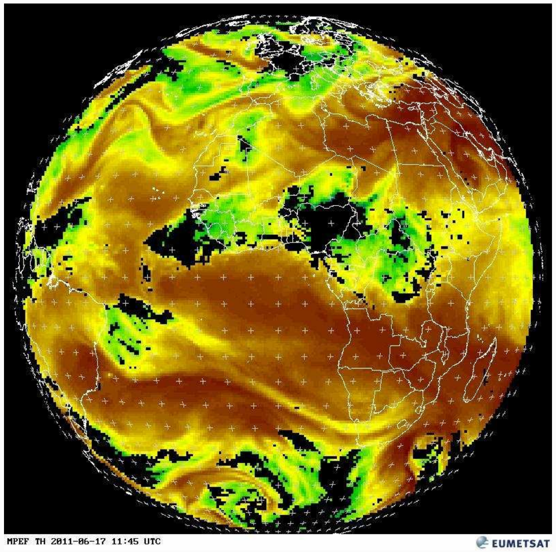
Energy and Enstrophy Cascades in Numerical Models

John Thuburn (University of Exeter),
James Kent (Now University of Michigan),
Nigel Wood (Met Office)

ECMWF, 20 June 2011

• Energy and (potential) enstrophy are conserved by the adiabatic, frictionless governing equations...

• ...but nonlinearity leads to systematic transfers between scales



Meteosat 'tropospheric relative humidity'
(red low, green high)

• How well do numerical models handle those transfers, especially near the truncation limit? ... Source of uncertainty.

Outline

- Explicit subgrid models vs ILES
- Barotropic vorticity equation as a model problem

  Effect of unresolved scales on enstrophy and energy spectra

  Effect of some numerical schemes on enstrophy and energy spectra

  Parameterization of energy backscatter

Numerical representation of energy and potential enstrophy transfers

Foremost, *need to remove potential enstrophy*. Typically either

(a) use conservative numerics supplemented by some scale-selective dissipation such as $\kappa\nabla^{2n}$ (but note its multiple roles)

or

(b) use inherently dissipative numerics such as semi-Lagrangian or non-oscillatory finite volume (ILES).

May also include some representation of *energy backscatter*.

Implicit Large Eddy Simulation (ILES)

Finite resolution => need to represent effects of unresolved scales: SG model.

At the same time, all numerical methods have truncation errors.

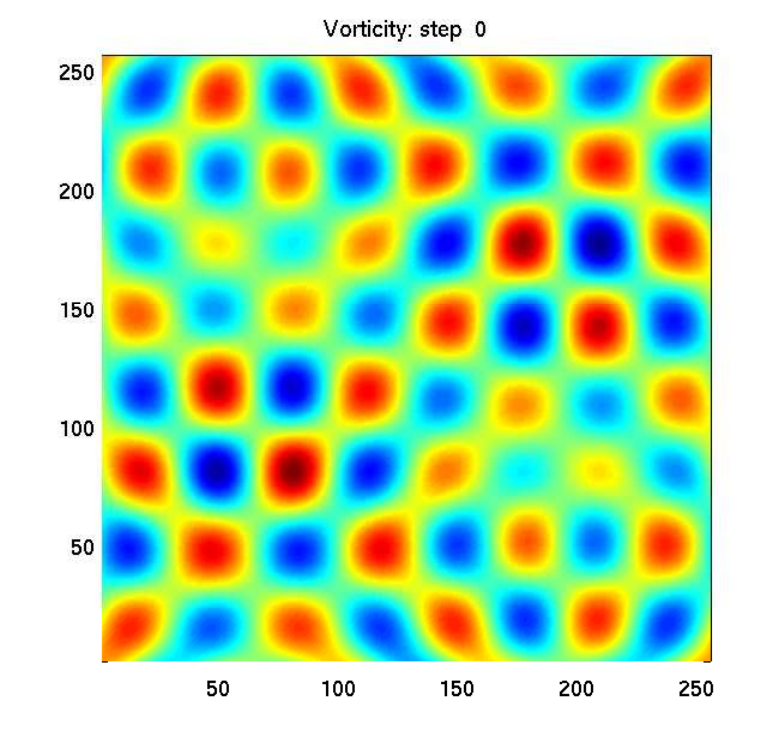
Can truncation errors play the role of a SG model?

