Initial Perturbations
in
Ensemble Prediction

Martin Ehrendorfer

Institut für Meteorologie und Geophysik

Universität Innsbruck

Presentation at

First Workshop of TIGGE (THORPEX Interactive Grand Global Ensemble)

1-4 March 2005
ECMWF, Reading, UK
1 March 2005

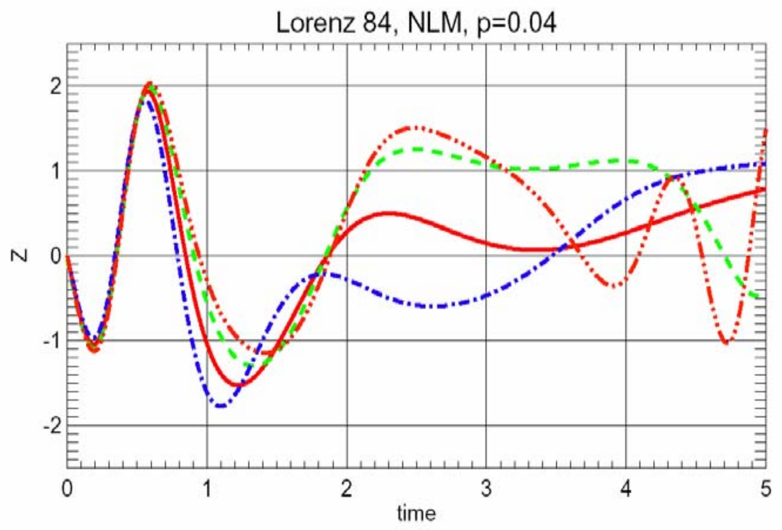
http://www2.uibk.ac.at/meteo

Predictability: error growth

Ensemble Prediction: rationale

Initial Perturbations: Methods

Overview of Methods Used



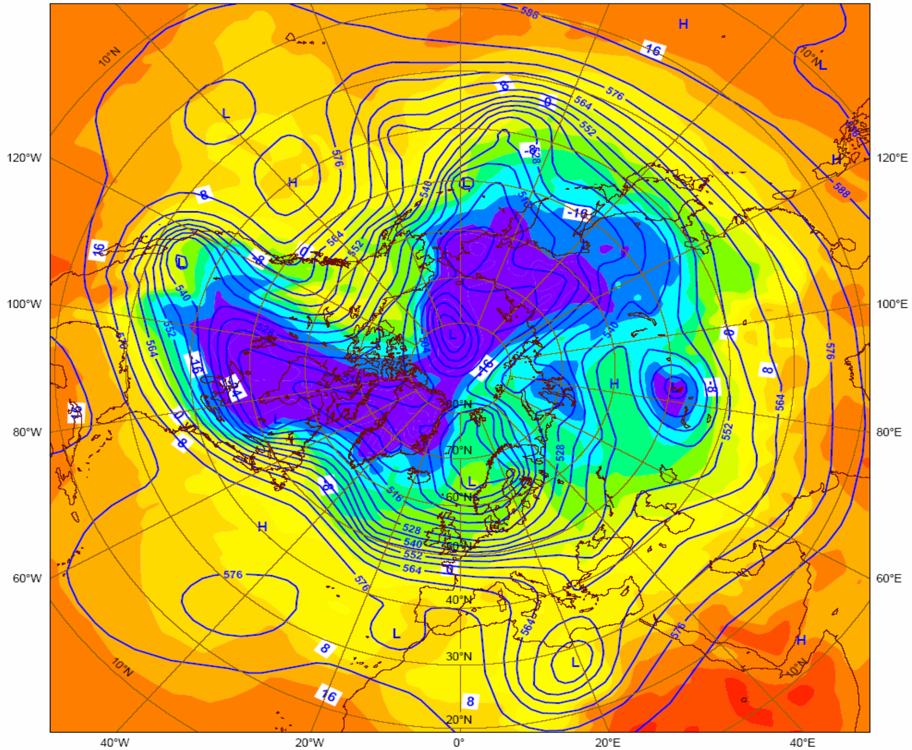
Discussion and Recommendations

do perturbation properties matter?

Conclusion

D+7 and D+6 ECMWF forecast for 20041224/12

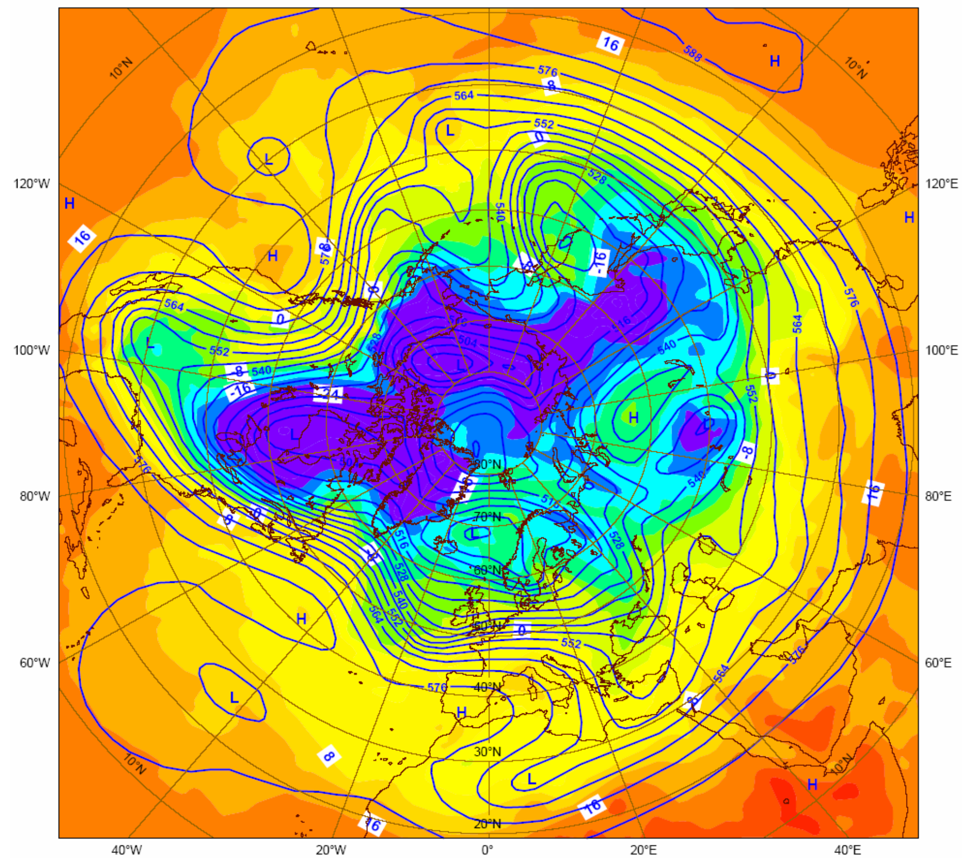
Friday 17 December 2004 12UTC ECMWF Forecast t+168 VT: Friday 24 December 2004 12UTC
500 hPa Height / 850 hPa Temperature

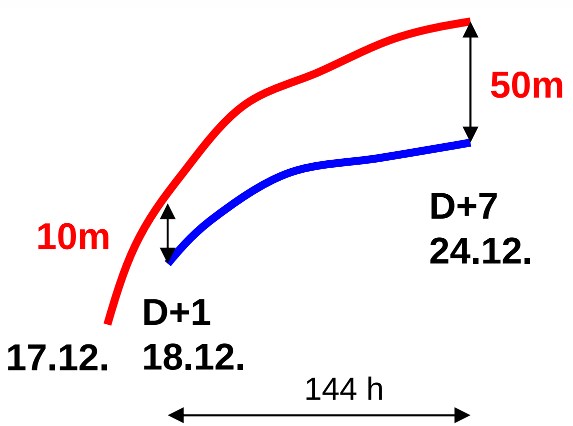


1-day forecast error nonzero

NWP model depends sensitively on I.C.

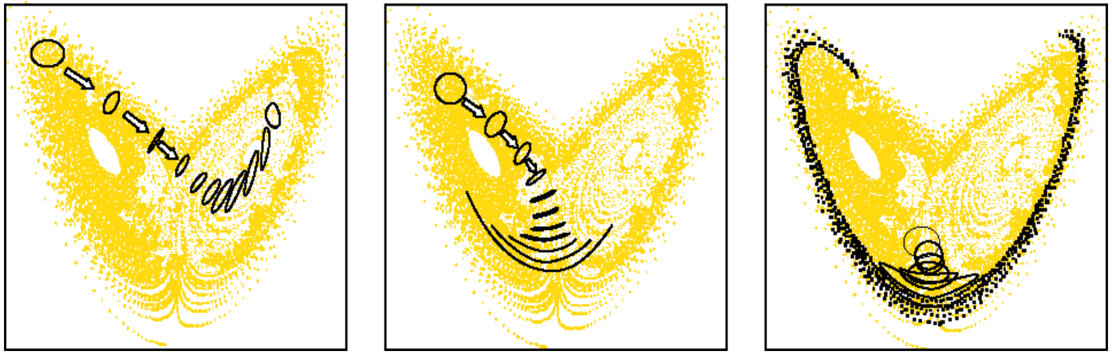
Saturday 18 December 2004 12UTC ECMWF Forecast t+144 VT: Friday 24 December 2004 12UTC
500 hPa Height / 850 hPa Temperature





Early Predictability Work

- Thompson (1957)
- Lorenz (1963)
- Lorenz (1965) SVs
- Charney et al. (1966)
- Epstein (1969)
- Leith (1974)
- ECMWF
- NMC/NCEP



T. Palmer, R. Buizza

– **Liouville equation**
– **Stochastic-dynamic equations**
– ***Monte Carlo approach***

View on predictability

nonlinearity of dynamics

and

instability with respect to small perturbations

$\rightarrow$

sensitive dependence on present condition

chaos

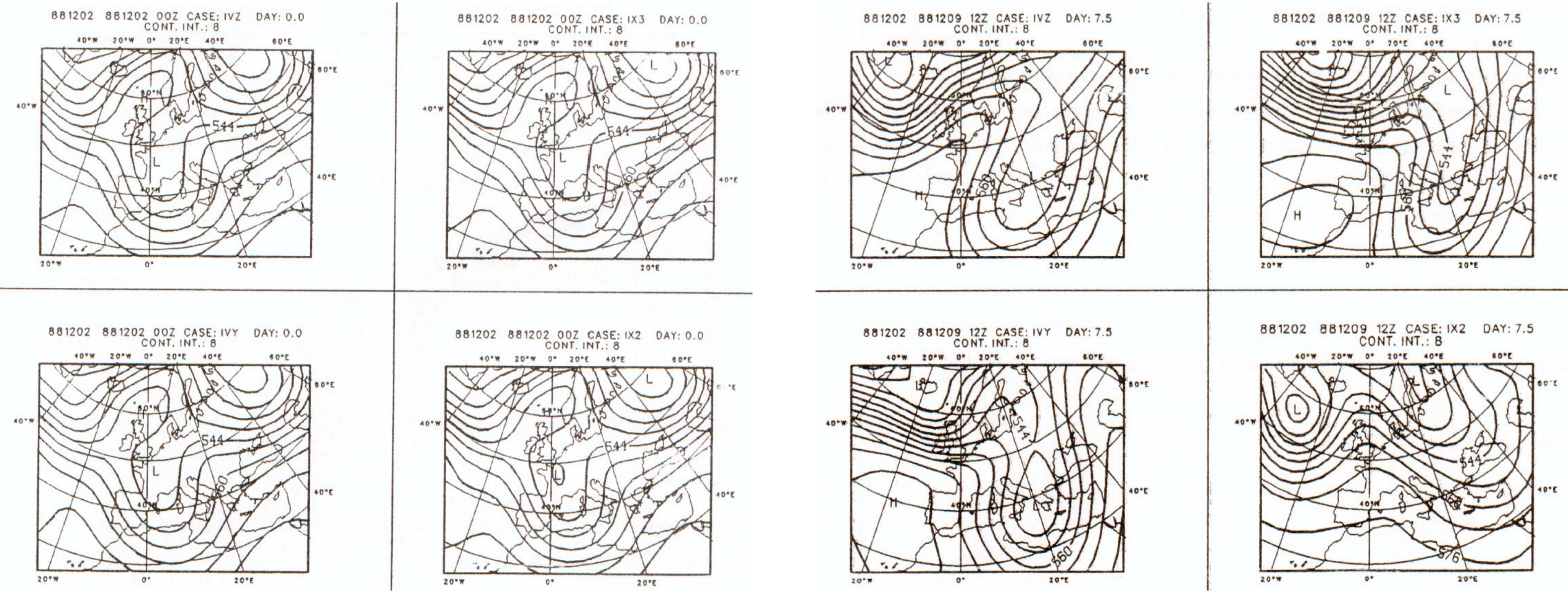
irregularity and nonperiodicity

unpredictability and error growth

ECMWF Seminar 1989

t=0

t = 7.5 d



perturbations generated from short-term forecast error

strong sensitivity to initial condition (19881202/00)

Palmer et al. 1990

„Predictability in the Medium Range and Beyond“

NMC LAF Method

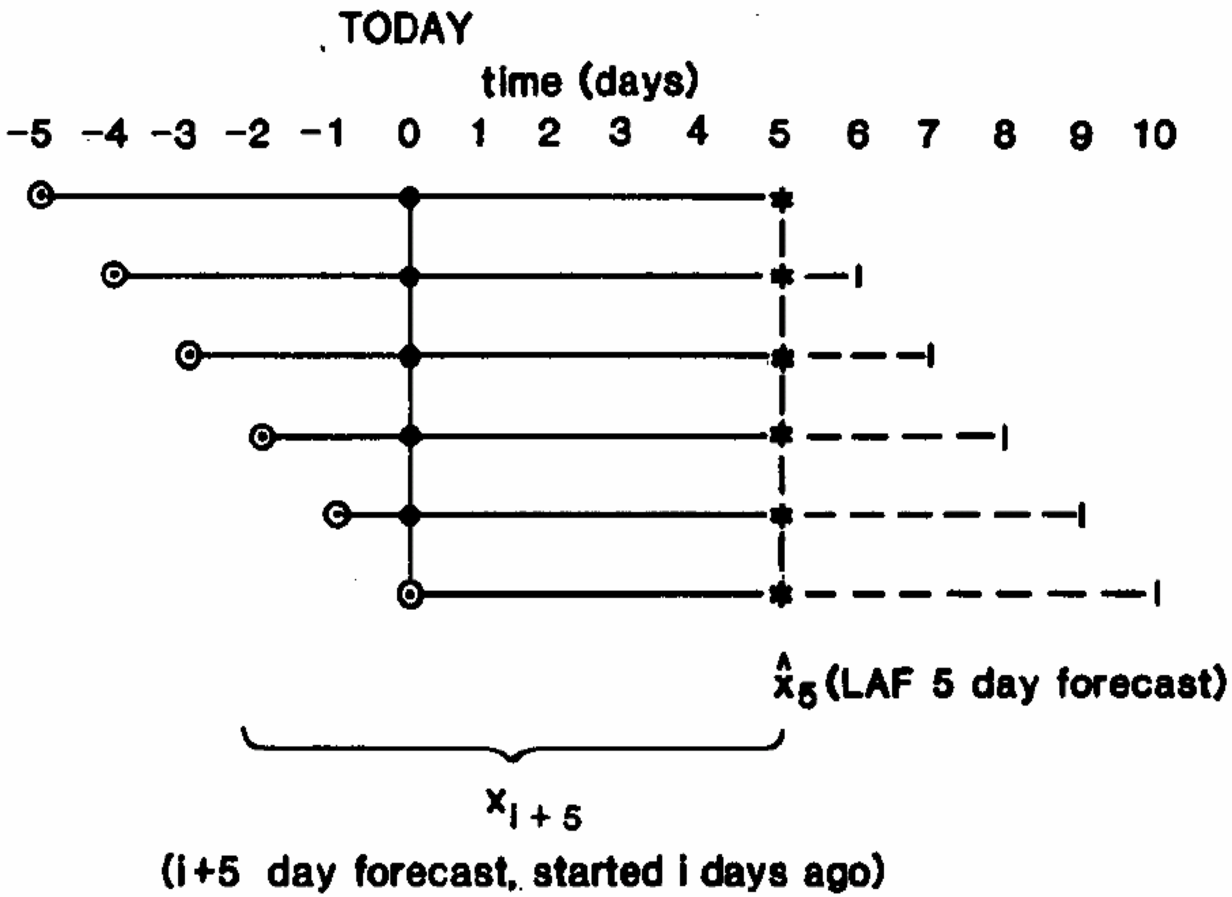


FIG. 1. Schematic description of the LAF method.

forecasts verifying at the same time with lagged initial times

Dalcher et al. 1988, MWR

Conclusions Seminar 1989

8. CONCLUSIONS

With projected upgrades in computer power, it will become technologically feasible to run Monte Carlo forecasts operationally in a few years. With estimates of optimal mode growth, it appears that a strategy for choosing the initial pertubations can be formulated. Finally, development of a probabilistic analysis of forecast flow fields will allow a synthesis of the ensemble forecasts to be given to the user. It can therefore be anticipated that there will be a significant change in the perception of the medium range forecast as a purely deterministic prediction. As a result it is hoped that the perceived skill of the medium range forecast will improve significantly.

eigenmode of L+L* … SV

Palmer et al. 1990

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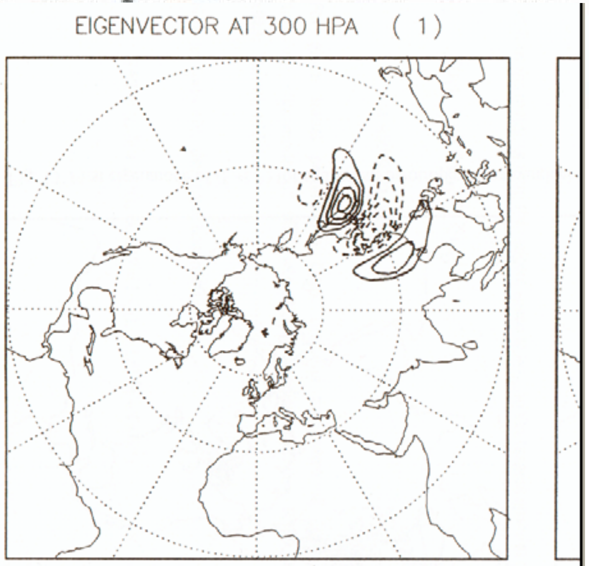


Fig. 14 First eigenmode of (L+L*) for baroclinic climatological basic state.

