Global Climate Diagnosis in a Linear Stochastically Forced Framework

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ECMWF Talk : 7 September 2009

$$\frac{dx}{dt} = A\,x + f_{ext} + B\,\eta$$

x = N-component anomaly state vector
η = M-component gaussian noise vector
$f_{ext}(t)$ = N-component external forcing vector
$A(t)$ = N x N matrix
$B(t)$ = N x M matrix

1. **Under this approximation, the system responds linearly to external forcing, and the prediction of a future state, given an initial state, is a linear prediction**

2. **The approximation is surprisingly good, and very useful for both diagnosis and prediction**

3. **It can also be reconciled with the existence of non-Gaussian PDFs, by making *B* a linear function of *x***

This talk is mostly based on a paper by Sardeshmukh and Sura (*Journal of Climate, March 2009)*

Thanks also to Barsugli, Compo, Newman, Penland, and Shin

The Linear Stochastically Forced (LSF) Approximation

$$\frac{dx}{dt} = A\,x + f_{ext} + B\,\eta$$

x	=	N-component anomaly state vector
η	=	M-component gaussian noise vector
$f_{ext}(t)$	=	N-component external forcing vector
$A(t)$	=	N x N matrix
$B(t)$	=	N x M matrix

Supporting Evidence

- Linearity of coupled GCM responses to radiative forcings
- Linearity of atmospheric GCM responses to tropical SST forcing
- Linear dynamics of observed seasonal tropical SST anomalies
- Competitiveness of linear seasonal forecast models with global coupled models
- Linear dynamics of observed weekly-averaged circulation anomalies
- Competitiveness of Week 2 and Week 3 linear forecast models with NWP models
- Ability to represent observed second-order synoptic-eddy statistics

Seasonal Predictions of Eastern Tropical Pacific SSTs at NCEP

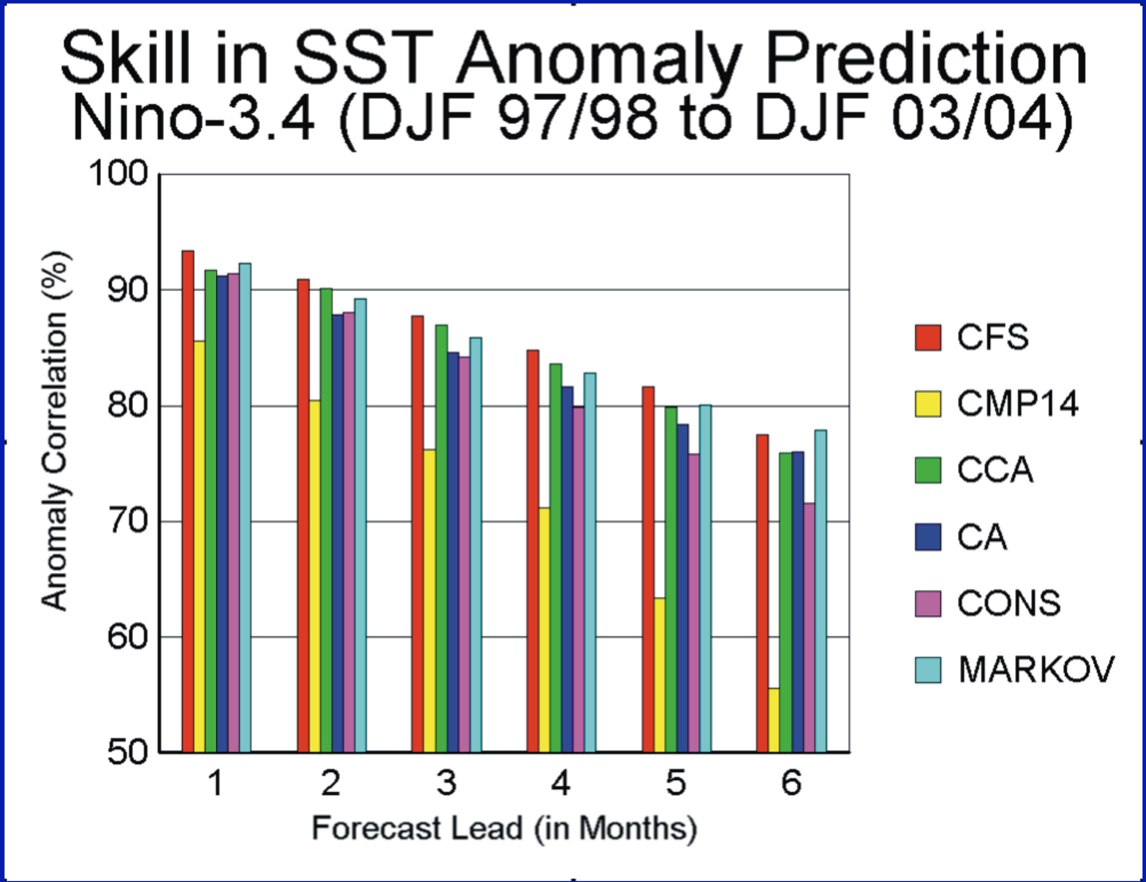
Skill Comparison of Nonlinear GCMs (CFS, CMP14)
and Linear empirical models (CCA, CA, CONS, MARKOV)

The simple linear empirical models are apparently just as good at predicting ENSO

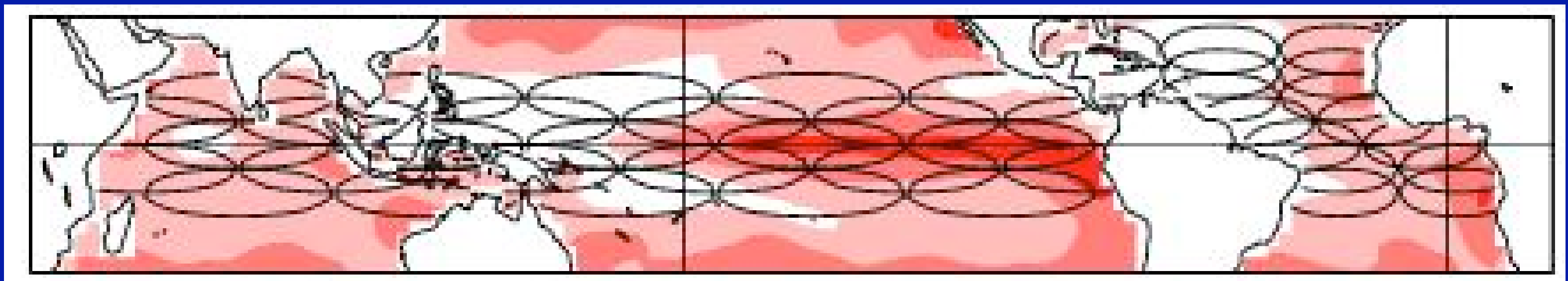
as are the

"state of the art" coupled GCMs

(From Saha et al, 2006)

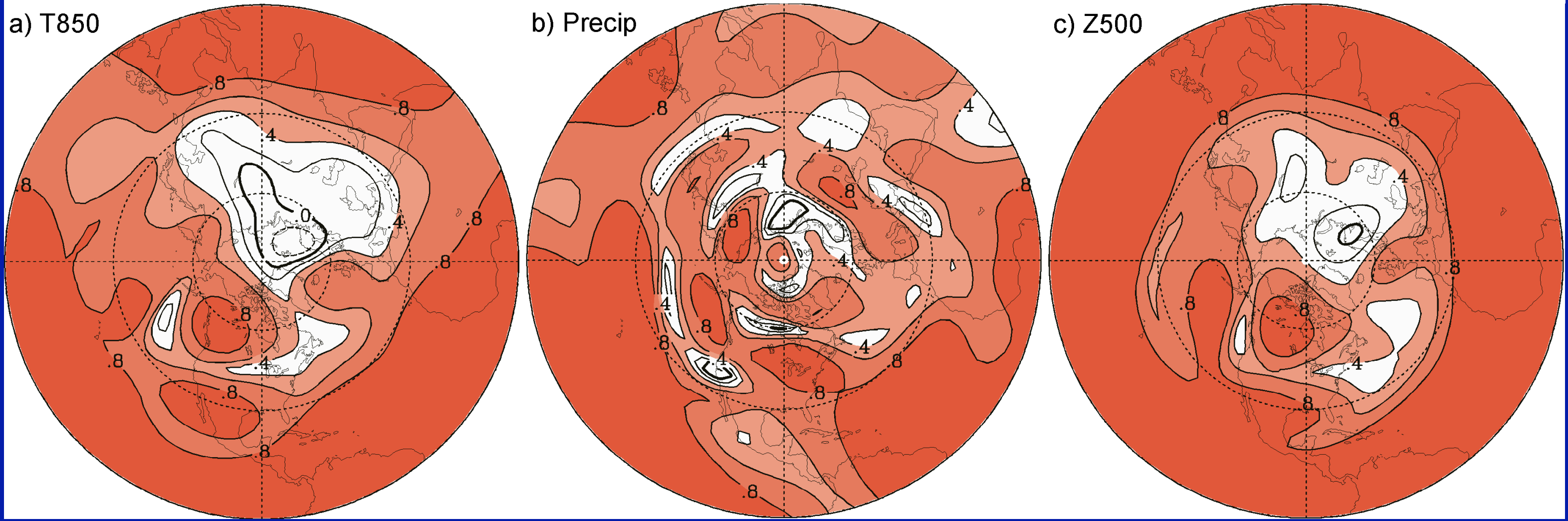


DOMINANCE and LINEARITY of Tropical SST influences on global climate variability



BASIC POINT: The nonlinear NCAR/CCM3 atmospheric GCM's responses to prescribed global SST changes over the last 50 years are well -approximated by linear responses to just the Tropical SST changes, obtained by linearly combining the GCM's responses to SSTs in the 43 localized areas shown above.

