Conservation issues

Clive Temperton
ECMWF

Overview

- Why worry about conservation?
- Conserving Eulerian schemes
- Non-conserving semi-Lagrangian schemes
- A posteriori fixes
- Inherently conserving semi-Lagrangian schemes
- =>

Overview (continued)

- Cell-integrated schemes
- Cascade interpolation to the rescue!
- Some problems
- ECMWF plans
- Conclusions

Why worry about conservation?

- Mass conservation (e.g., in long integrations)
- Moisture (significant drift even in "dynamical core" experiments with semi-Lagrangian integration scheme)
- Other advected quantities (e.g. when chemistry is included)

Digression

A slightly heretical observation:

If the *continuous* equations conserve X, then if the numerical scheme is *accurate* it should conserve X reasonably well.

A scheme which conserves X exactly but is otherwise inaccurate is not very useful.

Conserving Eulerian schemes

e.g., shallow-water continuity equation:

$$\frac{\partial \phi}{\partial t} = -\left\{ \frac{\partial}{\partial x}(\phi u) + \frac{\partial}{\partial y}(\phi v) \right\}$$

C-grid (for example):

$$\frac{\partial \phi}{\partial t} = -\left\{ \delta_x\left(\overline{\phi}^{x} u\right) + \delta_y\left(\overline{\phi}^{y} v\right) \right\}$$

(Spectral: more or less automatic)

Problem (for some)

Eulerian integration schemes are *inefficient* compared with semi-Lagrangian schemes

BUT

In general, semi-Lagrangian schemes are *not formally conserving*.

Two ways to tackle the problem

(1) A posteriori fixes (compute the gain/loss of *X* after each timestep, then restore it).

(2) Modify the semi-Lagrangian scheme so that it becomes *inherently* conserving.

A posteriori fixes (1)

How do we decide where to modify the new field of X in order to restore conservation?

- We could simply add/subtract the same amount everywhere

- Better philosophy is to make adjustments in regions where we expect the original semi-Lagrangian solution to be most in error.

A posteriori fixes (2)

Priestley (MWR Feb 1993): adjustment depends on difference between linear and cubic interpolation.

Bermejo and Conde (MWR Feb 2002): similar but more sophisticated (& is proportional to the cube of the difference).

(Both combined with quasi-monotone version of the semi-Lagrangian scheme).

Cell-integrated schemes (1)

- An *inherently* conserving SL scheme:
- Instead of finding the departure point corresponding to each arrival gridpoint, find the departure points corresponding to *the corners of the cell* surrounding each arrival gridpoint
- Integrate over the “departure cell” (with assumed distribution)
- “Remap” (transport to “arrival cell”)

Cell-integrated schemes (2)

- Rancic (MWR July 1992)
- Laprise & Plante (MWR Feb 1995) –also *downstream* version
- Nair & Machenhauer (MWR March 2002) – on the sphere
- Lauritzen (PDEs on the Sphere 2004) – in three dimensions

Cell-integrated schemes (3)

- 1 dimension: OK
- 2 dimensions: complicated
- 3 dimensions: very complicated!
- (Complicated => expensive too)
- Is there a way out?

Cascade interpolation to the rescue! (1)

- In two dimensions (x,y with rectangular mesh)
- First find the departure points as usual, then use them to construct "Lagrangian" mesh
- Find the points at which the Lagrangian Y-lines intersect the Eulerian x-lines
- Interpolate (1-dim) along the Eulerian x-lines
- Then interpolate (1-dim) along the Lagrangian Y-lines for the values at the departure points.

Cascade interpolation to the rescue! (2)

- Purser & Leslie (MWR Oct 1991) – cascade interpolation
- Leslie & Purser (MWR Aug 1995) – conservative version
- Nair, Côté & Staniforth:
- (QJ, Jan 1999) – simpler version of cascade interpolation
- (QJ, Apr. 1999) – extension to sphere

Cascade interpolation to the rescue! (3)

- Zerroukat, Wood & Staniforth:
- (QJ, Oct 2002) – added conservation ("SLICE")
- (QJ 2004, in press) – extension to the sphere

Some problems

- Spherical geometry ("engineering" needed near the pole for lat-long grid)
- Reduced grid for ECMWF model (no longer have "tensor product" grid)
- Distributed memory - communication
- Icosahedral grids - ???

ECMWF plans

- Diagnostics of non-conservation
- Try “a posteriori fix” – what difference does it make? (moisture, interaction with physics etc.)
- Try cascade interpolation (could go back to “non-reduced” lat-long grid for special applications)

Conclusions

- Semi-Lagrangian schemes can be made conservative (but it's not easy)
- Choice between a posteriori fixes and inherently conserving versions
- Inherently conserving: cell-integrated or based on cascade interpolation
- Still some practical problems (sphere, reduced grid,…)