Ensemble Forecasting at NMC: The Generation of Perturbations

Zoltan Toth[*,+] and
Eugenia Kalnay[+]

breeding

Abstract

On 7 December 1992, The National Meteorological Center (NMC) started operational ensemble forecasting. The ensemble forecast configuration implemented provides 14 independent forecasts every day verifying on days 1–10. In this paper we briefly review existing methods for creating perturbations for ensemble forecasting. We point out that a regular analysis cycle is a "breeding ground" for fast-growing modes. Based on this observation, we devise a simple and inexpensive method to generate growing modes of the atmosphere.

The new method, "breeding of growing modes," or BGM, consists of one additional, perturbed short-range forecast, introduced on top of the regular analysis in an analysis cycle. The difference

UTC, by an ensemble of four 12-day forecasts, plus an extension to 12 days of the aviation 3-day forecast run at 1200 UTC (Tracton and Kalnay 1993). The operational configuration implemented at that time is such that there are 14 forecasts, originating from analyses within the most recent 48 hours, that verify over the same 10-day period. It replaces the previous configuration, where only one operational forecast and one experimental forecast were available for the 6–10-day forecast range. In order not to increase the total use of the CRAY YMP supercomputer, which is already saturated, a compromise had to be found, where the

BAMS 1993

Q. J. R. Meteorol. Soc. (1996), **122**, pp. 73–119

The ECMWF Ensemble Prediction System: Methodology and validation

By F. MOLTENI, R. BUIZZA, T. N. PALMER* and T. PETROLIAGIS

European Centre for Medium-Range Weather Forecasts, UK

(Received 1 August 1994; revised 24 May 1995)

SUMMARY

The European Centre for Medium-Range Weather Forecasts (ECMWF) Ensemble Prediction System (EPS) is described. In addition to an unperturbed (control) forecast, each ensemble comprises 32 10-day forecasts starting from initial conditions in which dynamically defined perturbations have been added to the operational analysis. The perturbations are constructed from singular vectors of a time-evolution operator linearized around the short-range-forecast trajectory. These singular vectors approximately determine the most unstable phase-space directions in the early part of the forecast period, and are estimated using a forward and adjoint linear version of the ECMWF numerical weather-prediction model. An appropriate norm is chosen, and relationships between the structures of these singular vectors at initial time and patterns showing the sensitivity of short-range forecast error to changes in the analysis are discussed. A methodology to perform a phase-space rotation of the singular vectors is described, which generates hemispheric-wide perturbations and renormalizes them according to analysis-error estimates from the data-assimilation system.

singular vectors

Ensemble Prediction

- Purpose
  - evolve probability density function
  - identify flow-dependent predictability
  - determine possible different flow evolution
  - essentially all methods are of MC type
  - possibly account for model error

- Initial Perturbations
  - critical to efficiently reflect analysis uncertainty P^a
  - in view of high short-term sensitive dependence

- Analysis error covariance P^a
  - known incompletely and high-dimensional
  - efficiency in reflecting known P^a features
  - perturbations representing P^a

Generating Initial Perturbations

- Basic requirement
  - the initial perturbations reflect the covariance structure contained in P^a
  - then, the model M will map these perturbations into realizations consistent with P^f
- Difficulties
  - limited number of perturbations affordable
  - limited knowledge about structure of P^a

Evolving a probability density function (pdf)

- Initial pdf described by (multivariate normal):

$$\mathbf{x}_0 \sim \mathcal{N}(\boldsymbol{\mu}_0, \mathsf{P}^{\mathsf{a}}) \tag{1}$$

- Evolved pdf is, when state evolves according to linear model M:

$$\mathbf{x}_t = \mathsf{M}\mathbf{x}_0 \tag{2}$$

- still multivariate normal:

$$\mathbf{x}_t \sim \mathcal{N}(\mathsf{M}\boldsymbol{\mu}_0, \mathsf{M}\mathsf{P}^{\mathsf{a}}\mathsf{M}^{\mathrm{T}}) \tag{3}$$

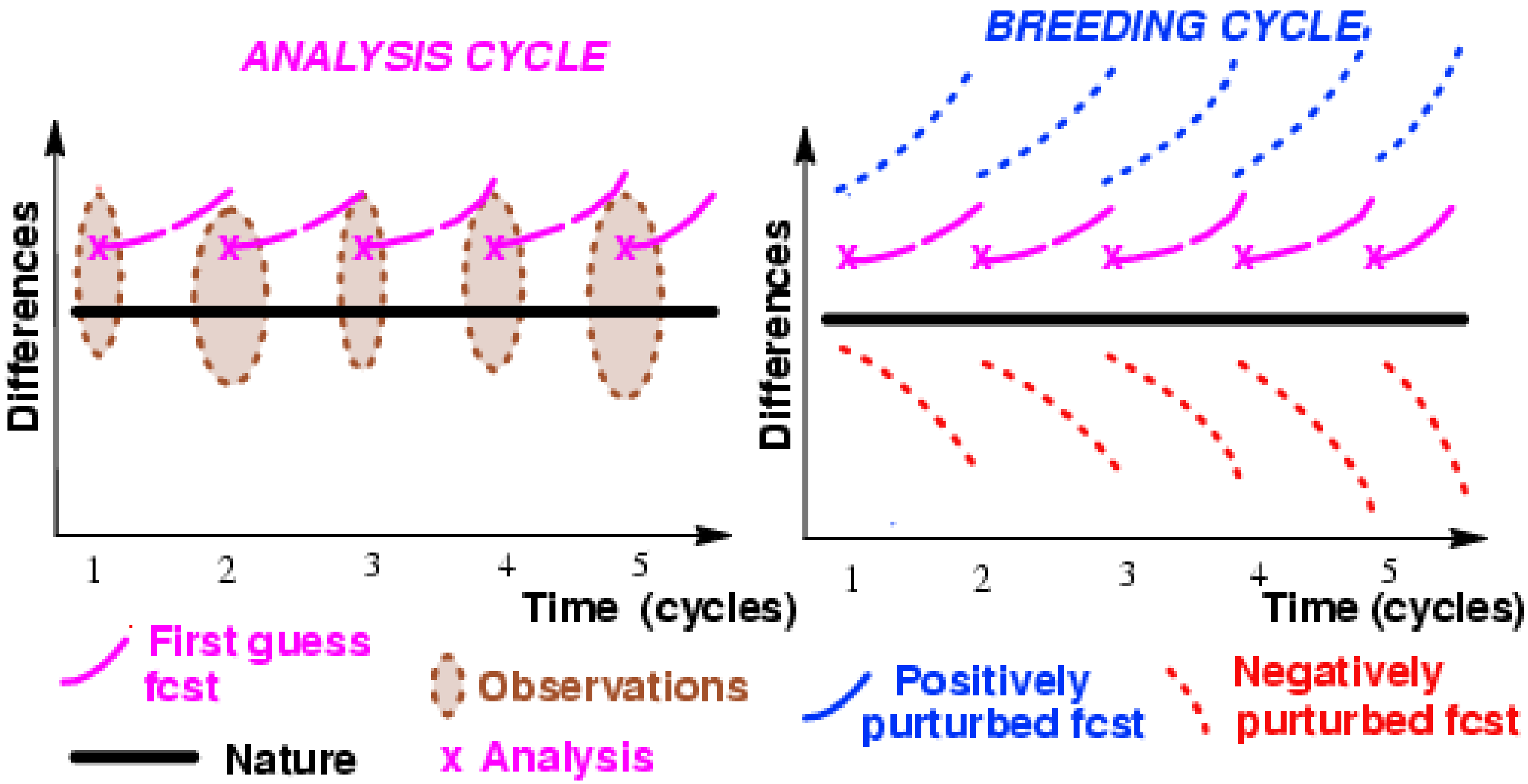
- with the forecast error covariance matrix:

$$\mathsf{P}^{\mathsf{f}} \equiv \mathsf{M}\mathsf{P}^{\mathsf{a}}\mathsf{M}^{\mathrm{T}} \tag{4}$$

Methods based on …

- Singular Vectors (SVs) – ECMWF (Palmer, Buizza, Barkmeijer)
  - total energy (TE) or analysis error covariance (AEC)
    - sample future dynamical instabilities given analysis uncertainty
- Breeding – NCEP (Kalnay, Toth)
  - regional rescaling
    - simulate analysis cycle, growth over past assimilation interval
  - ensemble transform (ET)/breeding
- Ensemble Kalman Filter – MSC (Houtekamer, Evensen, Hamill)
  - perturbed observations
    - parallel sets of data assimilation
  - ETKF (Bishop)
    - find T such that P^a=(I-KH)P^f is solved, where P^f is from evolved ensemble, P^a from transformed evolved ensemble and consistent with (new) observations
  - ExKF (Anderson, Hamill)
    - reforcasting and calibration

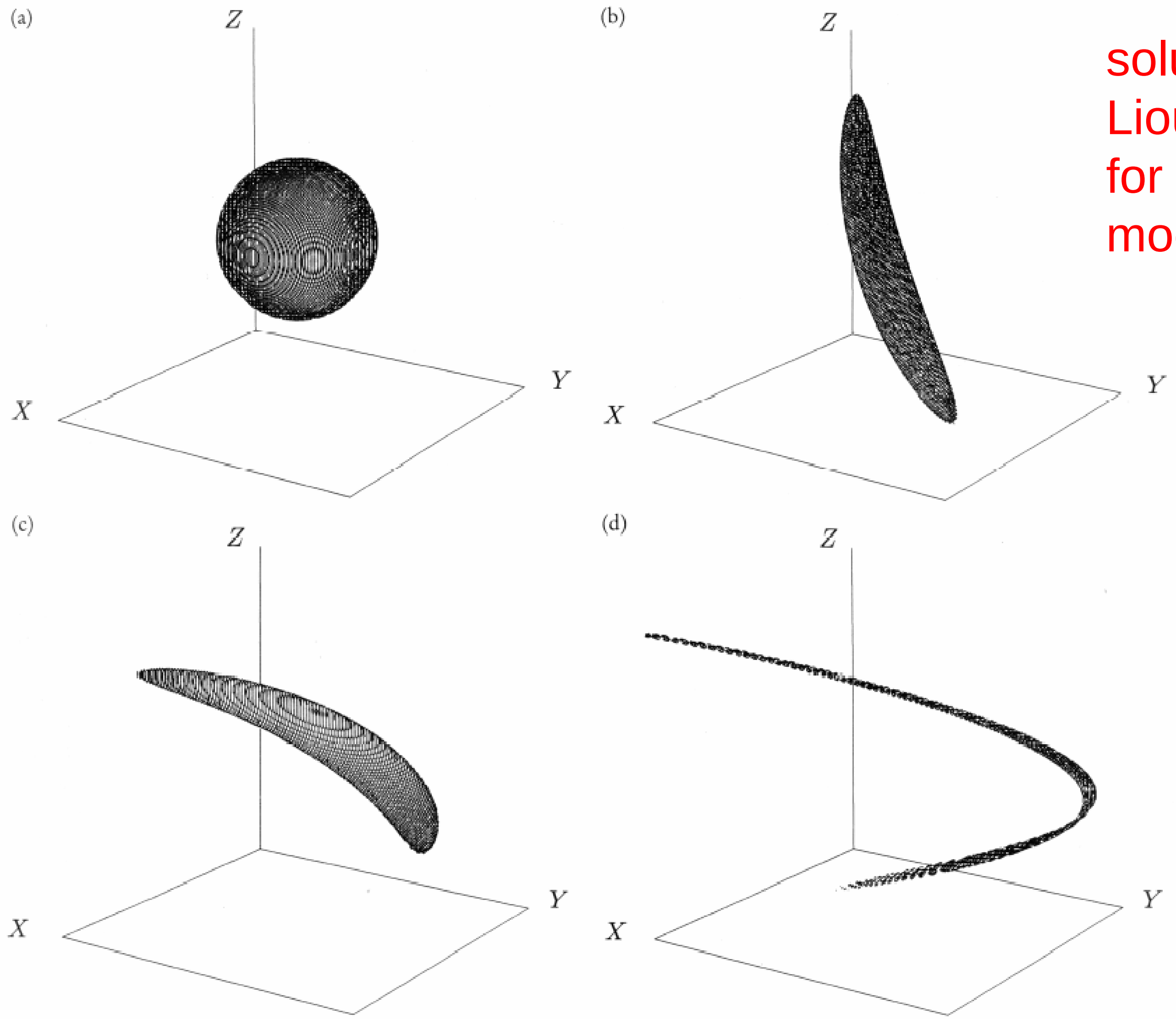
Breeding



Toth and Kalnay 1997

R. Buizza

Singular Vectors

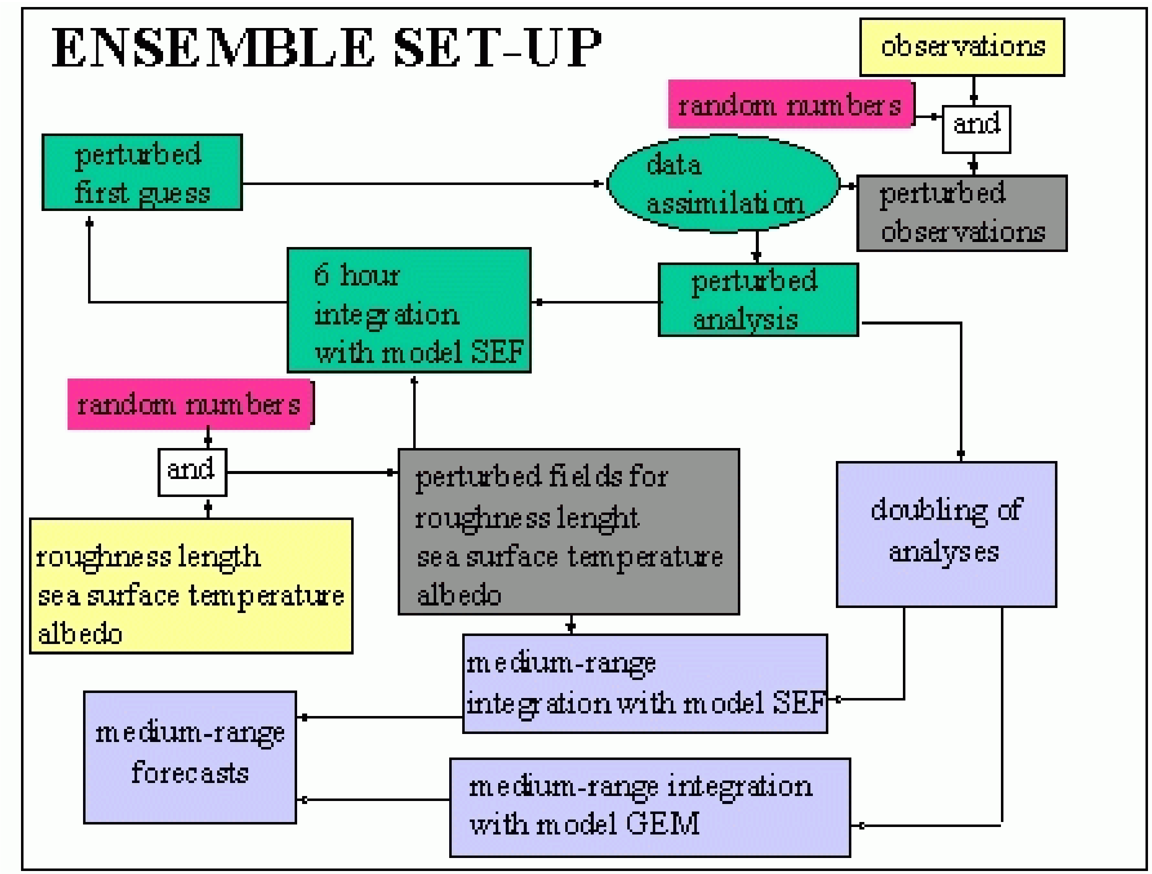


solution of the Liouville equation for Lorenz 1984 model

Ehrendorfer 1997

Ensemble Kalman filtering

MSC



perturbed observations

↓

assimilation

↓

perturbed analyses → EPS

↓

perturbed backgrounds

Houtekamer, Buizza

Singular Vectors - Definition

Singular Vectors (SVs)