Local correlation of annual mean "GOGA" and "Linear TOGA" responses



Sardeshmukh, Barsugli and Shin 2009

In LSF models, the nonlinear terms are not *ignored*

They are *approximated* as stochastic noise

Linear Anomaly Model of departures $x = X - \bar{X}$ of X from some background state $\bar{X}$

$$\frac{dx}{dt} \cong Ax + f_{ext} + B\eta$$

$A(t)$ and $B(t)$ are matrices; $f_{ext}(t)$ and η are vectors

The first- and second-moment equations for such models are :

$$\frac{d}{dt} < x > \;=\; A < x > \;+\; f_{ext} \qquad\qquad C(\tau) \;=\; < x(t+\tau)\, x^T(t) >$$

$$\frac{d}{dt} C(0) \;=\; A\, C(0) \;+\; C(0)\, A^T \;+\; BB^T \;+\; < x > f_{ext}^T \;+\; f_{ext} < x^T >$$

Linear Inverse Modeling (LIM)

If A and B are constant and $f_{ext} = 0$, then

$$< x > \;=\; 0$$

$$C(\tau) \;=\; e^{A\tau}\, C(0) \qquad \textbf{FDR - 1}$$

$$0 \;=\; A\, C(0) \;+\; C(0)\, A^T \;+\; BB^T \qquad \textbf{FDR - 2}$$

$$x(t+\tau) = e^{A\tau} x(t) \;+\; noise$$

A can be estimated empirically from FDR - 1 :

either using $C(\tau_0) = e^{A\tau_0} C(0)$ for some τ_0

or using $$A^{-1} = -\int_0^\infty C(\tau)\, C(0)^{-1} d\tau$$

B can then estimated from FDR - 2

(See Penland 1989, Penland and Sardeshmukh 1995)

Decay of lag-covariances of weekly anomalies is consistent with linear dynamics

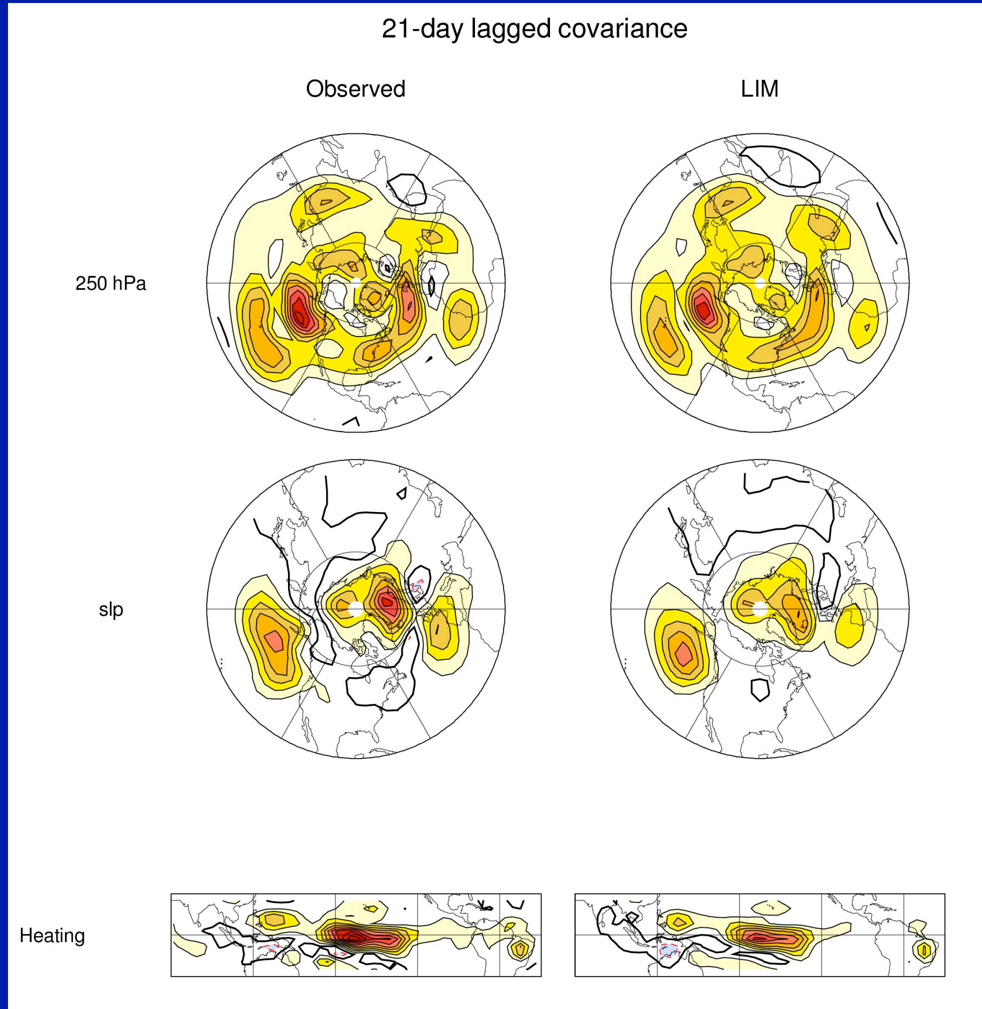
$$\frac{dx}{dt} = A\,x + B\,\eta$$

$$C(\tau) = \langle x(t+\tau)x^T(t) \rangle$$
$$C(\tau) = e^{A\tau}\,C(0)$$

A is first estimated using the observed $\boldsymbol{C(\tau = 5}$ days) and $\boldsymbol{C(0)}$ in this equation, and then used to "predict" $\boldsymbol{C(\tau = 21}$ days)

The components of the anomaly state vector ***x*** include the 7-day running mean PCs of 250 and 750 mb streamfunction, SLP, tropical diabatic heating and stratospheric height anomalies.

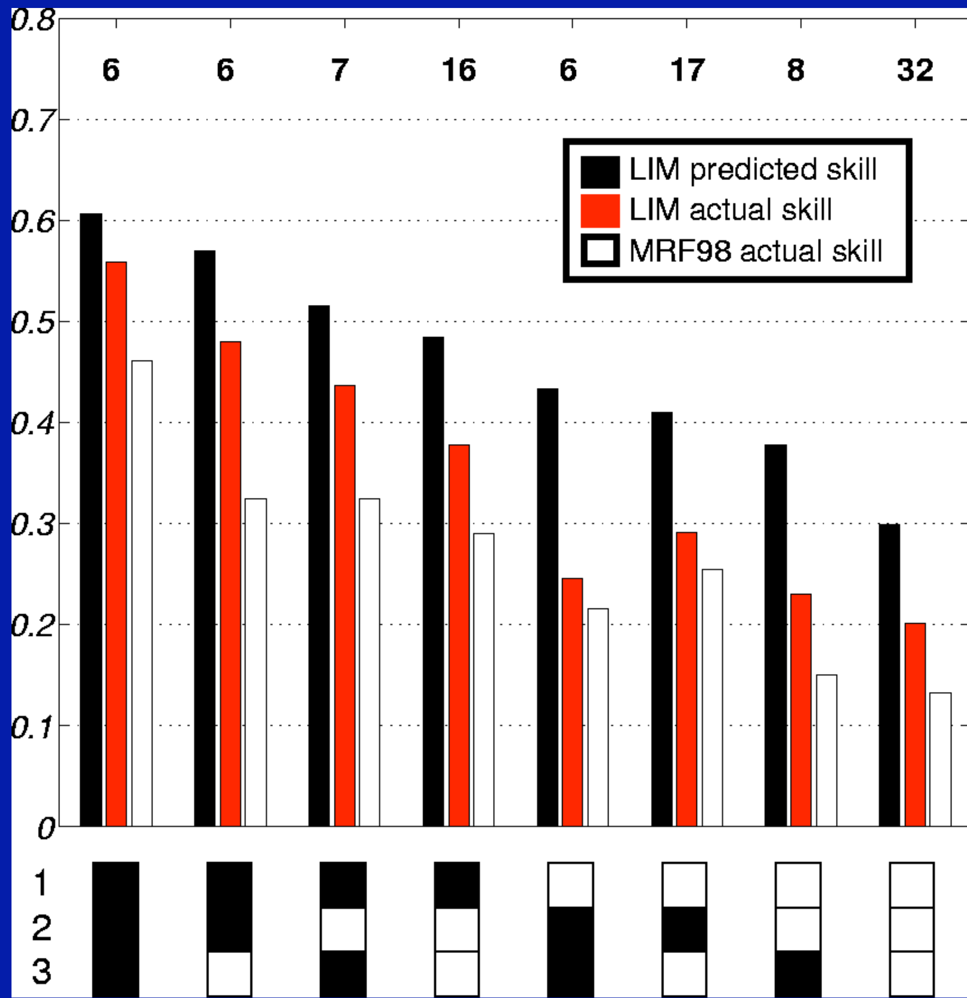
From Newman and Sardeshmukh (J. Climate 2008)



An example of the usefulness of LIM

The singular vectors of the empirically estimated G(τ) = *exp(*Aτ) operator can help identify relatively more skillful cases *a priori* . . .

