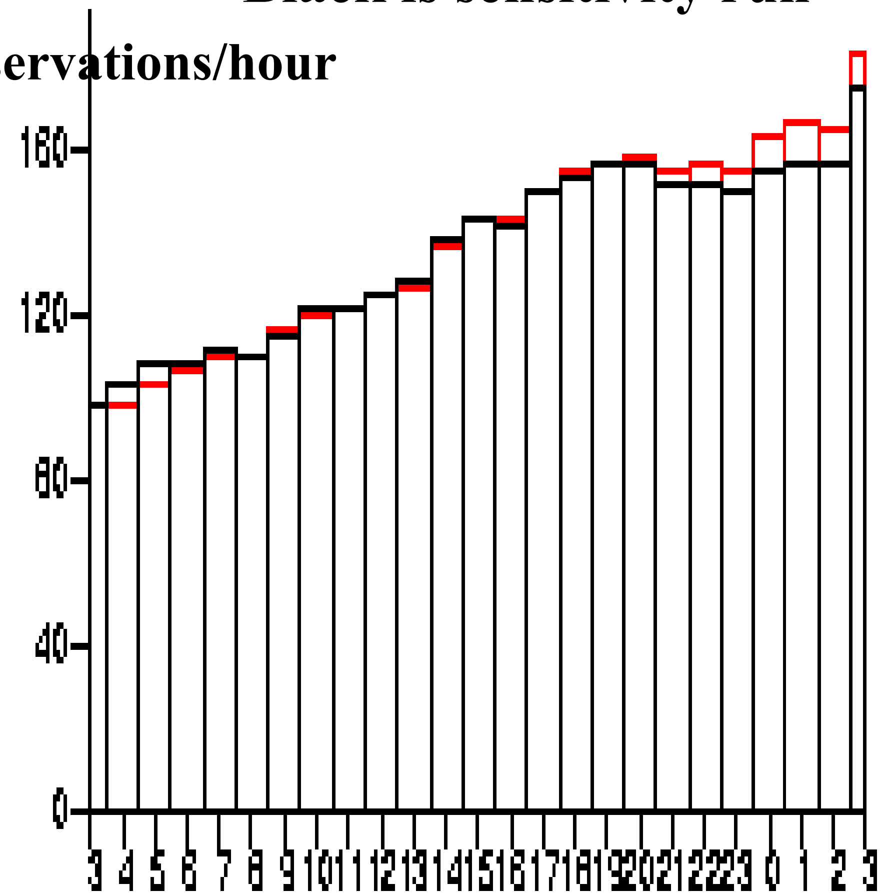
Red is control forecast

Black is sensitivity run

200-320 observations/hour



Energy norm sensitivity

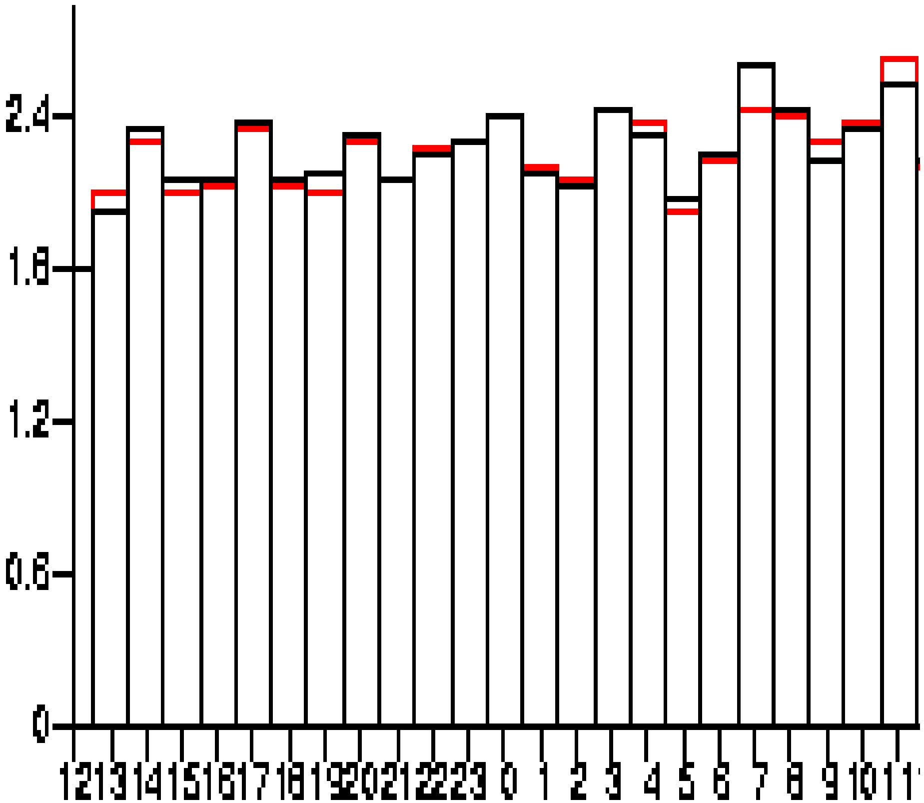


Hessian norm sensitivity

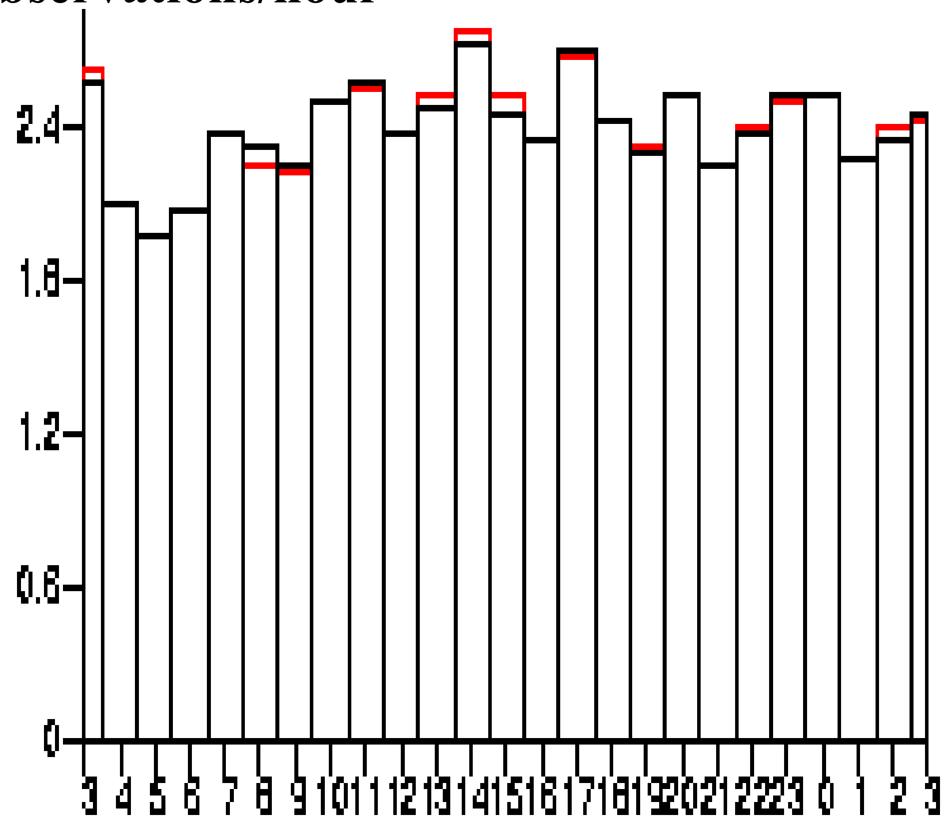
DRIBU wind speed

Red is control forecast **Black is sensitivity run**

200-320 observations/hour



Energy norm sensitivity



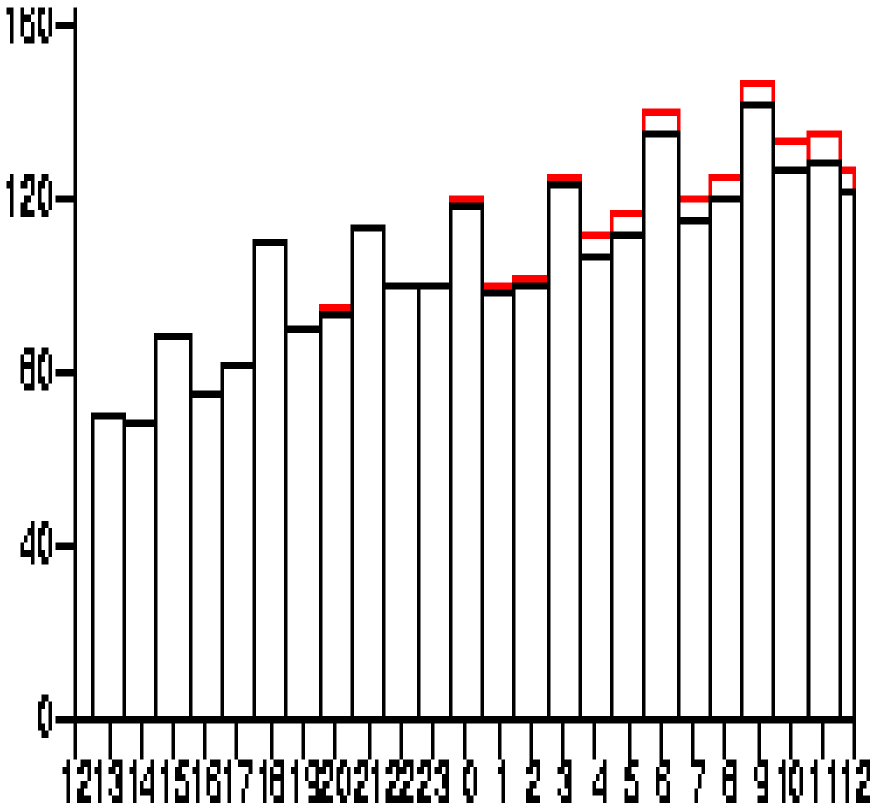
Hessian norm sensitivity

SYNOP surface pressure

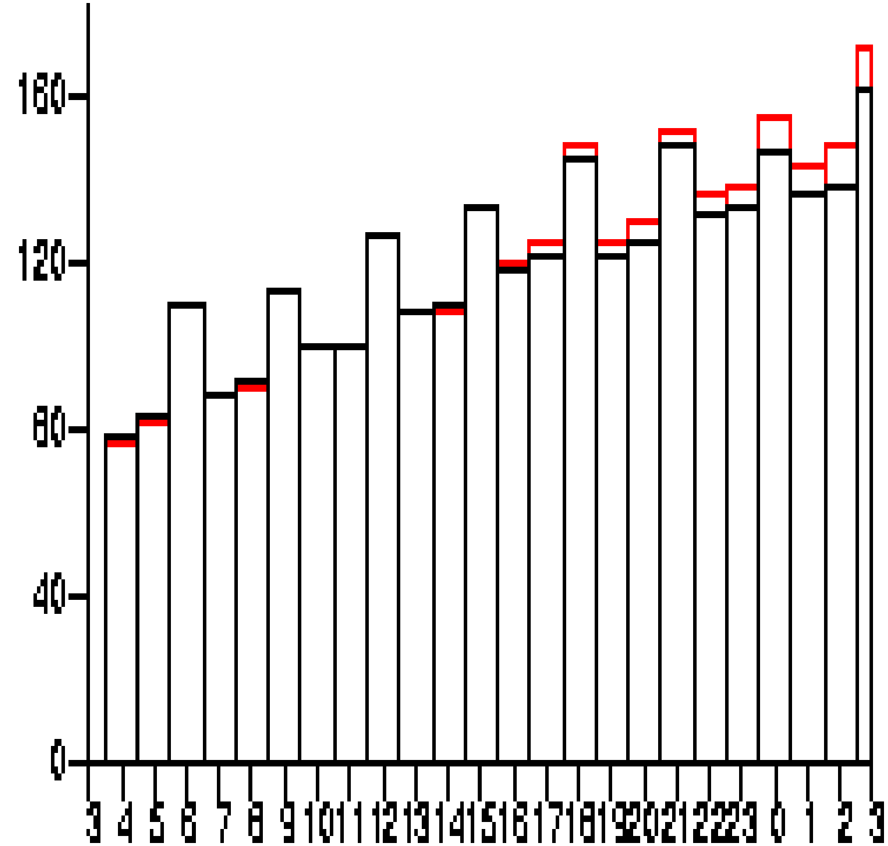
Red is control forecast

Black is sensitivity run

22000-96000 observations/hour



Energy norm sensitivity



Hessian norm sensitivity

Conclusions

- **Sensitivity patterns depends very much on the norm used**
- **Energy norm sensitivities are smaller scale and often very different in structure than J_b or Hessian norm sensitivities**
- **Energy norms are more closely associated with baroclinic regions than seen for J_b or Hessian norms**
- **J_b and Hessian norms give rather similar sensitivity patterns**
- **Forecasts from sensitivity pattern modified analyses are often further away from observations during the first 12 hours than is the case for the control forecasts**
- **From approximately 12 forecast hours and onwards the sensitivity forecasts are closer to the observations than is the case for the control forecast – as expected**
- **These results of relevance for: understanding poor Reduced Rank Kalman Filter performance, targeting, restructuring of observing systems and estimating the benefit of new satellite instruments**