Definition $\rightarrow$ Maximize: $\mathsf{C}^{\mathrm{T}}\mathsf{C} > 0$ and $\mathsf{A}^{\mathrm{T}}\mathsf{A} > 0$

$$J(\mathbf{x}) = \left(\mathsf{CM}_{\Xi,t}\mathbf{x}\right)^{\mathrm{T}}\left(\mathsf{CM}_{\Xi,t}\mathbf{x}\right) \qquad \text{s.t.} \qquad \left(\mathsf{A}\mathbf{x}\right)^{\mathrm{T}}\left(\mathsf{A}\mathbf{x}\right) = 1 \tag{5}$$

Solution $\mathbf{x}$ is called **first singular vector**, obtainable from eigenproblem:

$$\mathsf{M}_{\Xi,t}^{\mathrm{T}}\mathsf{C}^{\mathrm{T}}\mathsf{CM}_{\Xi,t}\mathbf{y} = \lambda\mathsf{A}^{\mathrm{T}}\mathsf{A}\mathbf{y} \qquad \text{s.t.} \qquad \mathbf{y}^{\mathrm{T}}\mathsf{A}^{\mathrm{T}}\mathsf{A}\mathbf{y} = 1 \tag{6}$$

Property $\rightarrow$ Note that:

$$J(\mathbf{x} = \mathbf{y}_k) = \lambda_k \tag{7}$$

Computation $\rightarrow$ Through a change of variable according to:

$$\mathbf{z} \equiv \mathsf{A}\mathbf{y} \qquad \Leftrightarrow \qquad \mathbf{y} \equiv \mathsf{A}^{-1}\mathbf{z} \tag{8}$$

(6) may completely equivalently be rewritten in the form:

$$\left(\mathsf{CM}_{\Xi,t}\mathsf{A}^{-1}\right)^{\mathrm{T}}\left(\mathsf{CM}_{\Xi,t}\mathsf{A}^{-1}\right)\mathbf{z} = \lambda\mathbf{z} \qquad \text{s.t.} \qquad \mathbf{z}^{\mathrm{T}}\mathbf{z} = 1 \tag{9}$$

The vectors $\mathbf{z}_k$ are the *right singular vectors* of the matrix $\mathsf{CM}_{\Xi,t}\mathsf{A}^{-1}$ with associated *singular values* $\sigma_k \equiv \sqrt{\lambda_k}$ (see, e.g., Golub and Van Loan 1989).

Hessian SVs

Hessian SVs (HSVs) / Analysis Error Covariance SVs (AECSVs)

The HSVs Z_0 solving the eigenvector problem: $\quad A^T A \equiv (P^a)^{-1}$

$$M^T C^T C M Z_0 = (P^a)^{-1} Z_0 \Lambda \qquad \text{s.t.} \quad Z_0^T (P^a)^{-1} Z_0 = I \tag{10}$$

are, when time–evolved, eigenvectors of P^f, because:

$$\Big(C\,\underbrace{M P^a\Big) M^T\, C^T}_{\equiv P^f}\, \underbrace{C M Z_0}_{\equiv Z_t} = \Big(C M P^a\Big)(P^a)^{-1} Z_0 \Lambda \quad \longrightarrow \quad \boxed{\Big(C P^f C^T\Big)\, Z_t = Z_t \Lambda} \tag{11}$$

The evolved HSVs Z_t are the eigenvectors of CP^fC^T – which is the forecast error covariance in the "final–time norm" C. Note the final–time orthogonality relationship:

$$Z_t^T Z_t = \Big(C M Z_0\Big)^T \Big(C M Z_0\Big) = Z_0^T\, \underbrace{M^T C^T C M Z_0} = \overbrace{Z_0^T (P^a)^{-1} Z_0}\, \Lambda = \Lambda \tag{12}$$

SVs – Properties ECMWF

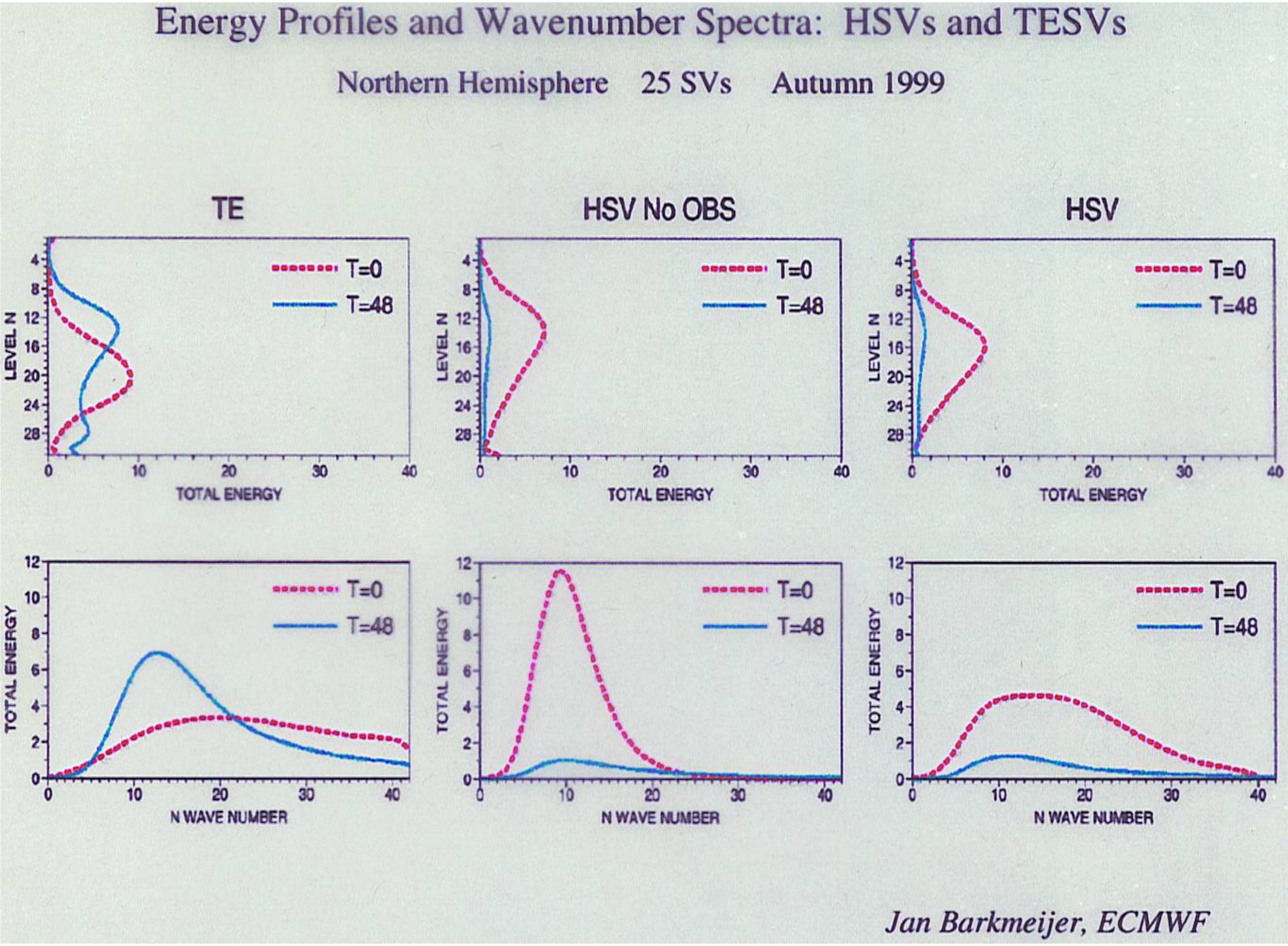


figure from R. Errico (GMAO)

SVs – Properties NRL/NAVDAS

GELARO ET AL.

10 m

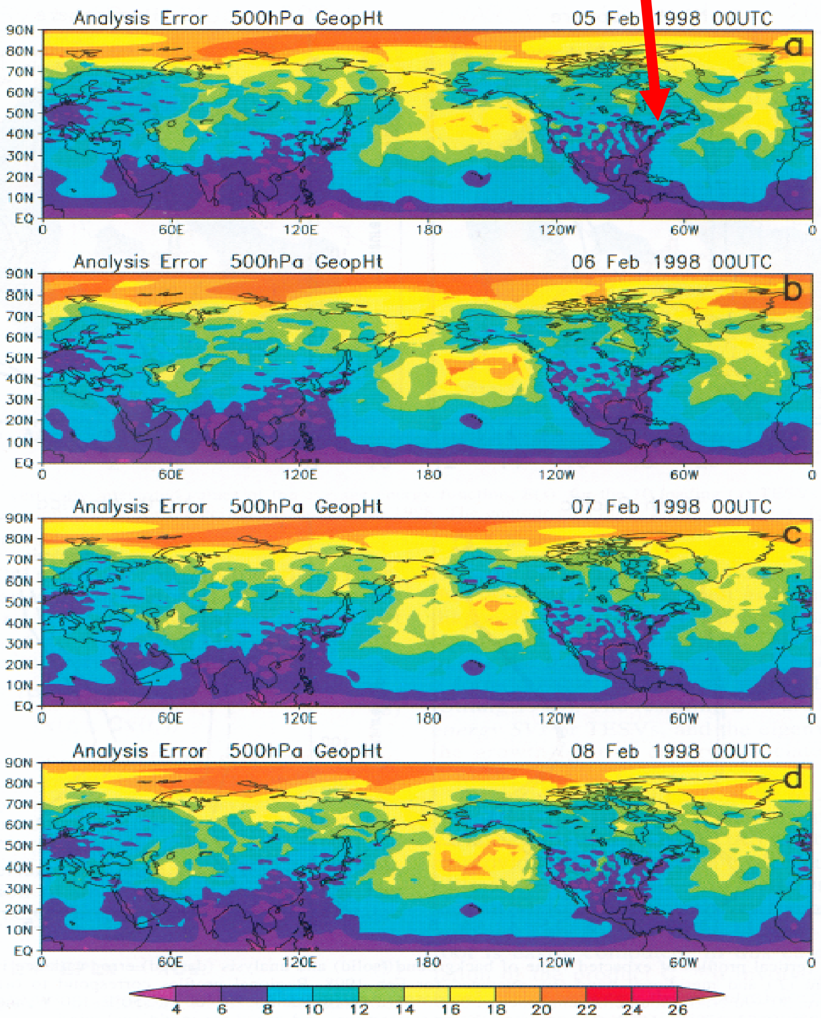


FIG. 2. Square root of expected value of analysis error variance in terms of 500-hPa geopotential height for 0000 UTC 5–8 Feb 1998, (a)–(d), respectively. The contour interval is 2 m.

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TE **VARSV**

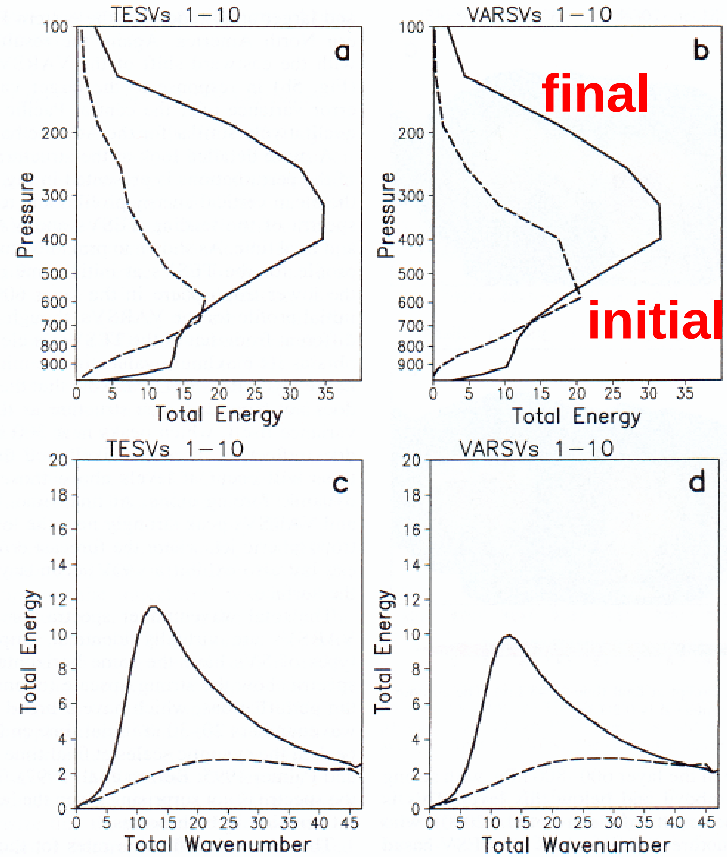


FIG. 7. Mean (a),(b) vertical energy profiles and (c),(d) total wavenumber spectra for the 10 leading (a),(c) TESVs and (b),(d) VARSVs for the period 31 Jan–20 Feb 1998. Dashed (solid) curves correspond to SVs at initial (final) time. For display purposes, the values at initial time have been multiplied by 100.