Expected and *actual* pattern correlation skill of Week-3 N.H. forecasts, stratified by initial state projections on the right singular vectors of G (τ=21 days)



For wintertime Week 3 Forecasts of 250 mb streamfunction

From Newman et al 2003

Observed and Simulated Spectra of Tropical SST Variability are basically Red Noise spectra

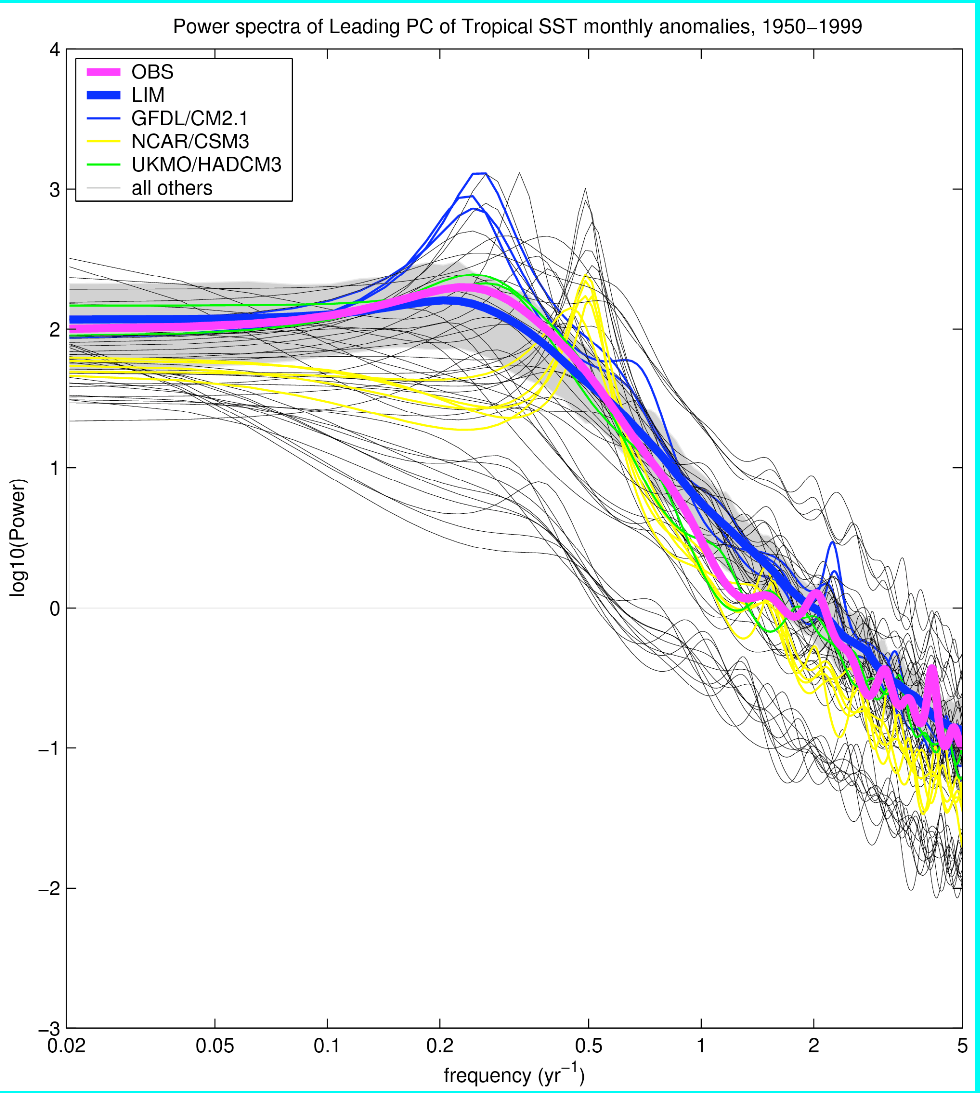
Spectra of the projection of tropical SST anomaly fields on the dominant pattern (1st EOF) of observed monthly SST variability in 1950-1999.

Observations (Purple)

IPCC AR4 coupled GCMs
(20th-century (20c3m) runs)
(thin black, yellow, blue, and green)

A linear inverse model (LIM) constructed from 1-week lag covariances of weekly-averaged tropical data in 1982-2005 (Thick Blue)

Gray Shading :
95% confidence interval from the LIM, based on 100 model runs with different realizations of the stochastic forcing.



From Newman, Sardeshmukh and Penland (J. Climate 2009)

A Coupled Linear Inverse Model (C-LIM) of Tropical Weekly Averages

derived from observed data for the 1982-2005 period

The Coupled Model

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{B}\eta = \mathbf{Ax} + \xi \qquad \mathbf{x} = \begin{bmatrix} \mathbf{T}_O \\ \psi \\ \mathbf{H} \\ \chi \end{bmatrix} = \begin{bmatrix} \text{Sea Surface Temperature (20 Patterns)} \\ \text{Atmospheric Streamfunction (7 Patterns)} \\ \text{Atmospheric Heating (17 Patterns)} \\ \text{Atmospheric Velocity Potential (3 Patterns)} \end{bmatrix} \qquad \mathbf{x}_A = \begin{bmatrix} \psi \\ \mathbf{H} \\ \chi \end{bmatrix}$$

Coupled and Uncoupled Versions of the model

$$\frac{d}{dt}\begin{bmatrix} \mathbf{T}_O \\ \mathbf{x}_A \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{OO} & \mathbf{A}_{OA} \\ \mathbf{A}_{AO} & \mathbf{A}_{AA} \end{bmatrix}\begin{bmatrix} \mathbf{T}_O \\ \mathbf{x}_A \end{bmatrix} + \begin{bmatrix} \xi_O \\ \xi_A \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} \mathbf{T}_O \\ \mathbf{x}_A \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{OO} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{AA} \end{bmatrix}\begin{bmatrix} \mathbf{T}_O \\ \mathbf{x}_A \end{bmatrix} + \begin{bmatrix} \xi_O \\ \xi_A \end{bmatrix}$$

$$\frac{d\mathbf{T}_O}{dt} = \mathbf{A}_{OO}\mathbf{T}_O + \xi_O$$

$$\frac{d\mathbf{x}_A}{dt} = \mathbf{A}_{AA}\mathbf{x}_A + \xi_A$$

A neat result: The eigenvectors of L separate cleanly into Coupled and Uncoupled (Internal Atmospheric) modes

$$\mathbf{U} \approx \begin{bmatrix} \mathbf{u}_j^{\text{coup}} & \mathbf{u}_k^{\text{int}} \end{bmatrix} \approx \begin{bmatrix} \mathbf{u}_{jO}^{\text{coup}} & 0 \\ \mathbf{u}_{jA}^{\text{coup}} & \mathbf{u}_{kA}^{\text{int}} \end{bmatrix}$$

$$\mathbf{x}(t) = \mathbf{x}^{\text{coup}}(t) + \mathbf{x}^{\text{int}}(t)$$

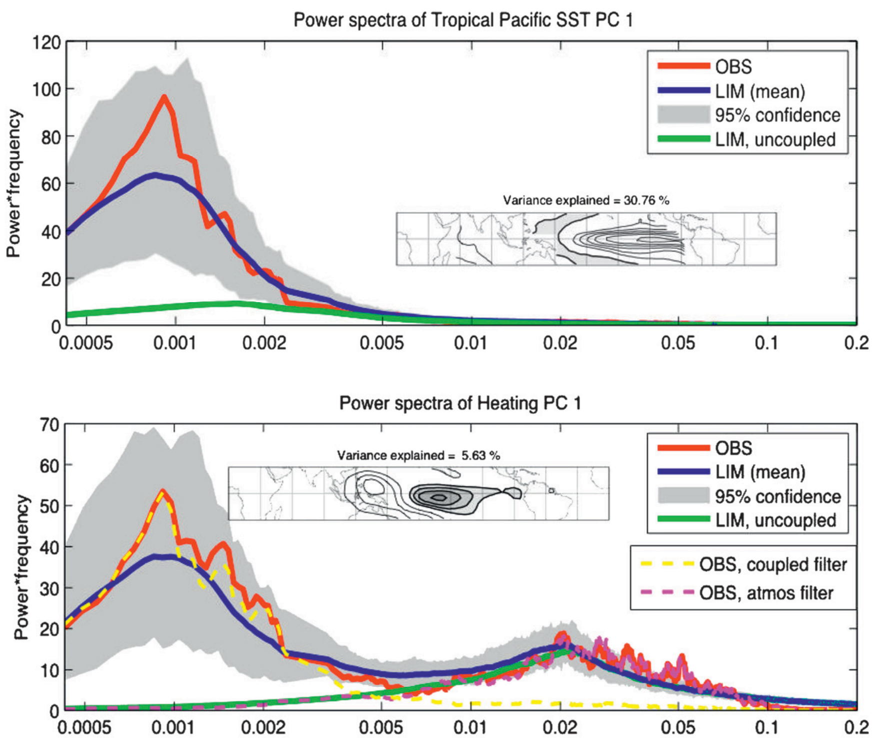
$$\mathbf{x}^{\text{coup}}(t) = \sum_j \mathbf{u}_j^{\text{coup}}\alpha_j^{\text{coup}}(t) \qquad \mathbf{x}^{\text{int}}(t) = \sum_k \mathbf{u}_k^{\text{int}}\alpha_k^{\text{int}}(t)$$

Another example of the usefulness of LIM : diagnosis of coupled interactions

Observed Power spectra of the leading Tropical SST and Atmospheric Diabatic Heating EOFs (red curves), compared to spectra predicted by the Coupled-LIM (blue curves) and by the Uncoupled-LIM (green curves)

SST EOF 1

Diabatic Heating EOF1



Gray shading represents 95% confidence intervals determined from a 2400 yr run of the C-LIM).