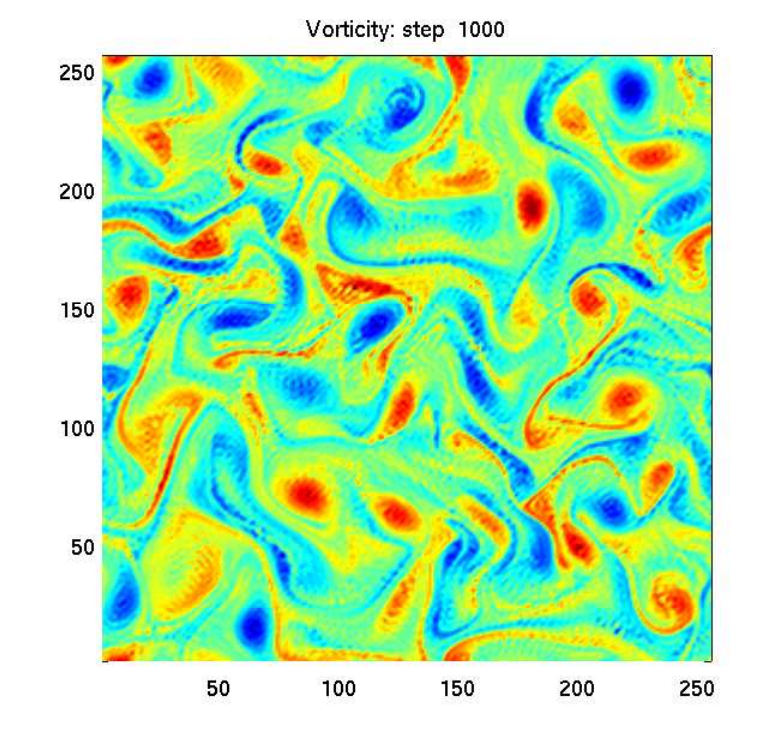
Some success claimed for 3D turbulence. (Except when upscale effects are important, e.g. near a wall.)

What about (layerwise) 2D turbulence?

Upscale energy transfers, but steeper spectrum so stronger slaving of small scales to large.

What if we don't remove resolved enstrophy?





There is evidence that models dissipate too much energy

If we remove enstrophy at horizontal wavenumber k_{diss} at a rate $\dot{Z}$ then we necessarily remove KE at a rate $\dot{E} = \dot{Z}/k_{\mathrm{diss}}^2 \geq \dot{Z}/k_{\mathrm{max}}^2$.

At current climate resolutions this is too large.

E.g. $\dot{Z} \sim 10^{-15}\,\mathrm{s}^{-3}$. Need $\dot{E} \sim 10^{-5}\,\mathrm{m}^{-2}\mathrm{s}^{-3}$ so $k_{\mathrm{diss}} \sim 10^{-5}\,\mathrm{m}^{-1}$.

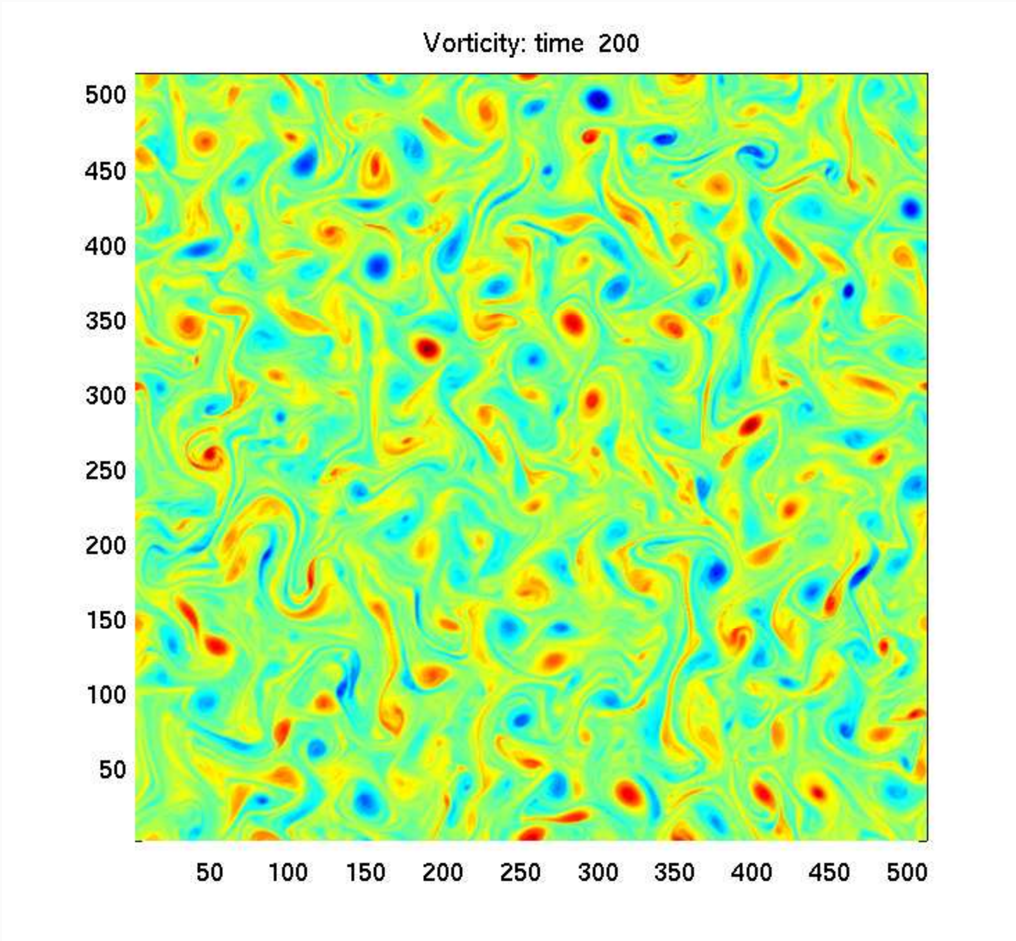
What does ILES or any explicit SG model need to capture?