Gelaro et al. 2002, MWR

Analysis error covariance and SVs

The SV–Decomposition of P^a

- Because the initial–time SVs satisfy (10), it is true that P^a can be written as:

$$\mathsf{P}^a = \mathsf{Z}_0 \mathsf{Z}_0^T \tag{13}$$

This is a special square–root for P^a (different from eigendecomposition and also not lower–triangular) $\rightarrow$ the **SV–decomposition of** P^a

- Under linear dynamics this SV–decomposition becomes the eigendecomposition of the forecast error covariance matrix, because (14) is the same as (11) together with (12):

$$\left(\mathsf{CM}\right)\mathsf{P}^a\left(\mathsf{CM}\right)^T = \left(\mathsf{CM}\right)\mathsf{Z}_0\mathsf{Z}_0^T\left(\mathsf{CM}\right)^T \quad \rightarrow \quad \boxed{\mathsf{CP}^f\mathsf{C}^T = \mathsf{Z}_t\mathsf{Z}_t^T} \tag{14}$$

- non–eigendecomposition $\rightarrow$ eigendecomposition (non–modality)

- ECMWF: use of SV–decomposition in EPS

- *"... k time–evolved HSVs are the leading k eigenvectors of forecast error covariance ..."* Ehrendorfer and Tribbia (1997)

Multinormal sampling

Multinormal Sampling Based on SV–Decomposition of P^a

- Transforming random variables

$$\mathbf{q} \sim \mathcal{N}(0, \mathsf{I}) \qquad \Rightarrow \qquad \mathbf{x} = \mathbf{x}_0^c + \mathsf{V}^{1/2}\mathbf{q} \qquad \longrightarrow \qquad \mathbf{x} \sim \mathcal{N}(\mathbf{x}_0^c, \mathsf{V}) \qquad (15)$$

- Use **SV–decomposition of** P^a (possibly truncated to N SVs) in (13) – to describe square–root of P^a – in process of generating initial–time perturbed states $\mathbf{x}$:

$$(\mathsf{P}^\mathsf{a})^{1/2} = \mathsf{Z}_0 \quad \longrightarrow \quad \left(\mathsf{P}^{\mathsf{a}(N)}\right)^{1/2} = \mathsf{Z}_0{}^{(N)} \quad \longrightarrow \quad \mathsf{P}^{\mathsf{a}(N)} = \left(\mathsf{Z}_0{}^{(N)}\right)\left(\mathsf{Z}_0{}^{(N)}\right)^{\mathrm{T}} \qquad (16)$$

$$\mathbf{x}_i = \mathbf{x}_0^c + \mathsf{Z}_0^{(N)}\mathbf{q}_i \qquad i = 1, 2, ..., M \quad \Rightarrow \quad \boxed{\mathbf{x} \sim \mathcal{N}\left(\mathbf{x}_0^c, (\mathsf{P}^\mathsf{a})^{(N)}\right)} \qquad (17)$$

- Generating perturbations consistent with P^a knowledge based on N SVs
- Assumes normally distributed analysis errors
- Taking SV properties into nonlinear regime
- Strong similarity to operational *rotation* at ECMWF
- free parameters: N and M

Ehrendorfer and Beck (2003)

Use of methods to generate initial perturbations

- Breeding
  - National Centers for Environmental Prediction (NCEP) **OP**
  - Climate Diagnostics Center (CDC)-NCEP/National Oceanic and Atmospheric Administration (NOAA)
  - National Centre for Medium Range Weather Forecasting (NCMRWF)
  - Fleet Numerical Meteorological & Oceanographic Center (FNMOC) **OP**
  - China CMA, Brazil CPTEC, Japan JMA, Korea KMA **4 x OP**

- Singular Vectors
  - European Centre for Medium-Range Weather Forecasts (ECMWF) **OP**
  - Bureau of Meteorology Research Centre (BMRC) **OP**

- Ensemble Kalman Filter
  - Meteorological Service of Canada (MSC) **OP**
  - United Kingdom Meteorological Office (UKMO)

Initial Perturbations at NCEP

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on bred vectors
  - 24 hr cycling
  - Rescaling dependent on geographically and seasonally estimated analysis uncertainty
- Configuration
  - 2 control forecasts
  - 5 pairs of perturbed forecasts up to 16 days
  - 4 times per day
- Resolution
  - High-resolution control T254L64 up to 3.5 days
  - Control truncated to T170L42 up to 7.5 days, then T126L28
  - Perturbed integrations T126L28 up to 7.5 days, then T62L28
- Recent work
  - 6hr breeding cycle with ETKF
  - to generate initial perturbations using NCEP real-time observations
  - Wang and Bishop 2003, JAS
- Work in progress
  - Testing ensemble transform method
  - for generating initial perturbations using information on analysis error variance from 3DVAR
  - Increase ensemble size from 10 to 80
  - 80 orthogonal perturbations

Initial Perturbations at CDC-NCEP/NOAA

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on bred vectors / reforecast data set
  - 24 hour cycling
  - Rescaling dependent on geographically and seasonally estimated analysis uncertainty
  - Toth and Kalnay 1997, MWR
- Configuration / reforecast data set
  - 7 pairs of perturbed forecasts plus control from NCEP/NCAR reanalysis, 15 members
  - 00 UTC, 15 days projection
  - Forecasts from 1979 to present (consistent fixed version of model)
- Resolution and model
  - T62L28 NCEP MRF model (recently renamed GFS)
  - Forecast archive truncated at T36, variables: u, v, T, Z, …
- Work in progress
  - Testing efficacy of reforecasts from ERA-40 initial condition
  - Next generation reforecast with higher-resolution updated NCEP GFS
- Comments
  - 23 year data base of retrospective forecasts
  - Hamill et al. 2004, MWR
  - Data base used to calibrate EPS over training sample
  - Use of MOS technique

Initial Perturbations at NCMRWF

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on breeding of growing modes
  - 24 hour cycling
  - Geographically and seasonally dependent rescaling based on estimated analysis uncertainty
- 2 control forecasts
  - At T80 and T170
  - 4 pairs of perturbed forecasts at T80
  - At 00 UTC for 168 hours
- Tentatively operational by 1 April 2005
  - Need to improve control forecast

Initial Perturbations at FNMOC

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on Bred modes
  - 18 members
  - 8 plus, 8 minus: T119L30
  - current and 12-h lagged high-resolution forecasts T239L30 truncated to T119L30 after 6 days
- Perturbed integrations
  - NOGAPS (Navy Operational Global Atmospheric Prediction System)
  - Run daily at 00 UTC out to 10 days
- Work in Progress
  - Twice daily
  - Initial perturbations that sample analysis error variance as estimated by NAVDAS (NRL Atmospheric Variational Data Assimilation System)
  - Initial perturbations based on Ensemble Transform being tested
  - Model error
  - Perturbations in the tropics

Initial Perturbations at ECMWF

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on SVs
  - T42L40 (simplified physics)
  - use of evolved SVs
  - 48 h OTI SVs
- Perturbed initial conditions
  - Up to 8 target areas (including tropical SVs)
  - Gaussian sampling to combine SVs (25 E-Tropics, 10 T)
  - Scaling based on 4DVAR analysis error
  - Perturbed integrations TL255L40
  - 50+1 members
- Work in progress
  - Moist SVs
  - TL95L60 SVs
  - Shorter OTI (24 h)
  - Hessian initial norm
  - Use of Ensemble Data Assimilation (EDA)
- Details
  - Molteni et al. 1996, QJ
  - Bourke et al. 2004, MWR: ECMWF, BMRC
  - Buizza et al. 2005, MWR: ECMWF, MSC, NCEP

Initial Perturbations at BMRC

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on SVs
  - Initial-time 48h SVs
  - evolved SVs not used
  - T42L19 (simplified physics)
  - Localization: excluding tropics 20S to 20N
- Perturbed initial conditions
  - f_j = f_0 + a_j,k SV_k with f_0 the TL119L19 analysis
  - Resolution TL119L19 (operational is TL239L29)
  - 32+1 members
  - Rotation of SVs
  - perturbations in both hemispheres
  - Scaling: spread at D+2 similar to error of control at D+2
- Regional EPS
  - Randomly perturbed observations, stochastic physics
  - Tropical cyclone bogus data are perturbed if TC present
- Planned work
  - Increase in resolution TL159L29
  - 50 members
  - Investigate occasional appearance of spurious SVs
- Details
  - Bourke et al. 2004, MWR

Initial Perturbations at MSC

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

- Based on ensemble of assimilation cycles
  - Random perturbations to model error fields and observations
- Ensemble Kalman Filter
  - With 96 members, reduced to 16 members
  - Multimodel ensemble: 8 versions SEF, 8 versions GEM
- Configuration
  - 16 members plus unperturbed control forecast, 00 UTC, 10 days
  - 8 members TL149 spectral model (SEF)
  - 8 members 1.2 deg finite element model (GEM)
  - Vertical resolution: 28 levels GEM, 23 or 41 levels SEF
- Recent work
  - EnKF with improved accuracy of ensemble mean
  - Short-range high-resolution SV-based ensemble
    (OTI 48 h, 20 members, 35 km resolution)
  - Bayesian model averaging, Extreme forecast index
- Future
  - 15 days projection, twice daily 00 and 12 UTC

Initial Perturbations at UKMO

NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO

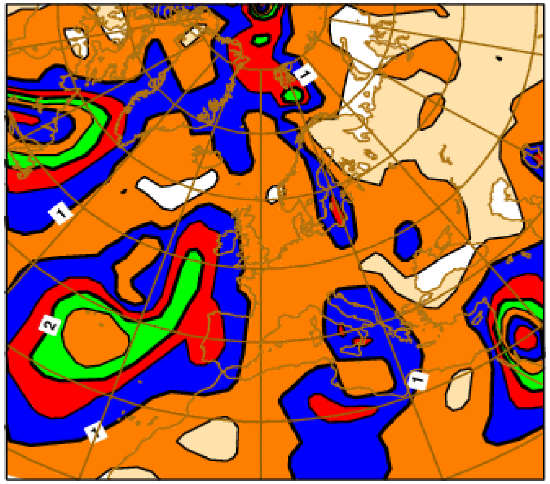
- Based on ETKF
  - ETKF
  - rescaling of evolved perturbations while observing $P^a=(I-KH)P^f$
  - an ensemble data assimilation method
- Model perturbations
  - RP (random parameter) scheme
  - Perturbing a selection of tunable parameters
  - SCV (stochastic convective vorticity) scheme
  - Based on conceptual dynamical model of mesoscale convective systems
- Present configuration
  - Global ensemble forecast (not operational)
- Future
  - Limited-area ensemble covering North-Atlantic and Europe
  - Initial perturbation from global ensemble
  - Localization within the ETKF
  - Stochastic kinetic-energy backscatter

Do perturbation properties matter?

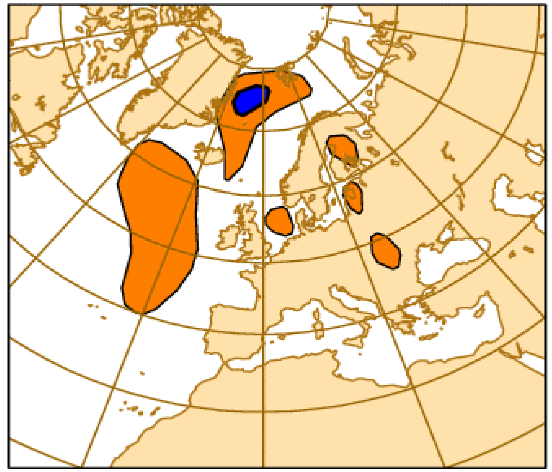
- **Yes**
  - – high short-term sensitivity of model to initial condition
  - – at least for short-term results
  - – most perturbations will (eventually) grow (in global model) due to presence of instability
    - thus not necessarily a sign of reflecting analysis error
  - – final pdf is direct result of initial-pdf formulation
  - – sample size is of secondary importance in comparison to reflecting P^a well

Z500 Europe STD T0 and D+2 for NCEP BMRC ECMWF

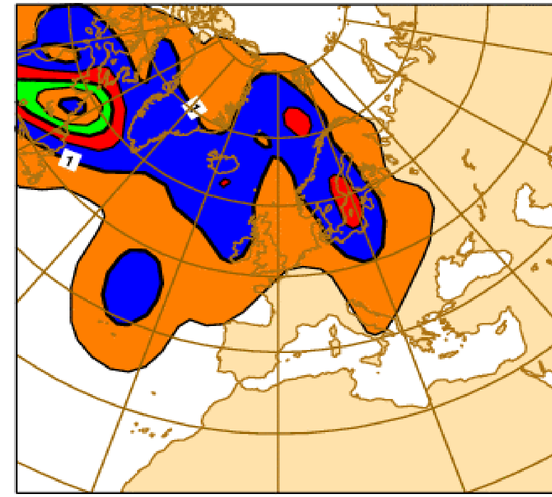
Z500 - 00UTC 15 Jan 2005 t0
NCEP 11m STD (ci=0.5dam)



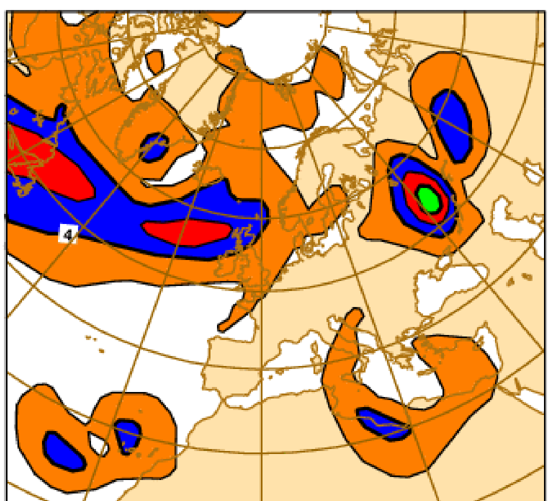
Z500 - 00UTC 15 Jan 2005 t0
BMRC 33m STD (ci=0.5dam)



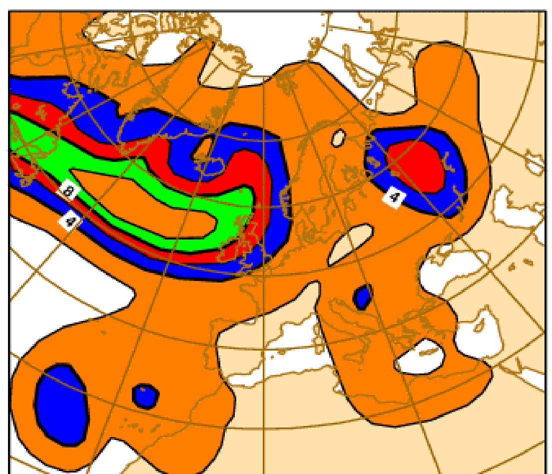
Z500 - 00UTC 15 Jan 2005 t0
ECMWF 51m STD (ci=0.5dam)



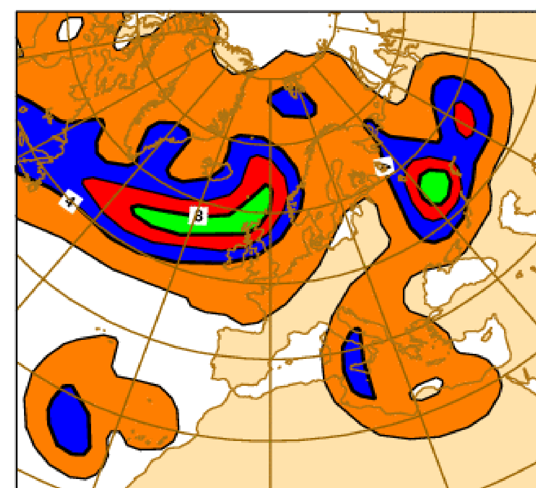
Z500 - 00UTC 15 Jan 2005 +48h
NCEP 11m STD (ci=2dam)



Z500 - 00UTC 15 Jan 2005 +48h
BMRC 33m STD (ci=2dam)



Z500 - 00UTC 15 Jan 2005 +48h
ECMWF 51m STD (ci=2dam)



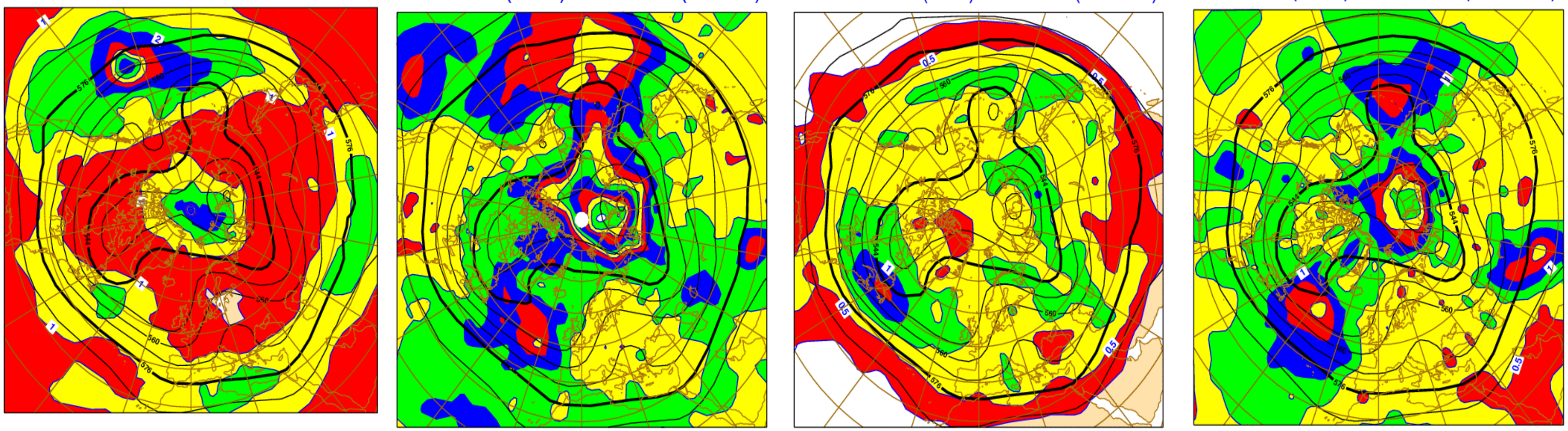
Z500 May 2002 EM+STD T0 for NCEP, MSC, ECMWF, Analysis