Insets in each panel show the corresponding EOF and the variance of weekly anomalies explained by that pattern.

Dashed curves: spectra of the observed heating PC 1 projected onto the subset of either the "Coupled" (yellow) or "Internal" (pink) eigenmodes of the full LIM operator.

From Newman, Sardeshmukh and Penland (J. Climate 2009)

NSP09_Fig_8

An attractive feature of the LSF Approximation

$$\frac{dx}{dt} = A\,x + f_{ext} + B\,\eta$$

Equations for the first two moments

(Applicable to both Marginal and Conditional Moments)

$$\frac{d}{dt} < x > \; = A < x > + f_{ext}$$

$$\frac{d}{dt} C = A\,C + C\,A^T + B\,B^T$$

$<x>$ = ensemble mean anomaly

C = covariance of departures from ensemble mean

***If A(t), B(t) , and f_{ext}(t)* are constant, then**

*First two **Marginal** moments* →

$$< x > \; = -A^{-1} f_{ext}$$

$$\frac{dC}{dt} = 0 = A\,C + C\,A^T + B\,B^T$$

*First two **Conditional** moments*
Ensemble mean forecast
Ensemble spread →

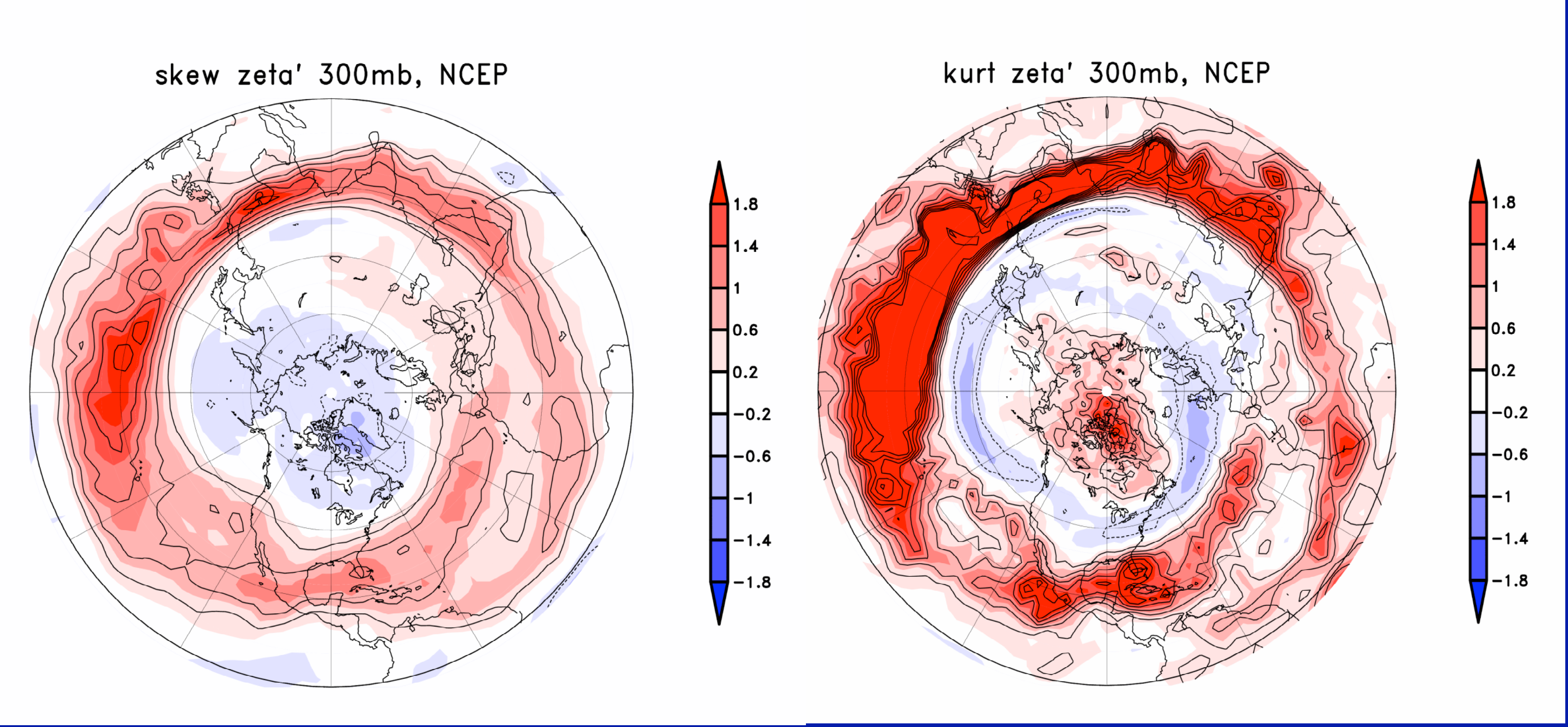
$$\hat{x}'(t) \equiv \; < x'(t) \mid x'(0) > \; = e^{At} x'(0)$$

$$\hat{C}(t) \equiv \; < (\hat{x}' - x')\,(\hat{x}' - x')^T > \; = C - e^{At} C e^{A^T t}$$

If *x* is Gaussian, then these moment equations **COMPLETELY** characterize system variability ***and*** predictability

But . . . atmospheric circulation statistics are not Gaussian . . .

Observed Skew *S* and (excess) Kurtosis *K* of daily 300 mb Vorticity (DJF)

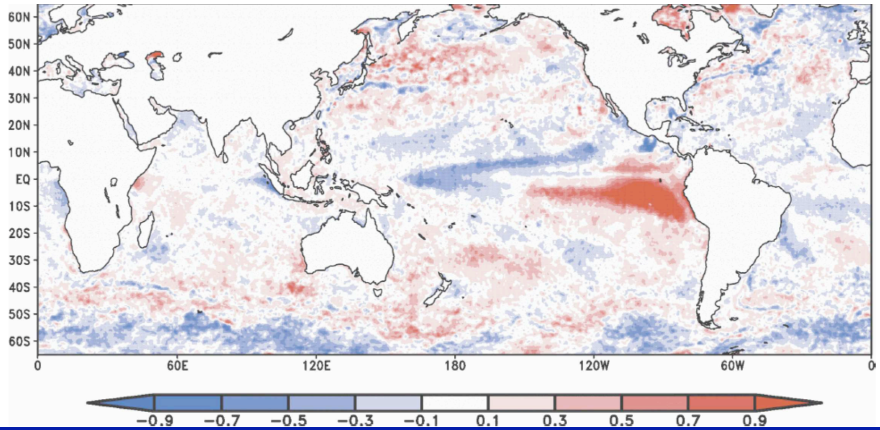


From Sardeshmukh and Sura 2008

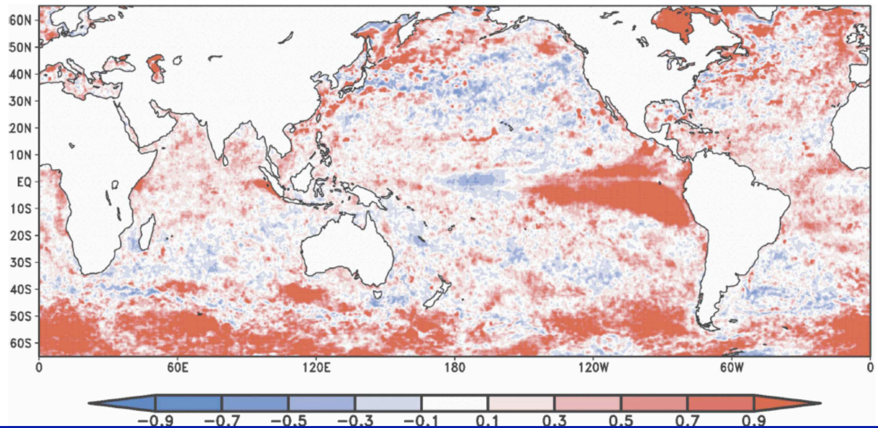
Sea Surface Temperature statistics are also not Gaussian . . .

Observed Skew *S* and (excess) Kurtosis *K* of daily SSTs (DJF)

Skew



Kurtosis



From Sura and Sardeshmukh 2008

Modified LSF Dynamics

$$Model\ 1: \quad \frac{dx}{dt} = Ax + f_{ext} + B\eta$$

$$Model\ 2: \quad \frac{dx}{dt} = Ax + f_{ext} + B\eta + (Ex)\xi$$

$$Model\ 3: \quad \frac{dx}{dt} = Ax + f_{ext} + B\eta + (Ex + g)\xi - \frac{1}{2}Eg$$