Barotropic vorticity equation as model problem:

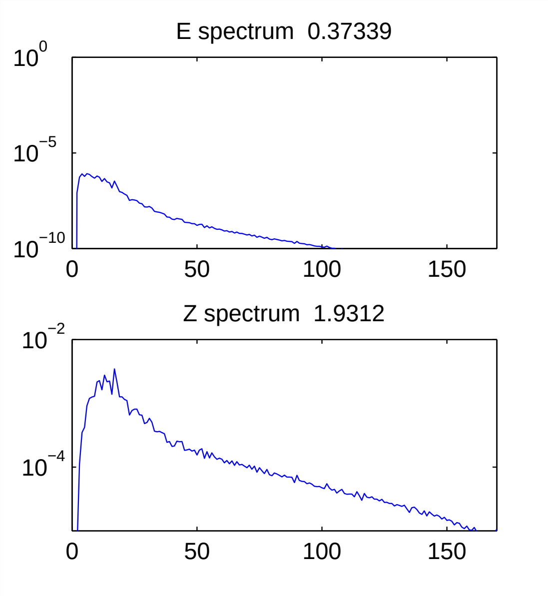
$$\frac{D\zeta}{Dt} = 0; \qquad \nabla^2\psi = \zeta; \qquad \mathbf{v} = \nabla^\perp\psi$$