Z500 - 00UTC May 2002 t0 (31d)
NCEP EM (ci=8) and STD (ci=0.5)

Z500 - 00UTC May 2002 t0 (31d)
MSC EM (ci=8) and STD (ci=0.5)

Z500 - 00UTC May 2002 t0 (31d)
ECMWF EM (ci=8) and STD (ci=0.25)

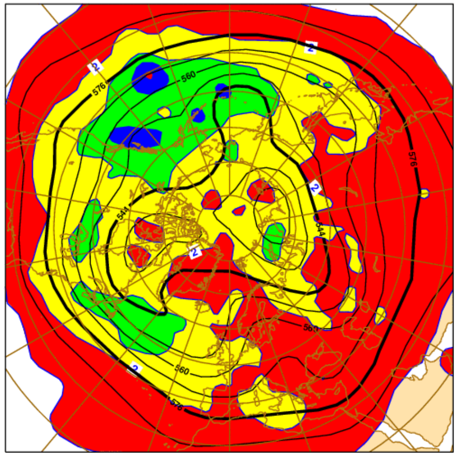
Z500 - May 2002 (31d) - t0
3C ANA (ci=8) and STD (ci=0.25)



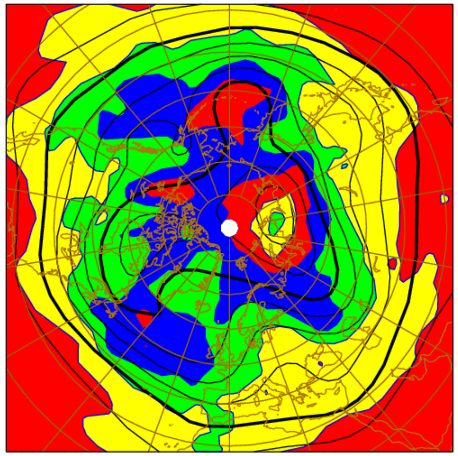
* NCEP and MSC ~ twice as large as ANA STD
* ECMWF has amplitude similar to ANA STD
* Differences in location

Z500 May 2002 EM+STD D+2 for NCEP, MSC, ECMWF, Analysis

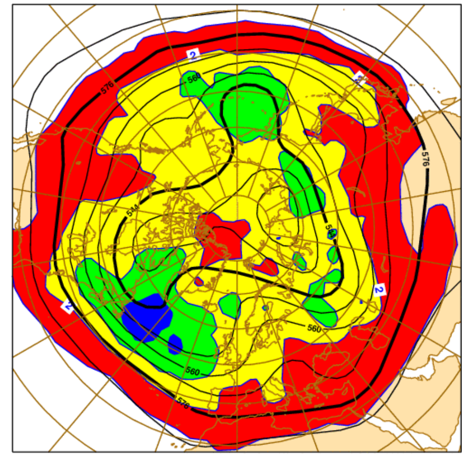
Z500 - 00UTC May 2002 t+48h (31d)
NCEP EM (ci=8) and STD (ci=1)



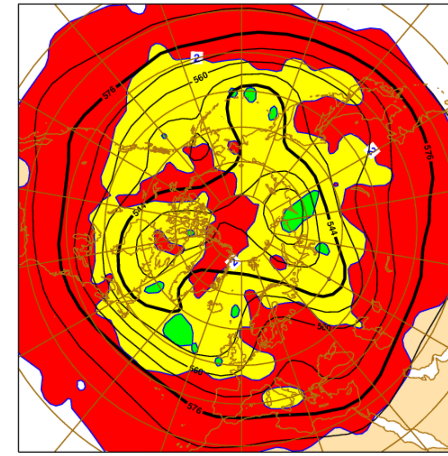
Z500 - 00UTC May 2002 t+48h (31d)
MSC EM (ci=8) and STD (ci=1)



Z500 - 00UTC May 2002 t+48h (31d)
ECMWF EM (ci=8) and STD (ci=1)



Z500 - May 2002 (31d) - t+48h
3C ANA (ci=8) and STD (ci=1)



* MSC has largest amplitude over NH
* ECMWF has smallest amplitude over tropics

Discussion of methods

- Thought experiment
  - if P^a were easily and fully available – how would we generate perturbations?

- Is it important to mimic analysis error?
  - Only the growing part?
  - A question of time scale?

- Are ensemble requirements different for
  - Generating 6-hour versus 3-day backgrounds?

- How do initial perturbations matter?
  - Does calibration offset initial perturbation deficiences and/or effects of ensemble sizes?

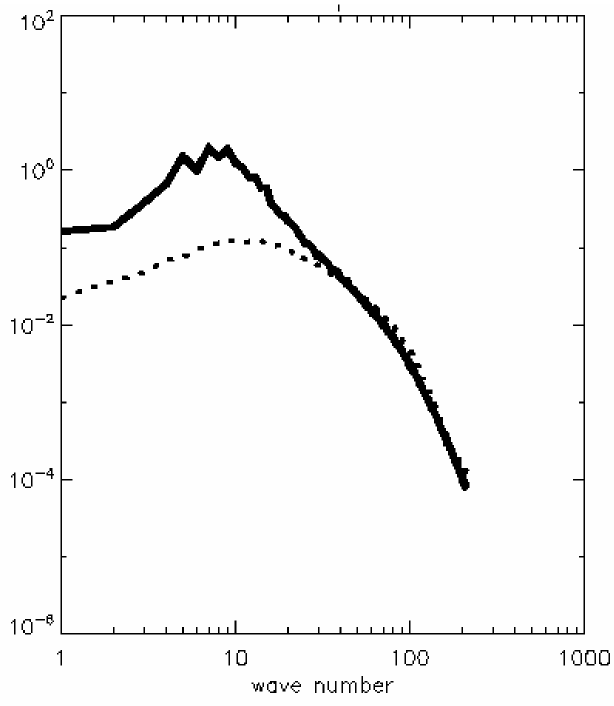
- Can/should we assess initial perturbations against analysis error?
  - What do we know about analysis error?
  - What are characteristics of analysis error in terms of scales, magnitude, balance, spectra?

Horizontal 2-D Spectra of Transient Fields at ~716 mb 12Z

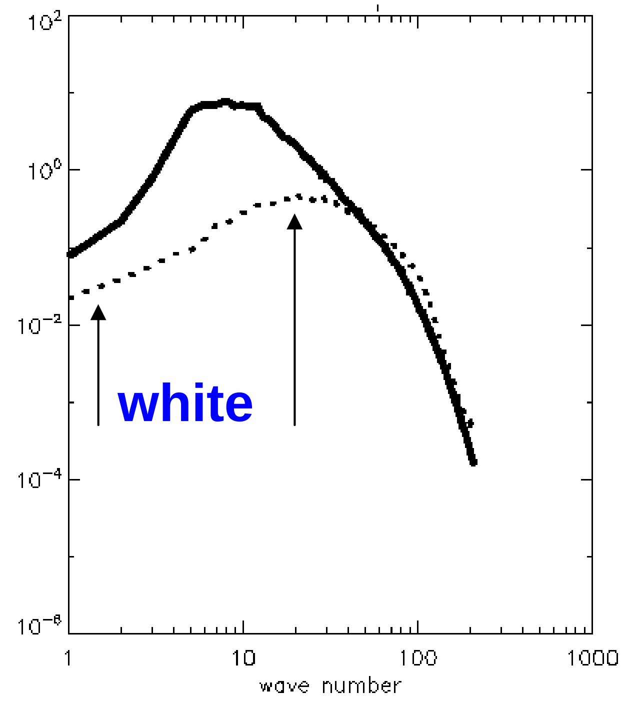
Solid=Nature Dotted = analysis error

NCEP-OSSEs

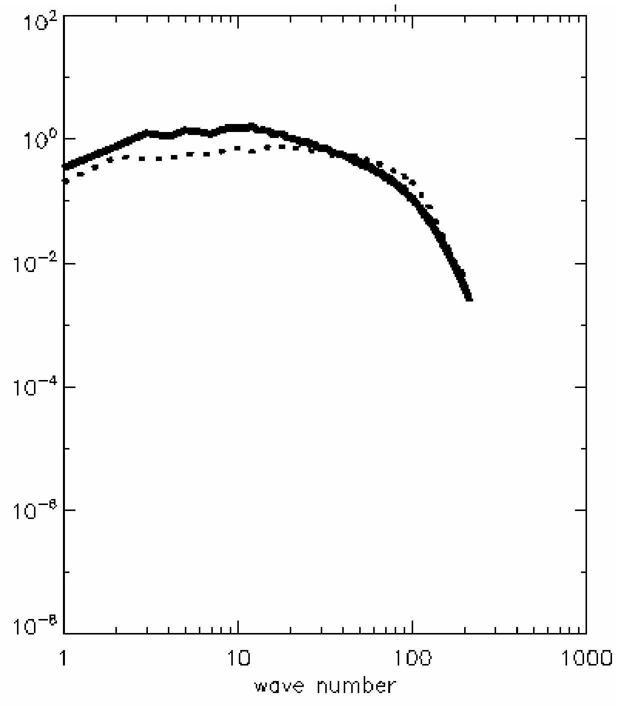
Temperature



Rotational KE



Divergent KE X 10



Ron Errico, GMAO

Differences between techniques

- SVs versus Bred Modes
  - The leading SVs explain very little of LVs
  - The leading SVs explain almost all growth
  - Gelaro/Reynolds/Errico QJ 2002
- ExKF techniques
  - Why is there a need for inflation? Why is the needed inflation factor small?
  - Which impact has the need for localization on balance issues?
  - Why seem/are small ensembles sufficient?
  - What are the implications of restriction to small subspaces?
  - If ensembles are representative of forecast errors – (why) can they also be made to be representative of analysis errors by transforming ensembles (as observations rotate spectra back to smaller scales thus whitening)?
- Is nonmodality important?

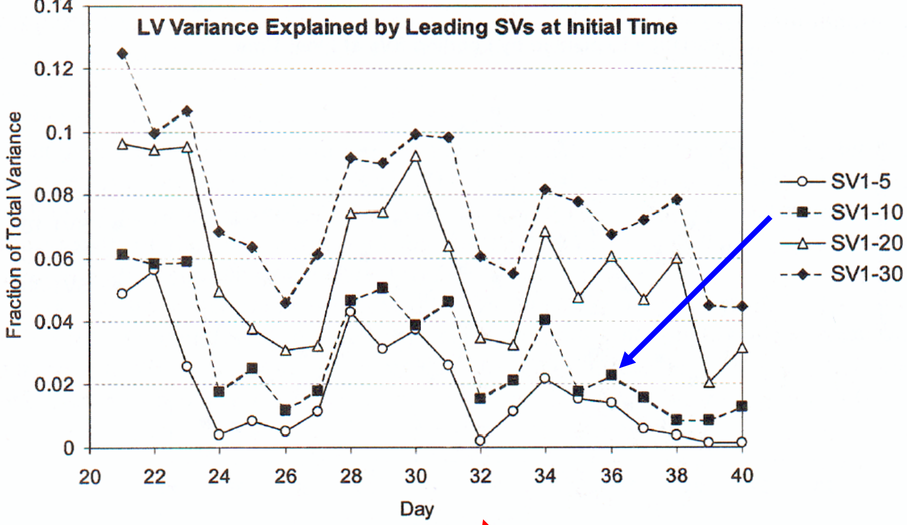


Figure 3. Fraction of variance of the leading Lyapunov vector (LV) explained by subsets of the initial-time, leading 24-hour singular vectors (SVs) on days 21–40. Results are shown for the leading 5 (SV1–5), 10 (SV1–10), 20 (SV1–20) and 30 (SV1–30) SVs. The incremental growth rate of the LV is shown above for comparison.

Gelaro et al. 2002, QJ

the leading SVs describe very little of LV

R. GELARO *et al.*

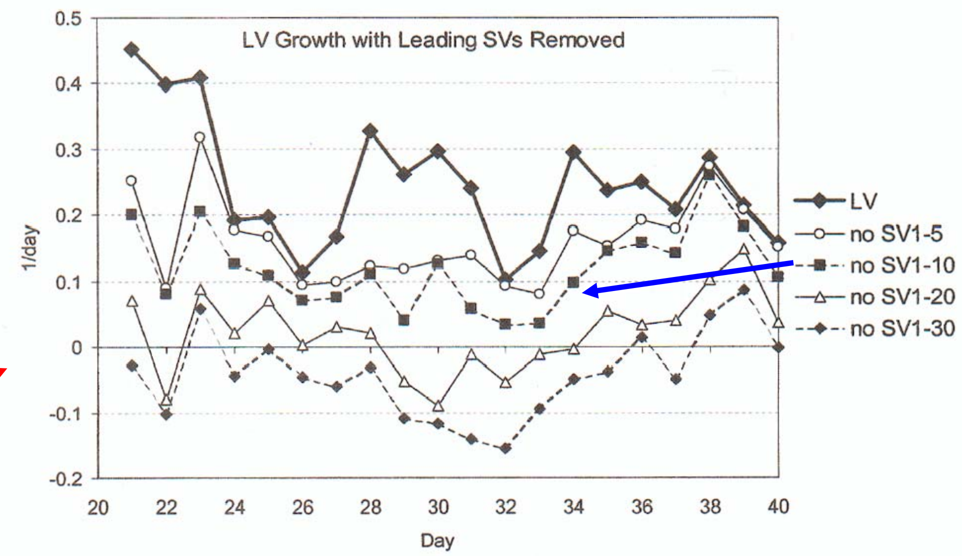


Figure 6. Incremental growth rates of the leading Lyapunov vector (LV) with various leading singular vector (SV) components of the perturbation removed. The 'filtered' incremental growth rates are based on Eq. (6). Results are shown corresponding to the removal of the leading 5 (no SV1–5), 10 (no SV1–10), 20 (no SV1–20) and 30 (no SV1–30) SV components from the LV (thin curves), and for the unfiltered LV (bold curve). Values less than zero indicate that the perturbation decays globally. See text for details.

leading SVs describe almost all growth

TIGGE Considerations / Recommendations

- Availability of different initial perturbations
  - in standard format (truncation)
  - to assess quantitative properties
  - to check against analysis error characteristics

- Standard set of perturbations
  - to be made available for use in different models
  - or to be easily generated by standard methods (given P^a)

- Model versus initial state error
  - What is today´s best estimate?
  - What was it ten years ago?
  - By which experiments can we refine the estimate?

- ?

- ?