For simplicity consider a scalar ξ here

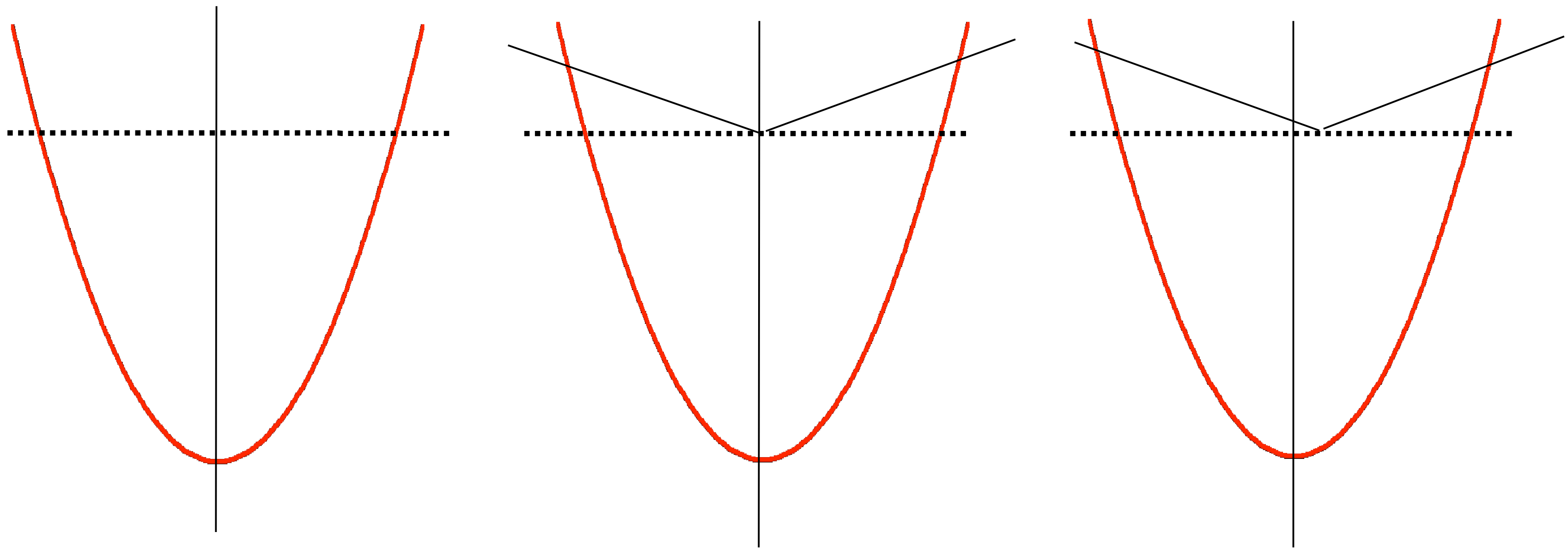
$A(t), B(t), E(t)$ are matrices; $g(t)$, $f_{ext}(t)$, η are vectors

Moment Equations :

$$\frac{d}{dt} < x > \; = \; M < x > \; + \; f_{ext} \qquad \text{where} \quad M \; = \; (A + \frac{1}{2}E^2)$$

$$\frac{d}{dt} C \; = \; M\,C \; + \; C\,M^T \; + \; B\,B^T \; + \; E\,\{\,C + <x><x>^T\,\}\,E^T \; + \; g\,g^T$$

A simple view of how additive and linear multiplicative noise can generate skewed PDFs even in a deterministically linear system



Additive noise only	Additive and uncorrelated Multiplicative noise	Additive and correlated Multiplicative noise
Gaussian		
No skew	Symmetric non-Gaussian	Asymmetric non-Gaussian

A 1-D system with Correlated Additive and Multiplicative ("CAM") noise

Stochastic Differential Equation :

$$\frac{dx}{dt} \cong Ax + (Ex + g)\eta + B\xi - \frac{1}{2}Eg$$

Fokker-Planck Equation :

$$Mxp = \frac{1}{2}\frac{d}{dx}\left[\left(E^2x^2 + 2Egx + g^2 + B^2\right)p\right]$$

Moments :

$$<x> \quad = \quad 0$$

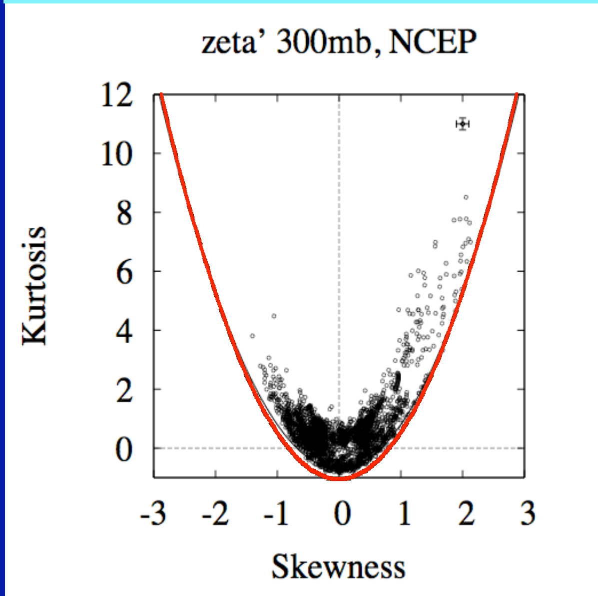
$$<x^n> \quad = \quad -\left(\frac{n-1}{2}\right)\left[2Eg <x^{n-1}> + (g^2 + B^2) <x^{n-2}>\right] / \left[M + \left(\frac{n-1}{2}\right)E^2\right]$$

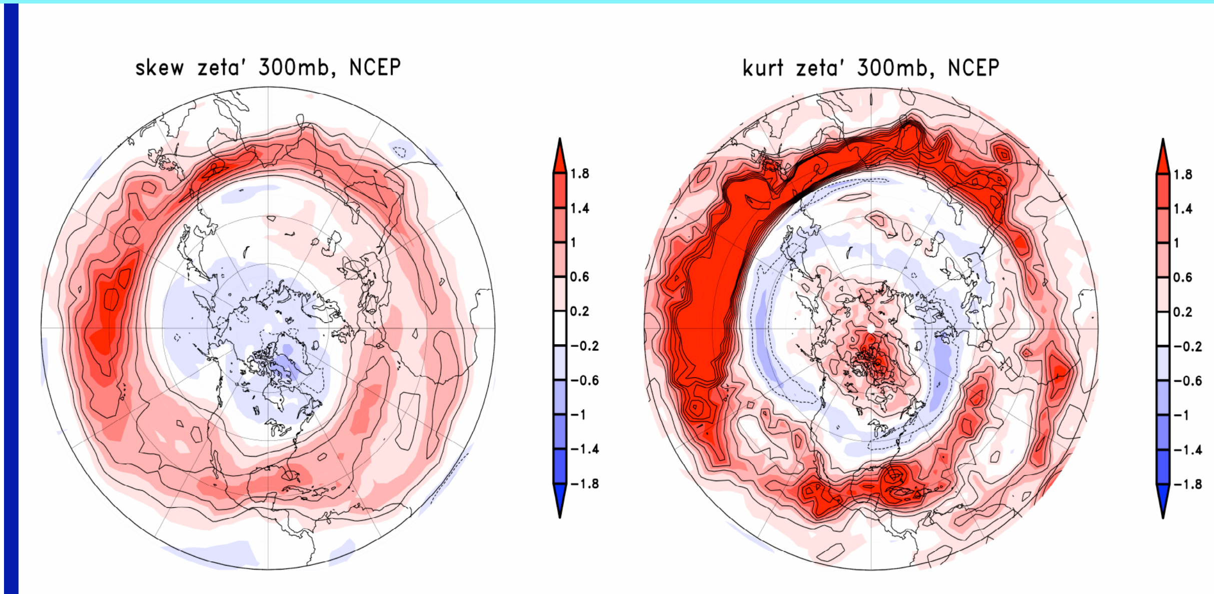
A simple relationship between Skew and Kurtosis :

Remembering that Skew $S = \frac{<x^3>}{\sigma^3}$ and Kurtosis $K = \frac{<x^4>}{\sigma^4} - 3$, we have

$$K = \frac{3}{2}\left[\frac{M + E^2}{M + (3/2)E^2}\right]S^2 + 3\left[\frac{M + (1/2)E^2}{M + (3/2)E^2} - 1\right] \geq \frac{3}{2}S^2$$

Observed Skew S and (excess) Kurtosis K of daily 300 mb Vorticity (DJF)

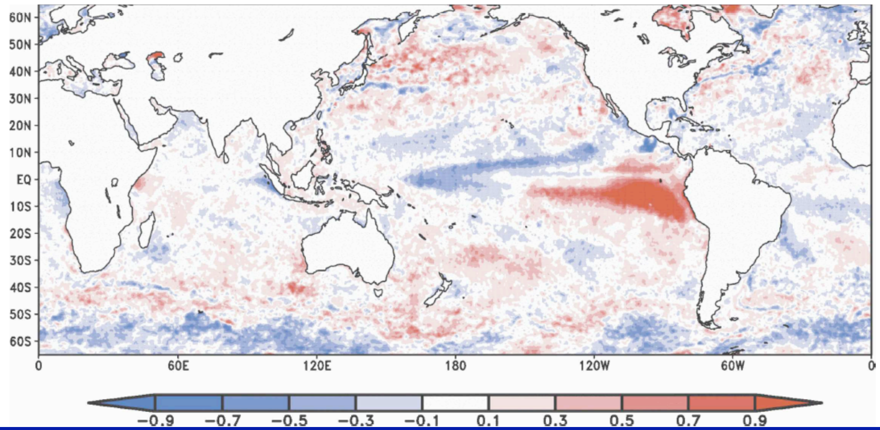




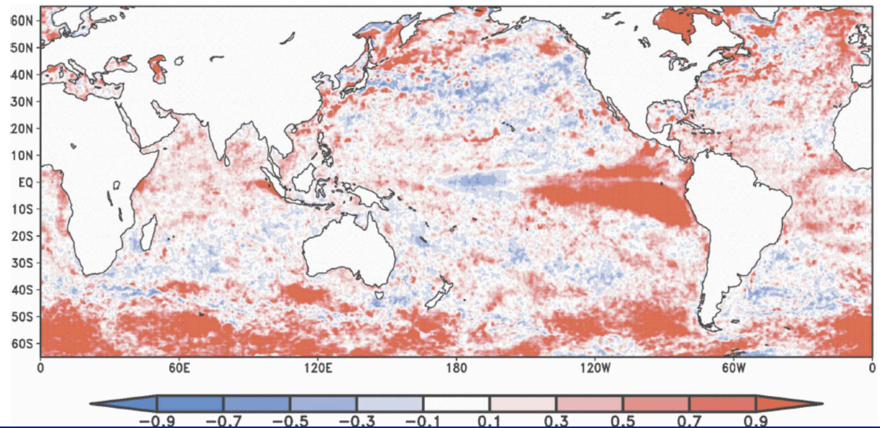
Note the quadratic relationship between K and S : $K \geq 3/2 \; S^2$