Summary

- Predictability
  - intrinsic error growth plus initial uncertainty
- Ensemble Prediction
  - flow-dependent uncertainty
- Generating Initial Perturbations
  - methods
    - Breeding, SVs, ensemble Kalman filter
  - analysis error and nonmodal finite-time growth of errors
- Use of Methods
    - NCEP, CDC, NCMRWF, FNMOC, ECMWF, BMRC, MSC, UKMO
- Assessment and Discussion of Methods
- Recommendations
  - availability/exchange of initial perturbations
  - assess initial-time perturbation properties
  - assume P^a given (simple setup)

Thanks for contributions …

- … in particular to
  - Mozheng Wei
  - Tom Hamill
  - Ashwini Bora
  - Carolyn Reynolds
  - Roberto Buizza
  - Bill Bourke
  - Kamal Puri
  - Laurie Wilson
  - Neill Bowler
  - Ron Errico

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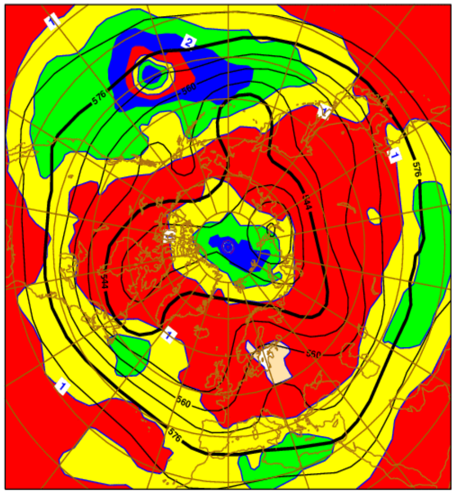
Additional Slides

Z500 May 2002 EM+STD D+2 for NCEP, MSC, ECMWF, Analysis

Z500 - 00UTC May 2002 t0 (31d)
NCEP EM (ci=8) and STD (ci=0.5)



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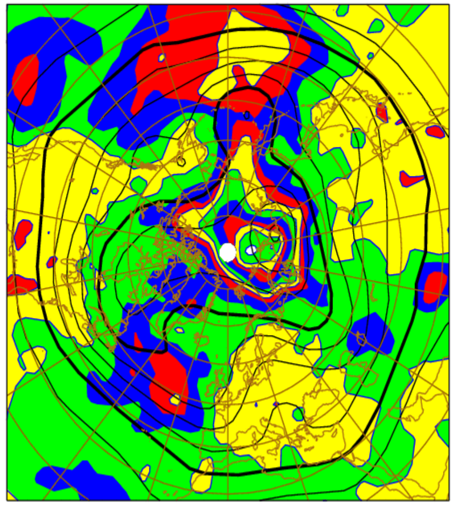
MSC EM (ci=8) and STD (ci=0.5)



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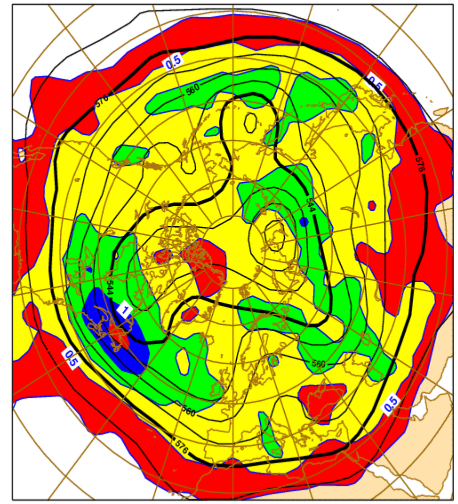
ECMWF EM (ci=8) and STD (ci=0.25)



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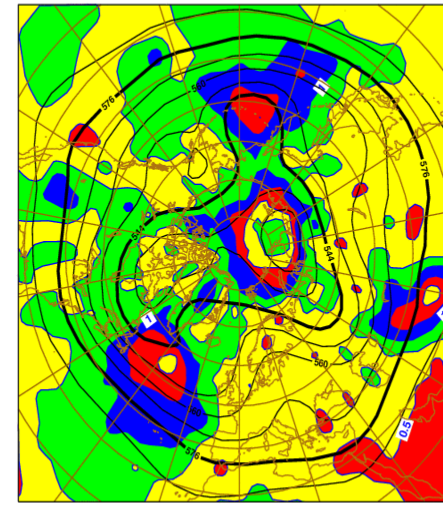
3C ANA (ci=8) and STD (ci=0.25)



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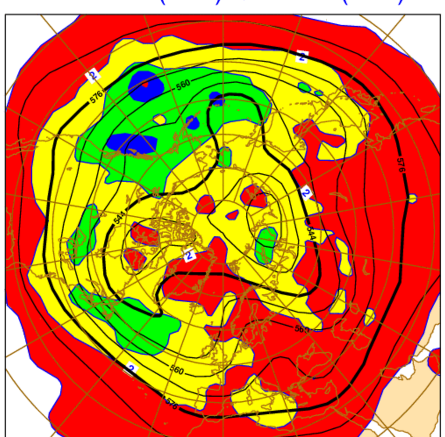
NCEP EM (ci=8) and STD (ci=1)



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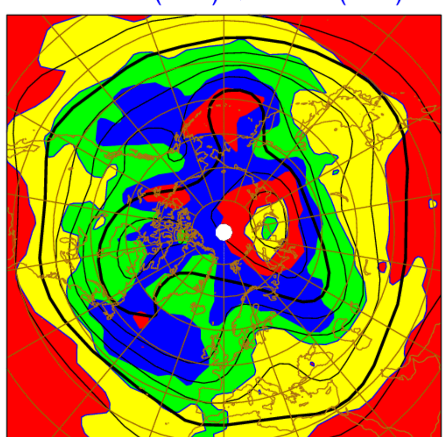
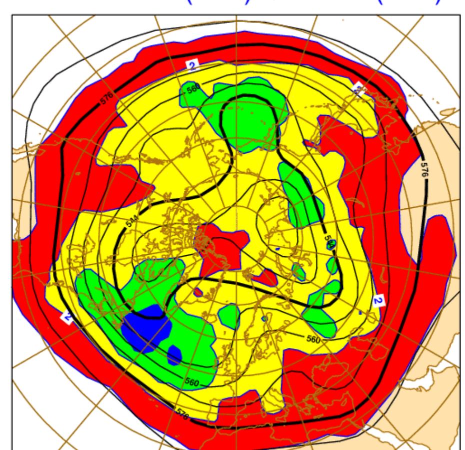
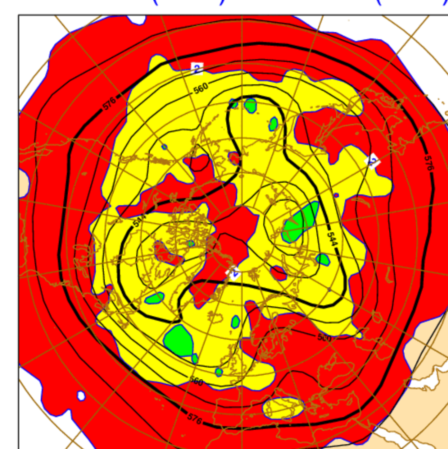
MSC EM (ci=8) and STD (ci=1)



Z500 - 00UTC May 2002 t+48h (31d)
ECMWF EM (ci=8) and STD (ci=1)



Z500 - May 2002 (31d) - t+48h
3C ANA (ci=8) and STD (ci=1)



Use of Methods

- National Centers for Environmental Prediction (NCEP)
  - Breeding
- Climate Diagnostics Center (CDC)-NCEP/ National Oceanic and Atmospheric Administration (NOAA)
  - Breeding
- National Centre for Medium Range Weather Forecasting (NCMRWF)
  - Breeding
- Fleet Numerical Meteorological and Oceanographic Center (FNMOC)
  - Breeding
- European Centre for Medium-Range Weather Forecasts (ECMWF)
  - SVs
- Bureau of Meteorology Research Centre (BMRC)
  - SVs
- Meteorological Service of Canada (MSC)
  - Ensemble Kalman filter
- United Kingdom Meteorological Office (UKMO)
  - Ensemble transform Kalman filter

Bayes´ Theorem

Data $\mathbf{y}$ and a priori estimate $\mathbf{x}^{\mathrm{b}}$ for the state $\mathbf{x}$ to be estimated are available, both with their respective uncertainties (assuming normal distributions):

$$p_{\mathbf{y}|\mathbf{x}}(\mathbf{y}) \sim \mathcal{N}(\mathsf{H}\mathbf{x}, \mathsf{R}) \tag{6.4}$$

$$p_{\mathbf{x}}(\mathbf{x}) \sim \mathcal{N}(\mathbf{x}^{\mathrm{b}}, \mathsf{B}) \tag{6.5}$$

B and R are assumed known. Using Bayes' Theorem the posterior pdf for the state given the data is obtained as:

$$p_{\mathbf{x}|\mathbf{y}}(\mathbf{x}) \propto p_{\mathbf{y}|\mathbf{x}}(\mathbf{y})\ p_{\mathbf{x}}(\mathbf{x}) \tag{6.6}$$

$$\boxed{p_{\mathbf{x}|\mathbf{y}}(\mathbf{x}) \propto \exp\left[-\frac{1}{2}\Big((\mathbf{y}-\mathsf{H}\mathbf{x})^{\mathrm{T}}\mathsf{R}^{-1}(\mathbf{y}-\mathsf{H}\mathbf{x}) + (\mathbf{x}-\mathbf{x}^{\mathrm{b}})^{\mathrm{T}}\mathsf{B}^{-1}(\mathbf{x}-\mathbf{x}^{\mathrm{b}})\Big)\right]} \tag{6.7}$$

The EKF Equations

Analysis Step

$$\mathbf{x}^a(t_i) = \mathbf{x}^b(t_i) + \mathsf{K}_i\Big(\mathbf{y}_i^o - \mathsf{H}_i(\mathbf{x}^b(t_i))\Big)$$

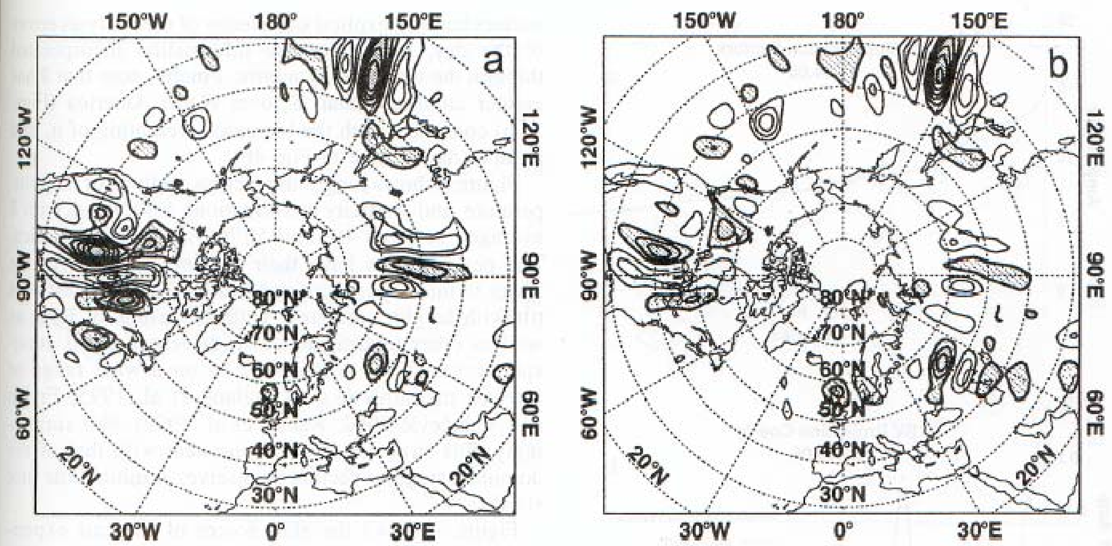
$$\mathsf{K}_i = \mathsf{P}^f(t_i)\mathsf{H}_i^{\mathrm{T}}\Big(\mathsf{H}_i\mathsf{P}^f(t_i)\mathsf{H}_i^{\mathrm{T}} + \mathsf{R}_i\Big)^{-1} = \mathsf{P}^a(t_i)\mathsf{H}_i^{\mathrm{T}}\mathsf{R}_i^{-1}$$

$$\mathsf{P}^a(t_i) = (\mathsf{I} - \mathsf{K}_i\mathsf{H}_i)\mathsf{P}^f(t_i) = \Big([\mathsf{P}^f(t_i)]^{-1} + \mathsf{H}_i^{\mathrm{T}}\mathsf{R}_i^{-1}\mathsf{H}_i\Big)^{-1}$$

Prediction Step

$$\mathbf{x}^b(t_{i+1}) = \mathcal{M}_i(\mathbf{x}^a(t_i)) \qquad \mathsf{P}^f(t_{i+1}) = \mathsf{M}_i\mathsf{P}^a(t_i)\mathsf{M}_i^{\mathrm{T}} + \mathsf{Q}(t_{i+1})$$

1.5 K



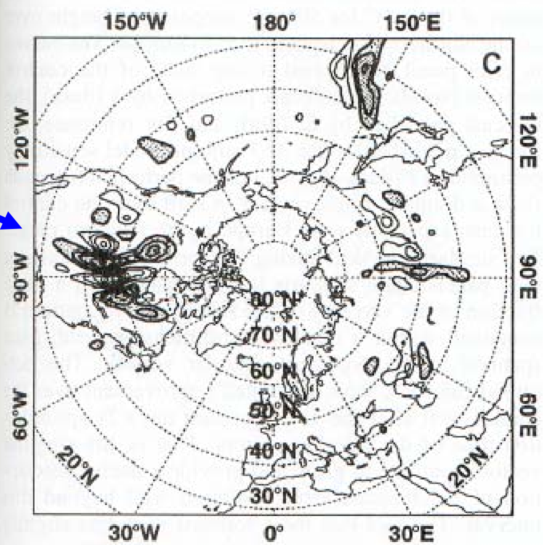
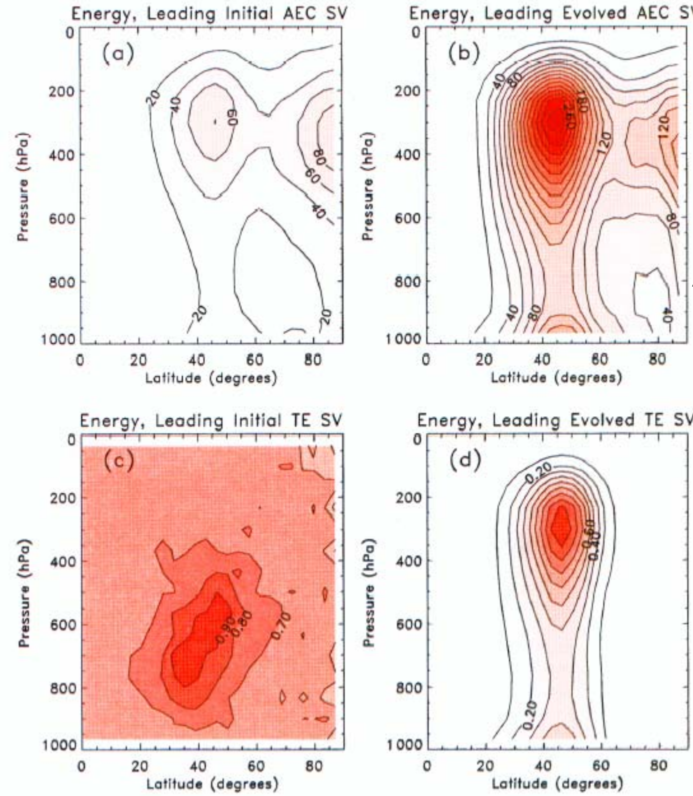


FIG. 3. Temperature perturbation at 500 mb for the case 6 April 1994 corresponding to (a) $\bar{s}$, (b) $\tilde{e}_0$, based on $n = 30$ singular vectors; and (c) s, based on a T63L31 adjoint model integration scaled according to the limited-area optimal factor technique described by R96. The contour interval is 0.2°C (zero contour omitted), with positive values shaded.

temperature analysis errors -> by inverse adjoint technique (ECMWF model)

Gelaro et al. 1998, JAS

Hamill et al. 2003, MWR



AEC

TE

FIG. 9. Zonal-mean profile of energy of the leading AEC SV averaged over the 33 cases: (a) initial time and (b) 48-h evolved; units are $m^2 s^{-2}$ (c), (d) As in (a), (b) but for 48-h evolved TE SV; amplitudes are nondimensional energy; (c) chosen to be normalized by the maximum initial-time amplitude and (d) by the maximum final-time amplitude.

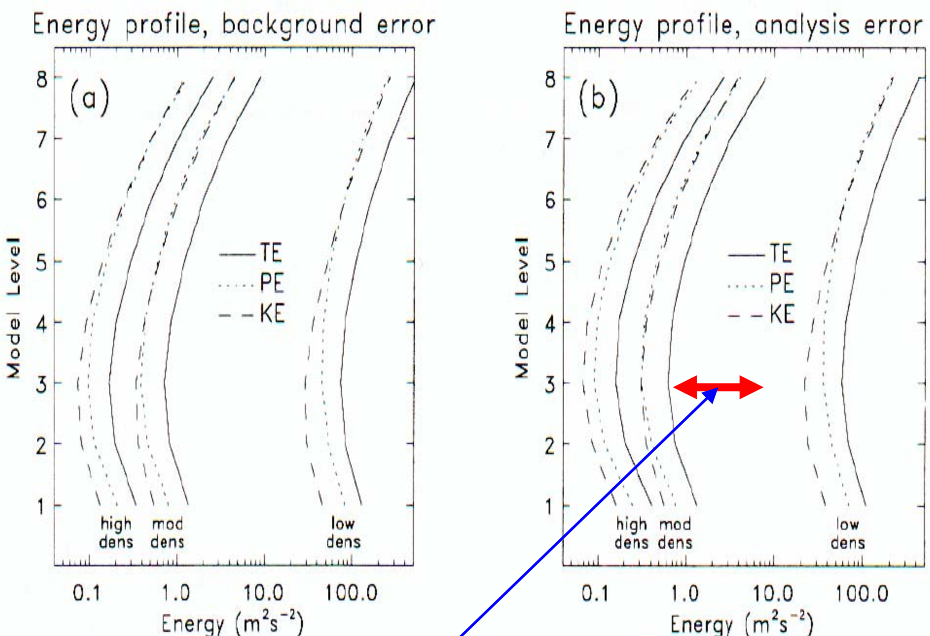


FIG. 6. Vertical profiles of total, kinetic, and potential energies of analysis and background errors, derived from an average over all case days and ensemble members. (a) Error in the background; (b) error in the analysis.