Observed Skew S and (excess) Kurtosis K of daily SSTs (DJF)

Skew



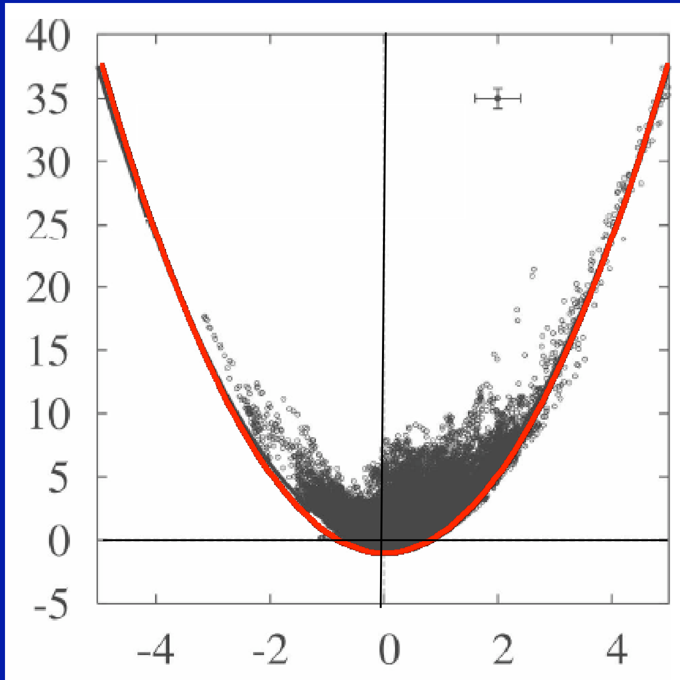
Kurtosis



Note the quadratic relationship

between K and S : $K \geq 3/2\ S^2$

From Sura and Sardeshmukh 2008



Understanding the patterns of Skewness and Kurtosis

Are diabatic or adiabatic stochastic transients more important ?

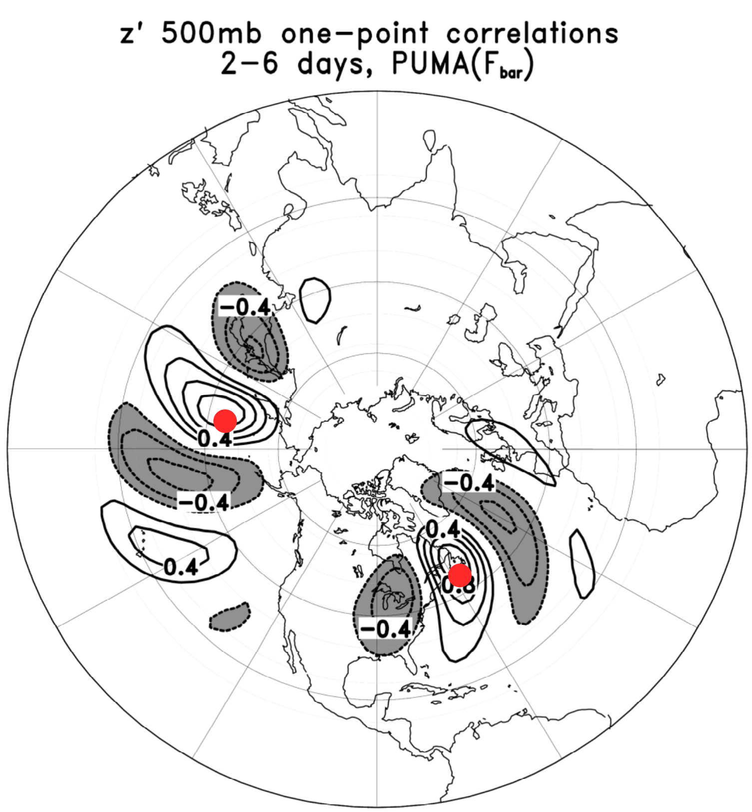
To clarify this, we examined the circulation statistics in a 1200 winter simulation generated with a T42 5-level dry adiabatic GCM ("PUMA") with the observed time-mean diabatic forcing specified as a **fixed** forcing.

There is thus NO transient diabatic forcing in these runs.

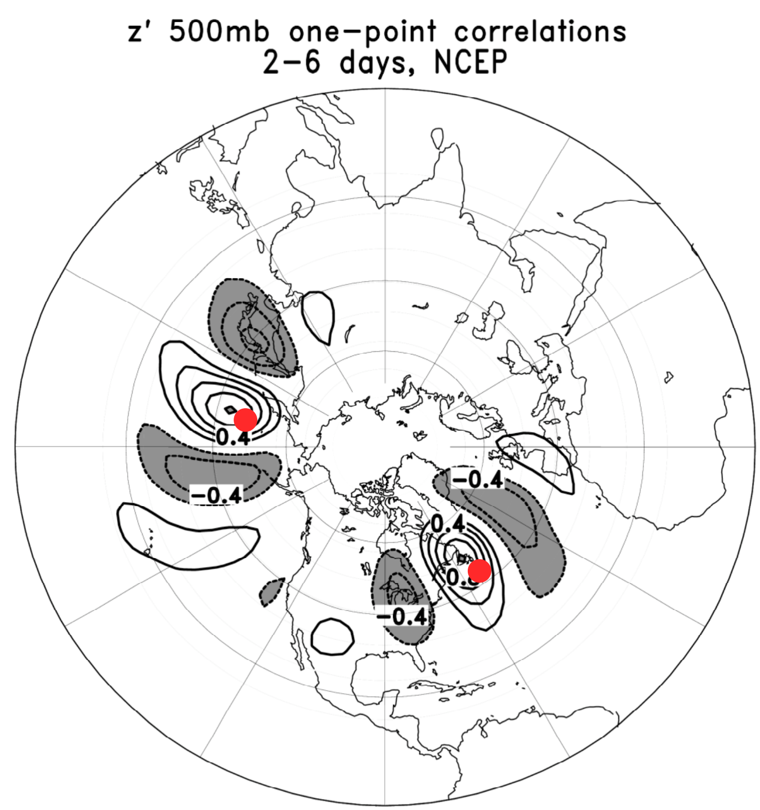
Sardeshmukh and Sura 2007, 2009

1-point anomaly correlations of synoptic (2 to 6 day period) variations with respect to base points in the Pacific and Atlantic sectors