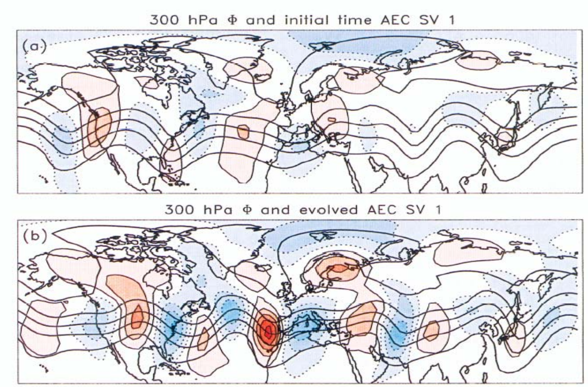


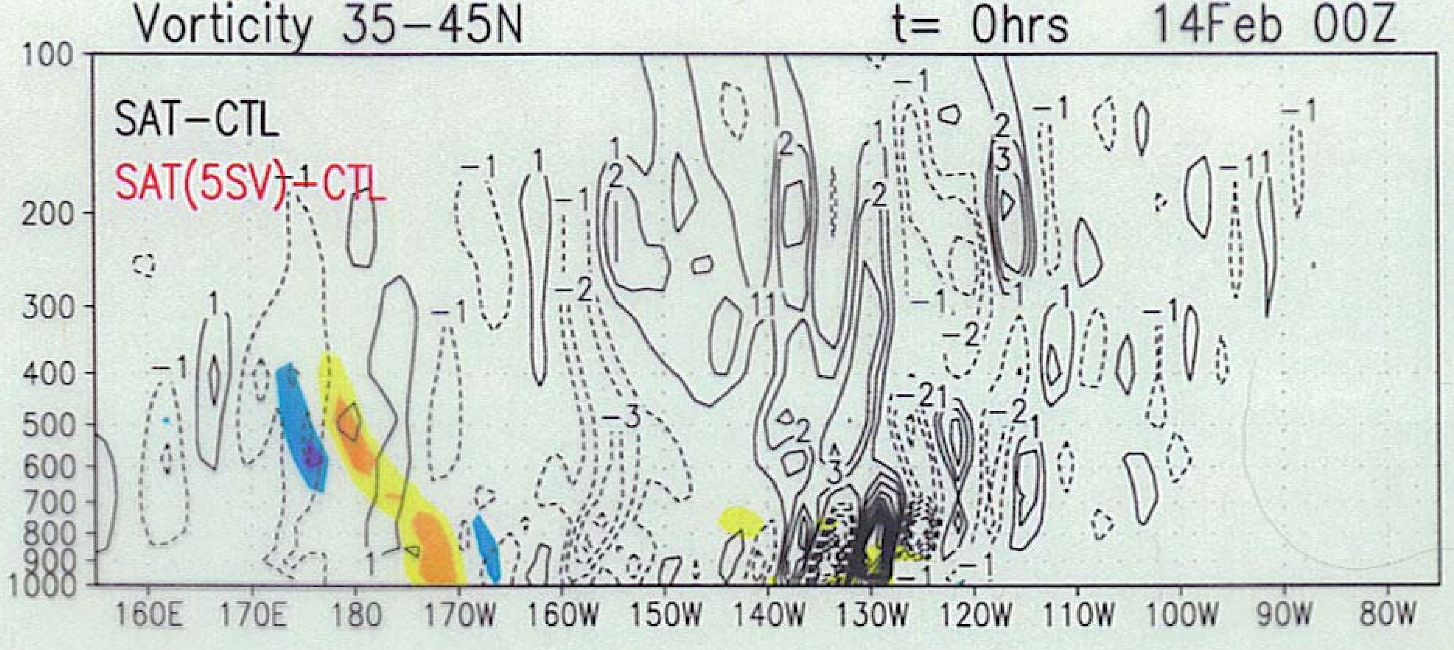
FIG. 13. The 300-hPa AEC SV structure for (a) initial time (here, day 32.5) and (b) 48-h evolved (here, day 34.5). Heavy solid lines denote streamfunction of true state at that time, and colored lines denote the perturbation to streamfunction from the first singular vector. Contours of true streamfunction are every $2. \times 10^7$ $m^2 s^{-1}$. Streamfunction perturbations are normalized by the largest perturbation from the forecast, with contours at [−0.9, −0.7, . . . , 0.7, 0.9]. Red perturbations are positive, blue perturbations negative.

factor 10 between levels

Hamill et al. 2002, MWR

Gelaro et al.
MWR 2000

Contour 1/10

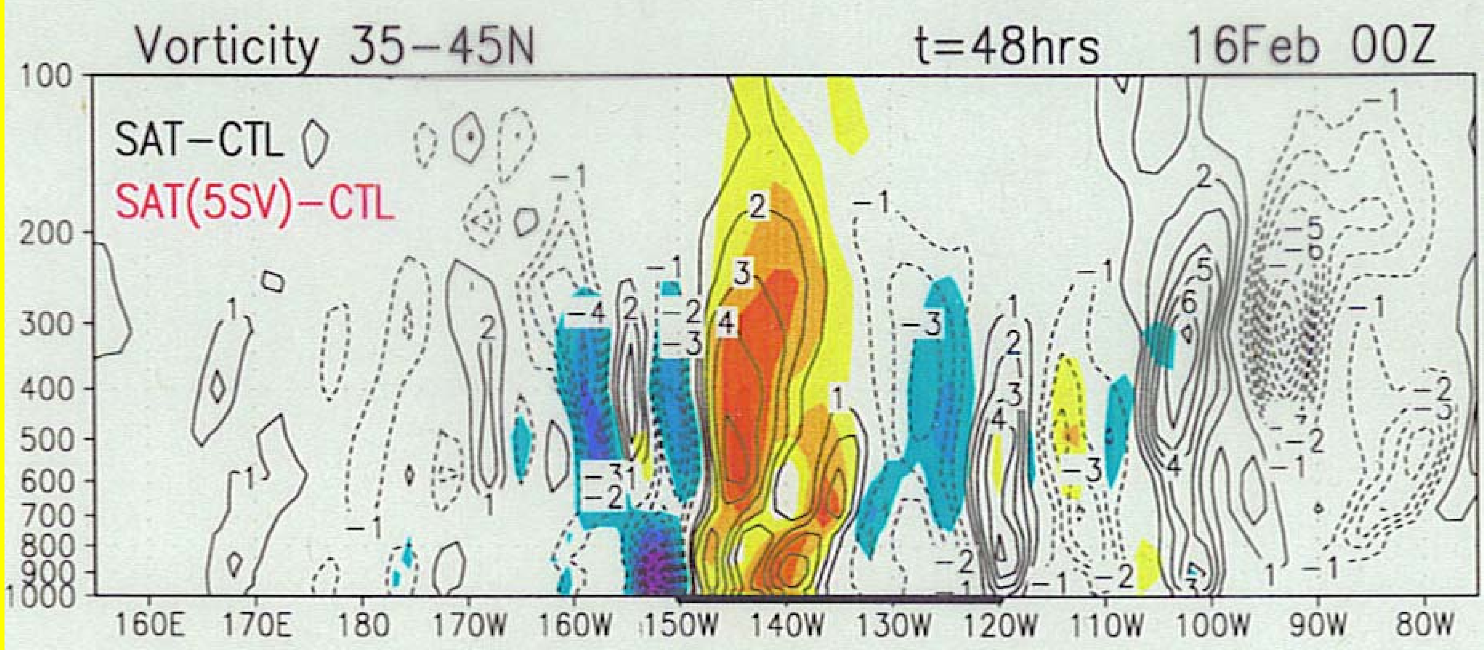


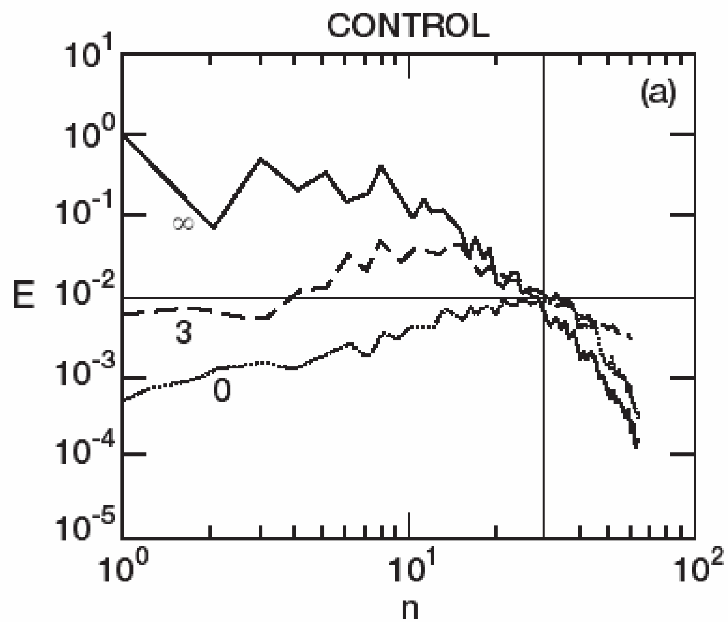
5SV-increment (TE)
small fraction of initial-time increment

large fraction of forecast improvement

located in lower/middle troposphere

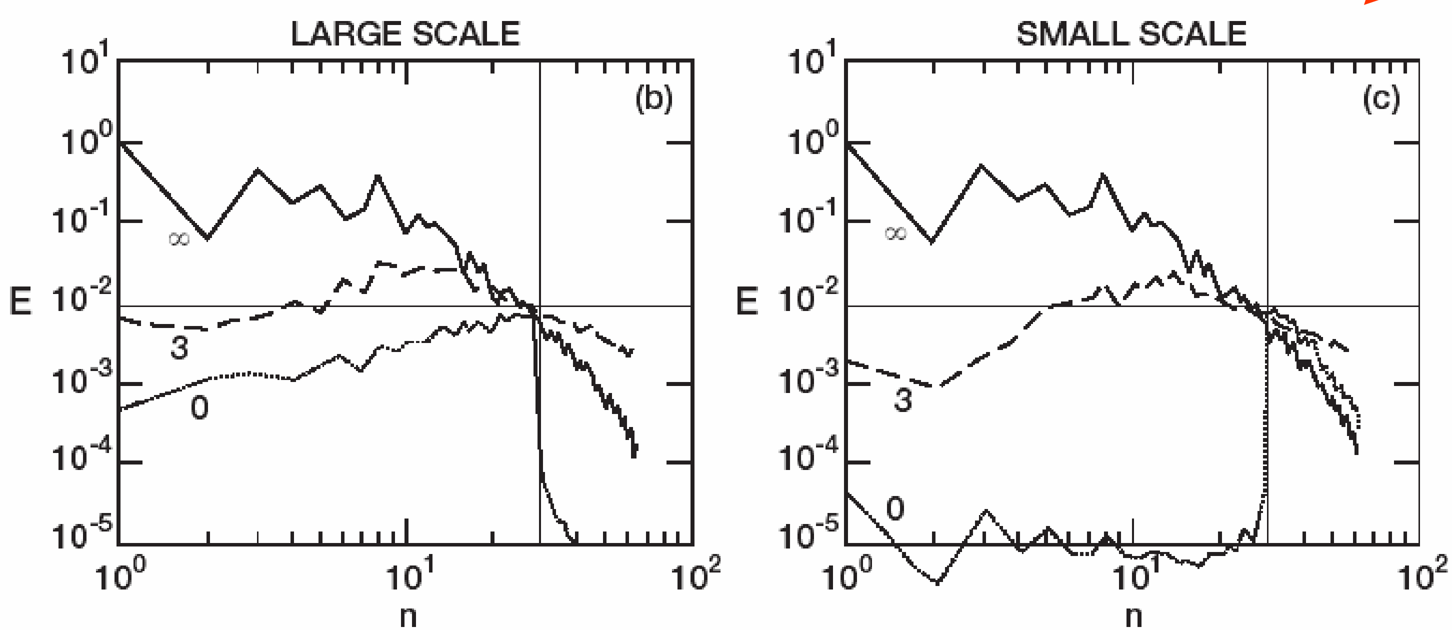
westward tilt

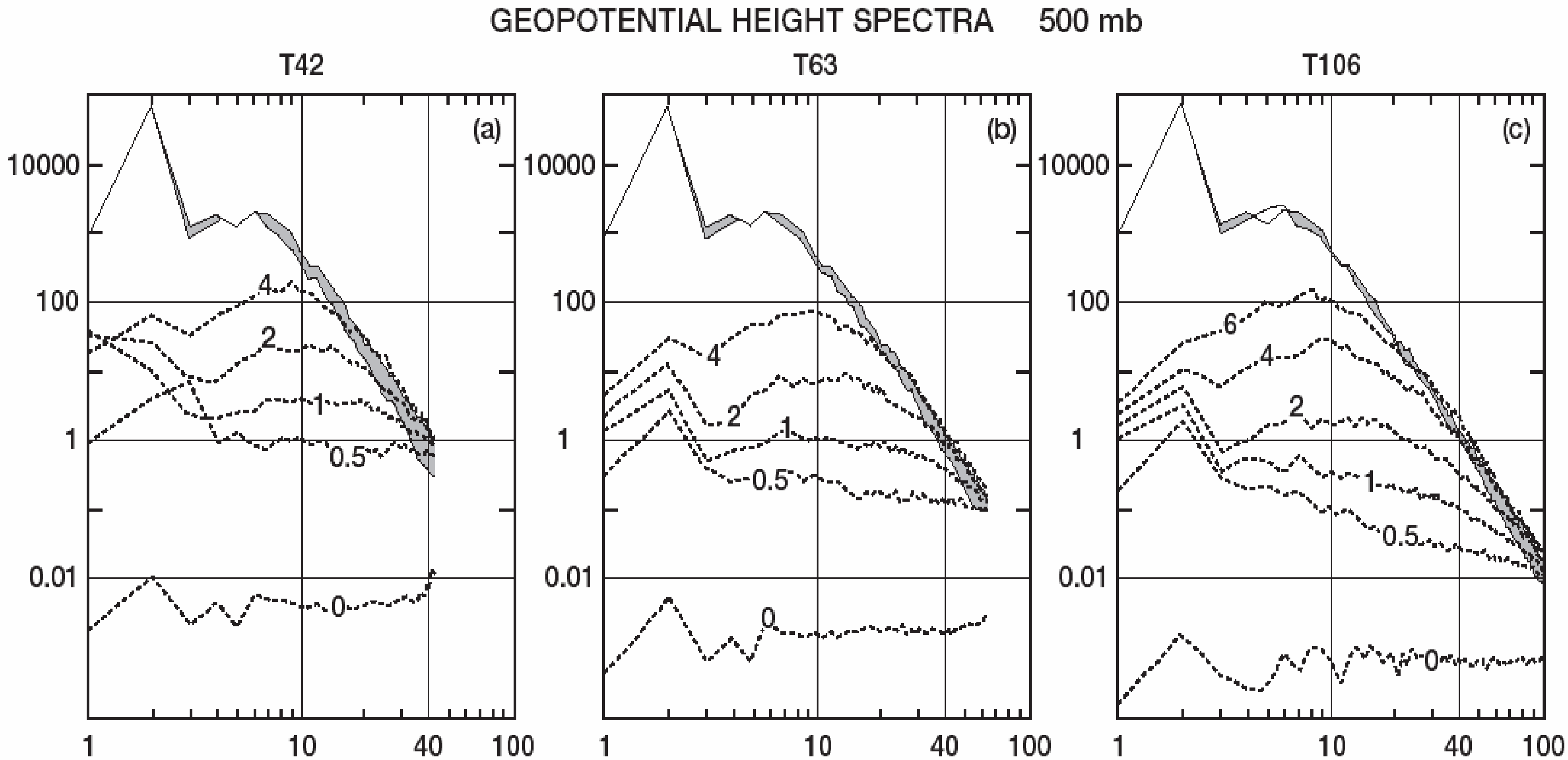




similarity of
spectra at day 3
→ spectrally local
error reduction
will not help

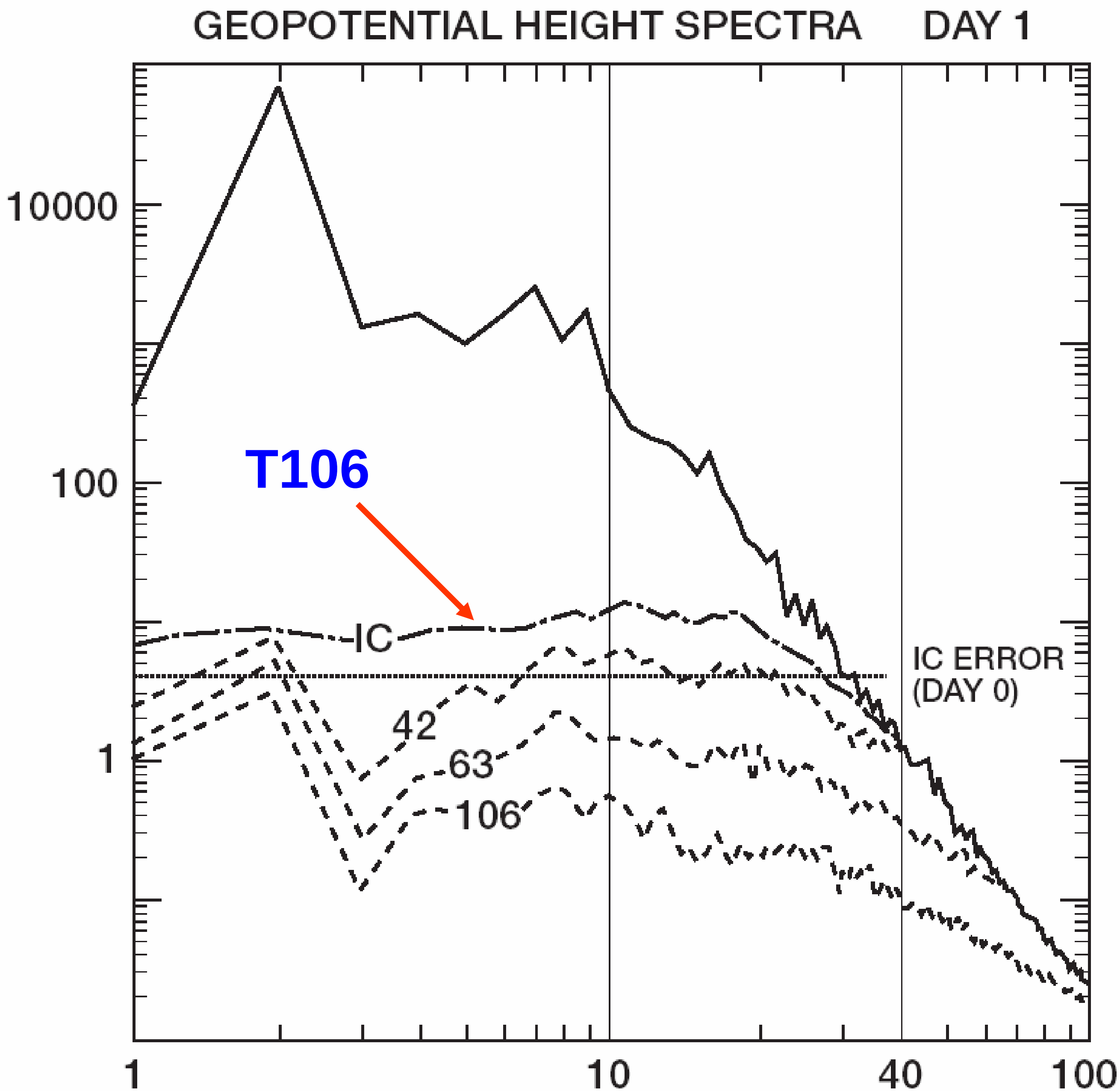
only small-scale error





error growth to due resolution differences (against T170):

D+1 error T42 = 10 x D+1 error T63 = 10 x D+1 error T106



even at T42 the D+1 truncation error growth has not exceeded D+1 IC T106 growth

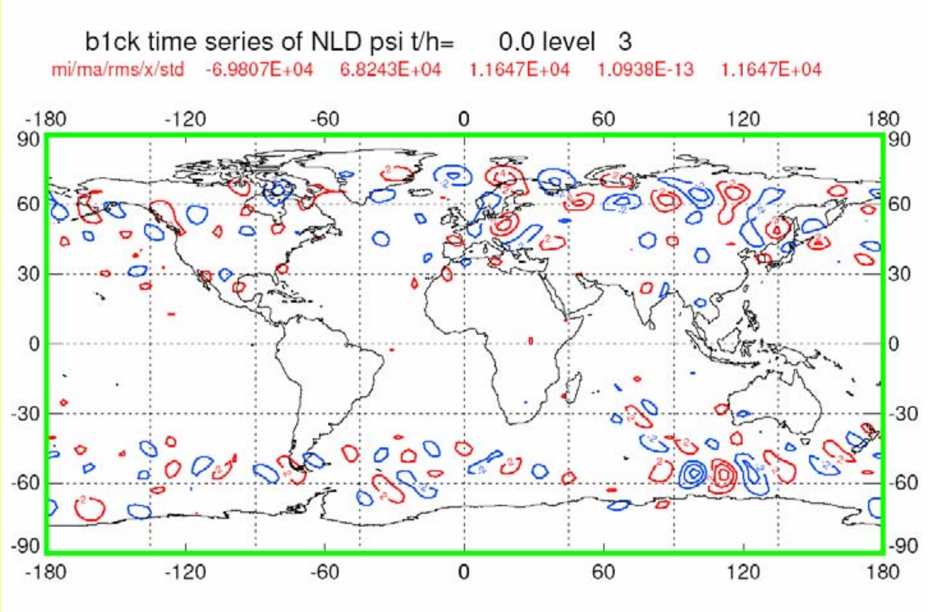
T106 truncation error growth is one order of magnitude smaller than D+1 T106 IC error growth

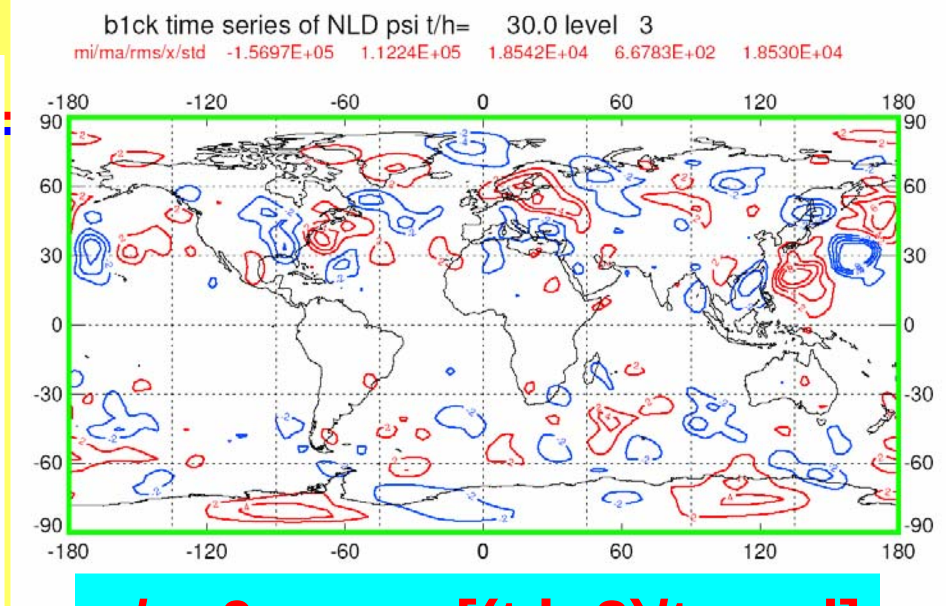
need IC/10 before going beyond T106

[IC analysis error growth exponential]

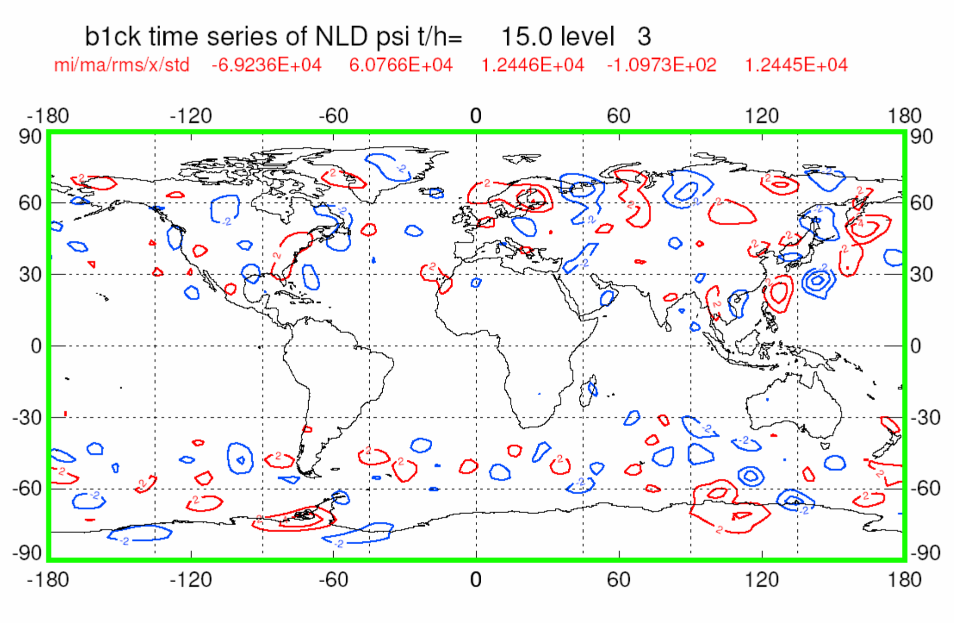
Tribbia Baumhefner 2004

10^-3/10^3 = 0.0001 %





e/e_0 = exp [(t ln2)/tau_d]



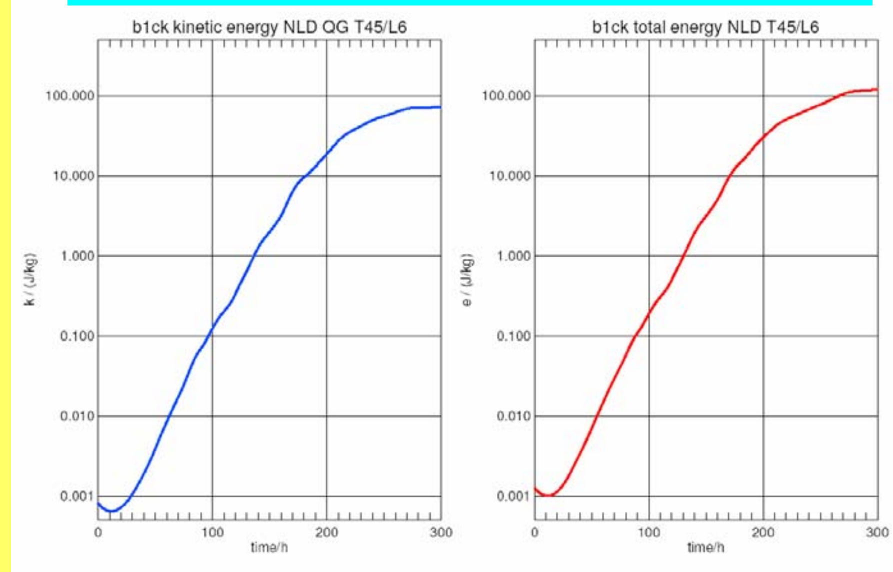


Figure 39: KE and TE for b1ck, NLD experiment 14 May 04.

tau_d = 13 h **2^13=8192**

25 hPa $\frac{\partial \psi}{\partial p} = 0$

1 100 hPa

2 250 hPa

3 400 hPa

4 550 hPa

5 700 hPa

6 850 hPa

925 hPa $\frac{\partial \psi}{\partial p} = 0$

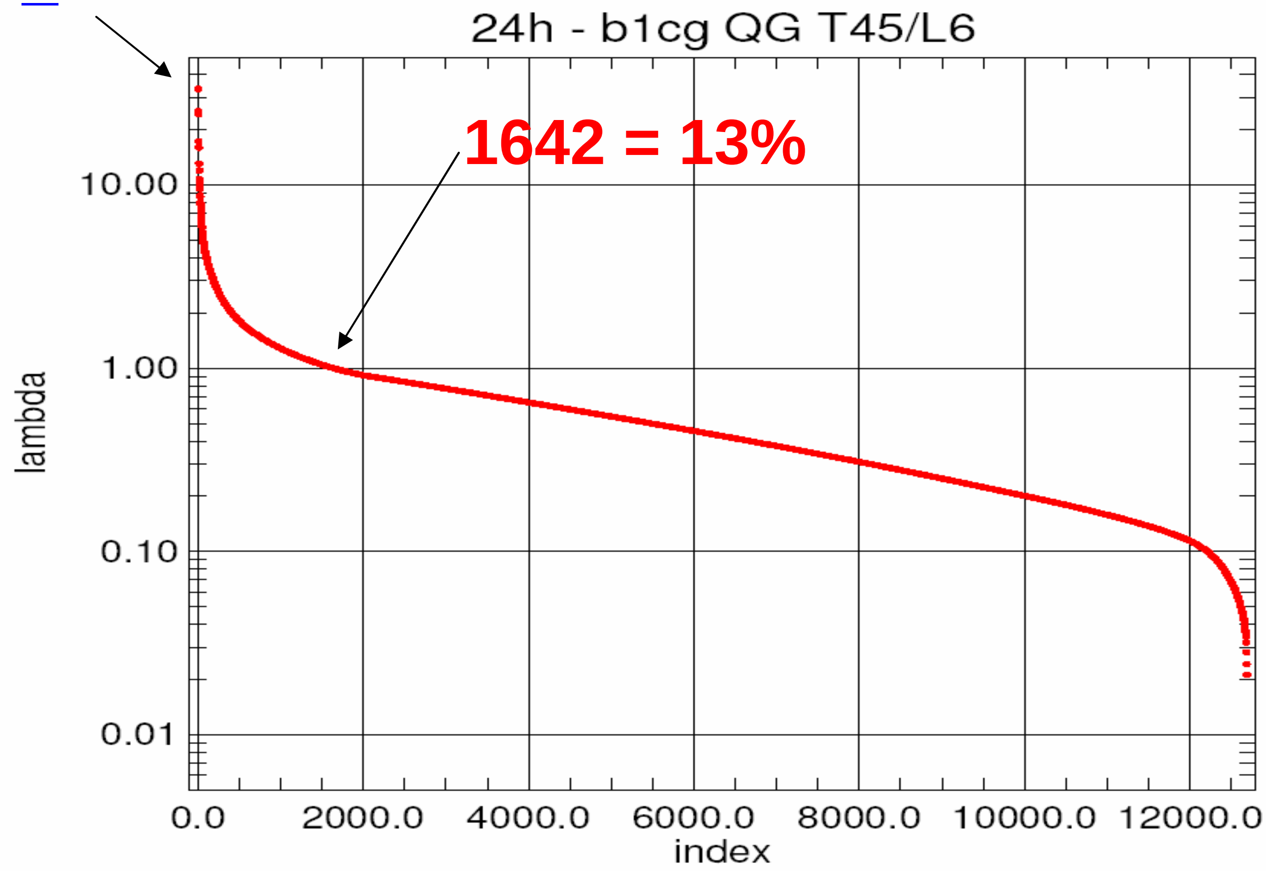
T45/L6 QG n=12690

$$\left(\frac{\partial}{\partial t} + (\mathbf{k} \times \nabla\psi) \cdot \nabla\right)\left(\nabla^2\psi + f + \frac{\partial}{\partial p}\left[\frac{f_0^2}{S}\frac{\partial \psi}{\partial p}\right]\right) = 0$$

QG TE SV spectrum

lambda_1=33.47

T45/L6



lambda= 0.0212

Figure 42: b1cg spectrum. OTI 24.0 h; $\lambda_1 = 0.334732129161647E+02$, $\lambda_{12690} = 0.212031927289761E-01$, $\lambda_{1642} = 0.100006329799706E+01$.