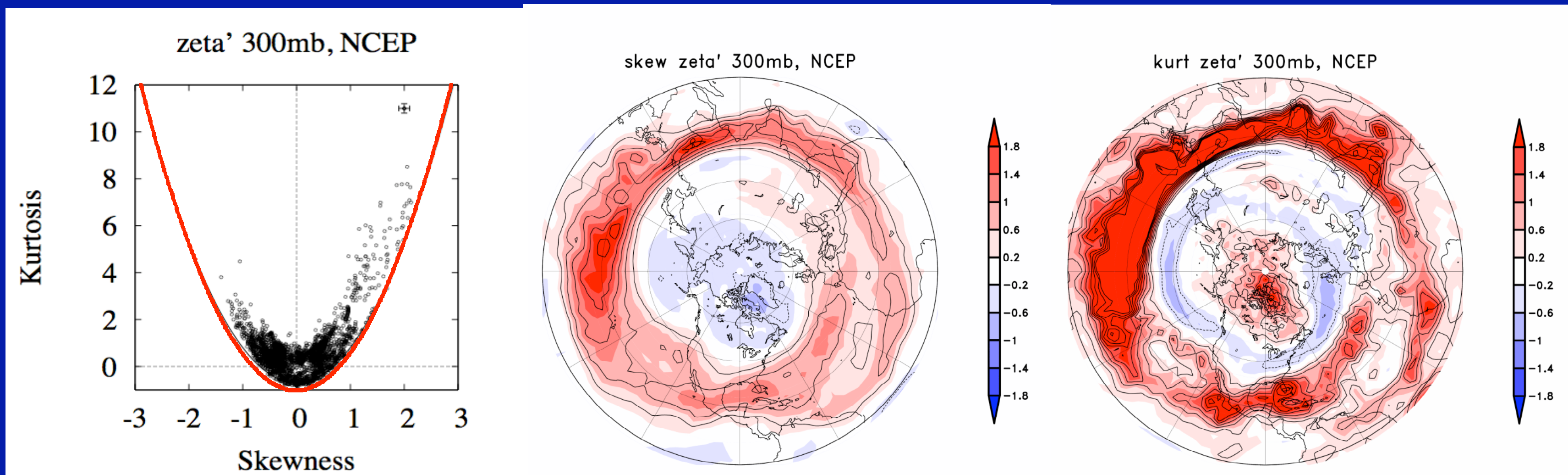
Simulated

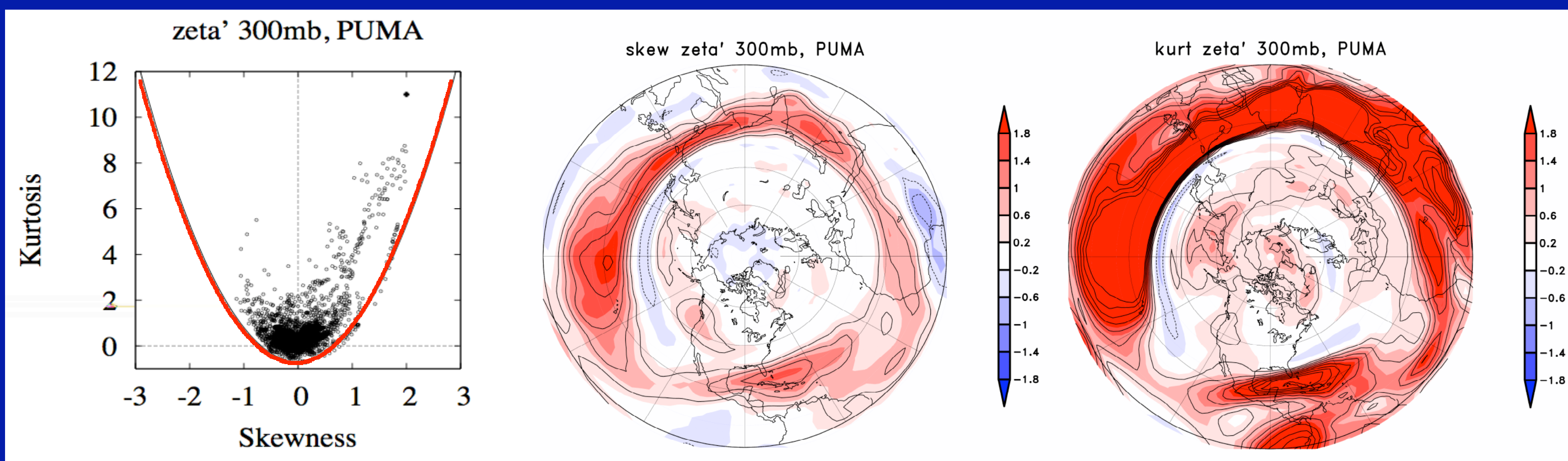


Observed



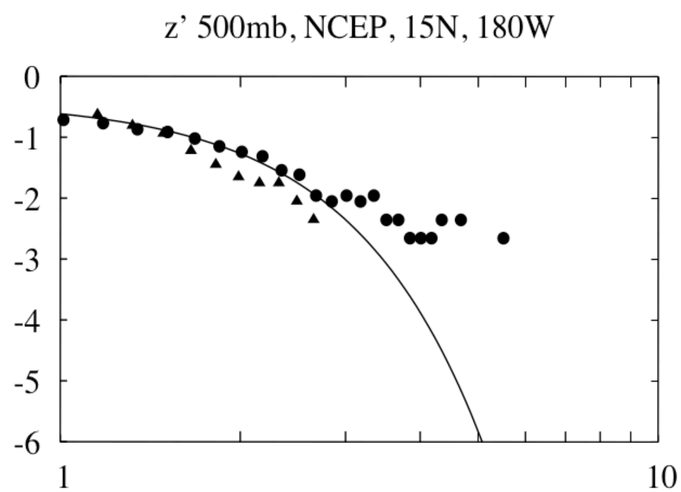
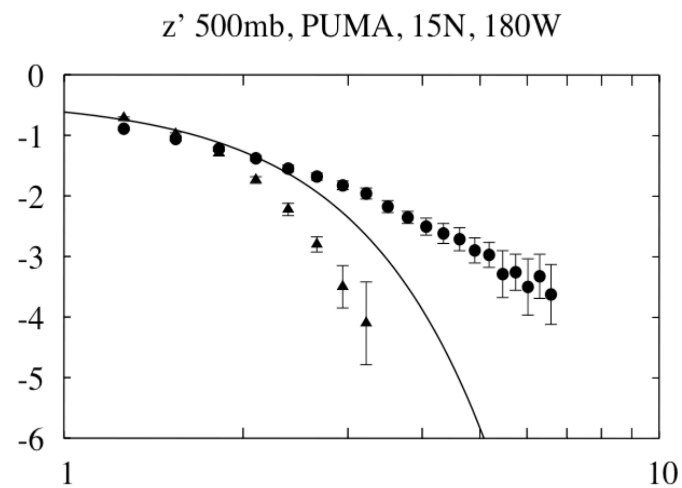
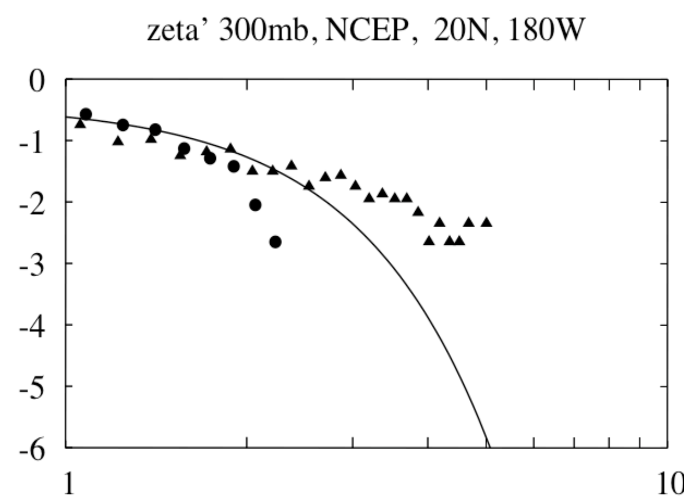
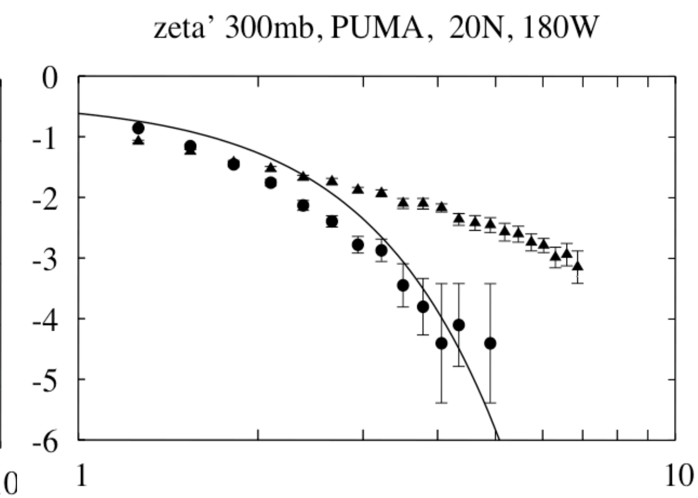
Observed (NCEP, Top) and Simulated (PUMA, Bottom) *S* and *K* of 300 mb Vorticity





Observed and Simulated pdfs in the North Pacific

(On a log-log plot, and with the negative half folded over into the positive half)

	Observed (NCEP Reanalysis)	**Simulated by a dry adiabatic GCM with fixed forcing**
500 mb Height		
300 mb Vorticity		

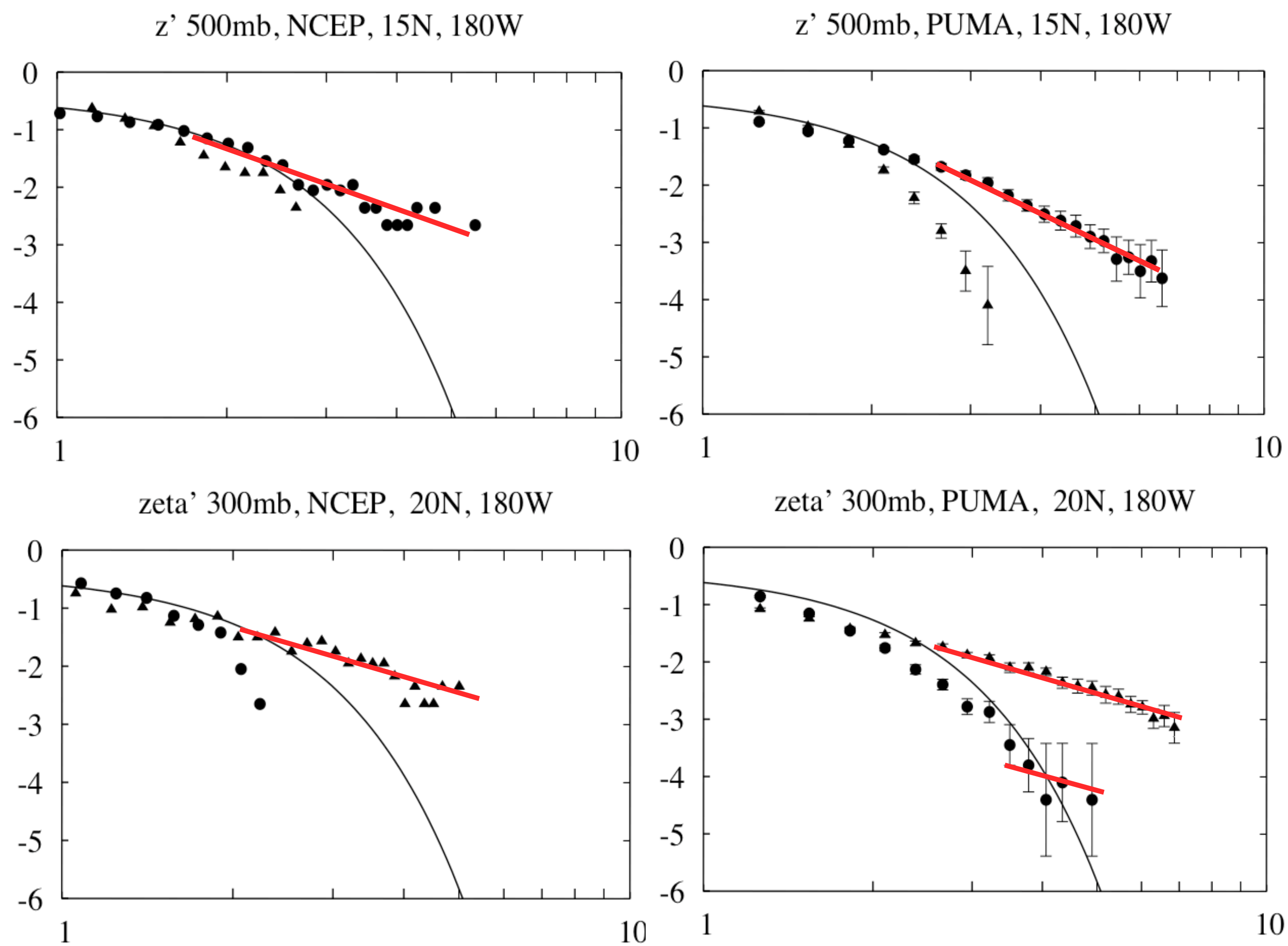
Observed and Simulated pdfs in the North Pacific

(On a log-log plot, and with the negative half folded over into the positive half)

Observed (NCEP Reanalysis)	Simulated by a dry adiabatic GCM with fixed forcing

500 mb Height

300 mb Vorticity



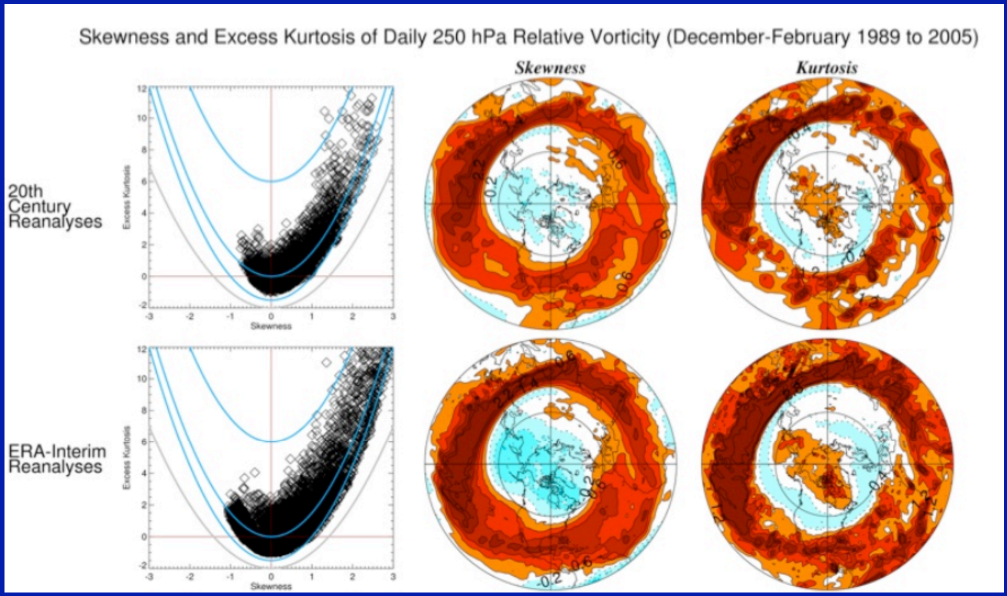
Skewness and Kurtosis are *robust* features of atmospheric circulation statistics.

They need to be accurately represented in models, because of their effect on PDF shape.

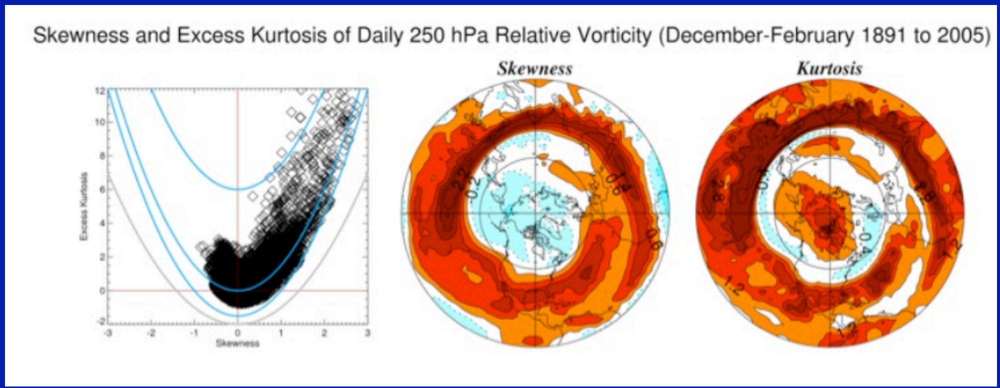
K - S statistics of daily 250 mb Vorticity in 17 recent winters (1989-2005) in two completely different reanalysis datasets :

the 20th Century Reanalysis (20CR) using ONLY surface pressure observations, and

the ERA-Interim Reanalysis using ALL observations



K - S statistics of daily 250 mb Vorticity in all 115 winters (1891-2005) of the 20CR dataset



Compo and Sardeshmukh (2009)

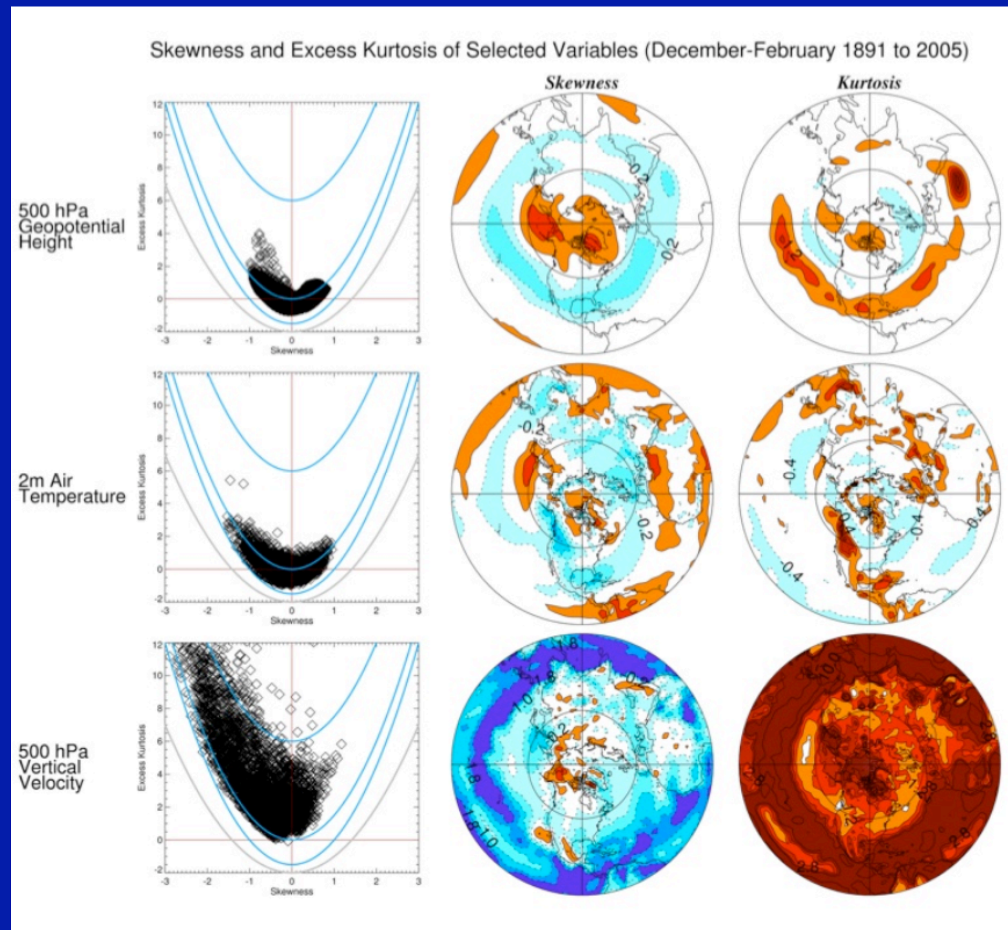
Skewness and Kurtosis are *robust* features of atmospheric circulation statistics.

They need to be accurately represented in models, because of their effect on PDF shape.

(For example, the PDF of 500 mb ω is highly skewed. This impacts the PDF of precipitation.)

K - S statistics in winter of some other important daily atmospheric variables

Based on all 115 winters (1891-2005) in the 20CR dataset



Compo and Sardeshmukh (2009)

Summary

1. Strong evidence for "coarse-grained" linear dynamics is provided by
   (a) the observed decay of correlations with lag
   (b) the success of linear forecast models, and
   (c) the approximately linear system response to external forcing.

2. The simplest dynamical model with the above features is a linear model perturbed by **additive** Gaussian stochastic noise. Such models have been proven to be very useful. **They cannot, however, generate non-Gaussian statistics**.

3. **Linear models with correlated additive and multiplicative ("CAM") noise *can* generate non-Gaussian statistics, and can also explain the remarkable observed quadratic K-S relationship between Kurtosis and Skew, as well as the Power-Law tails of the PDFs.**

4. Such extended linear models should be additionally useful for diagnosing extreme behaviour in reality and in weather and climate models.