Statistically steady turbulence

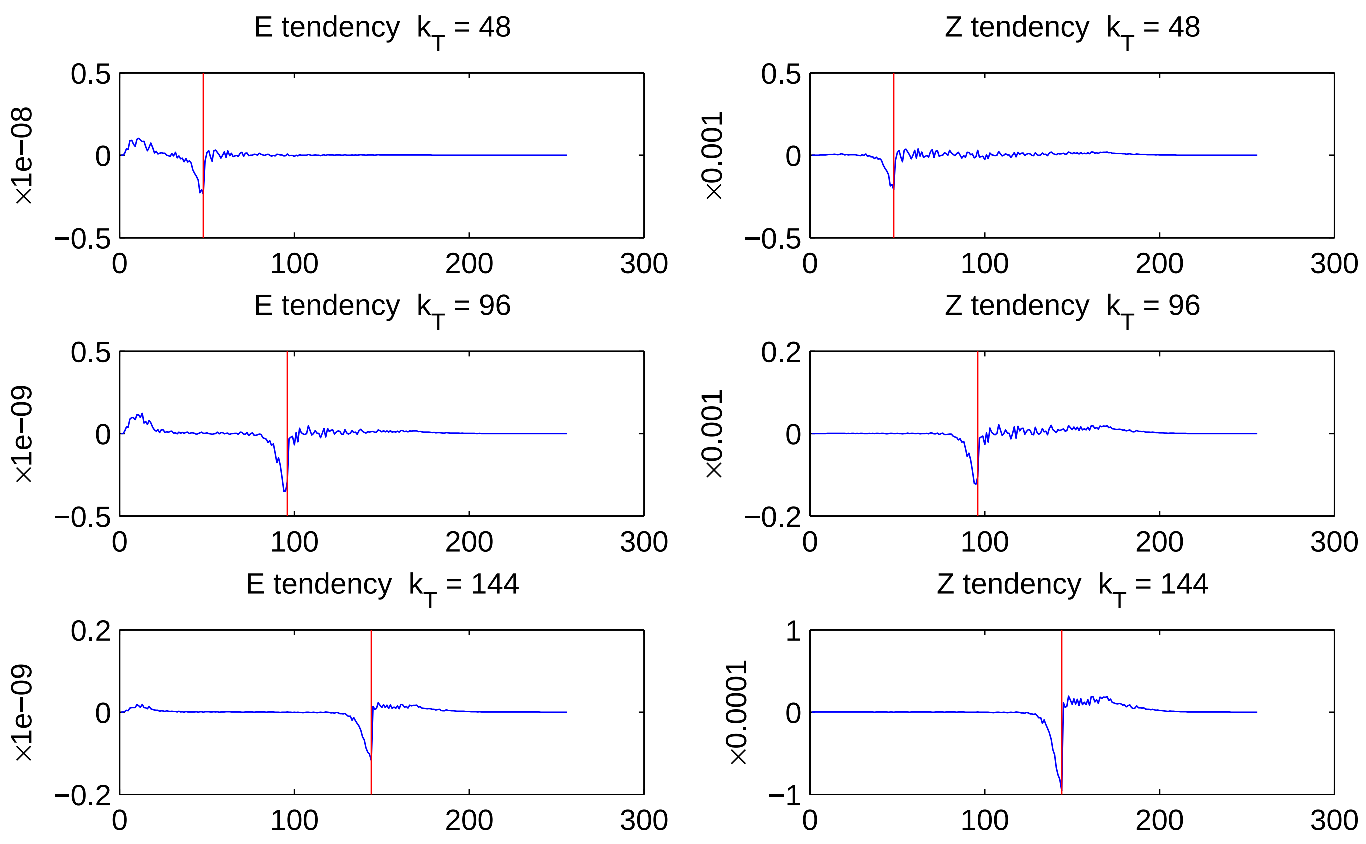
$t = 200$



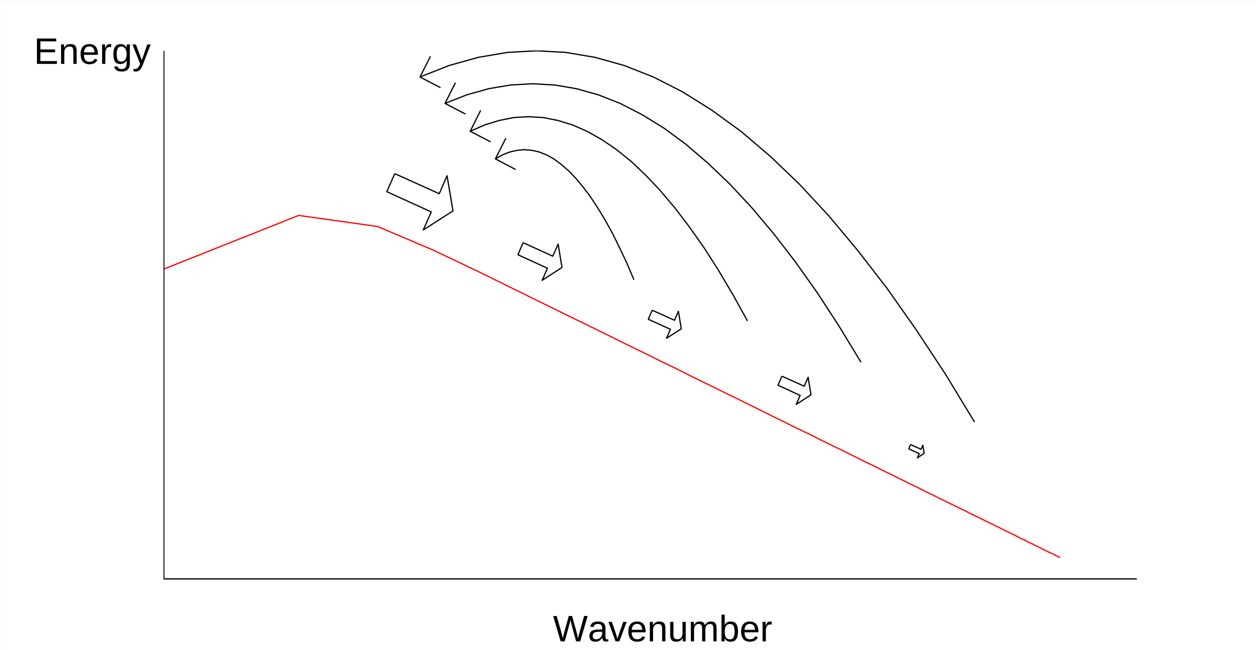
Forcing at $n = 16$; scale-independent dissipation; and ∇^8 small-scale dissipation.



Spectral interactions associated with truncated scales



Schematic of energy transfers



Cascade local in k; backscatter nonlocal.

(See also Huang and Robinson 1997; Thompson and Young 2007)

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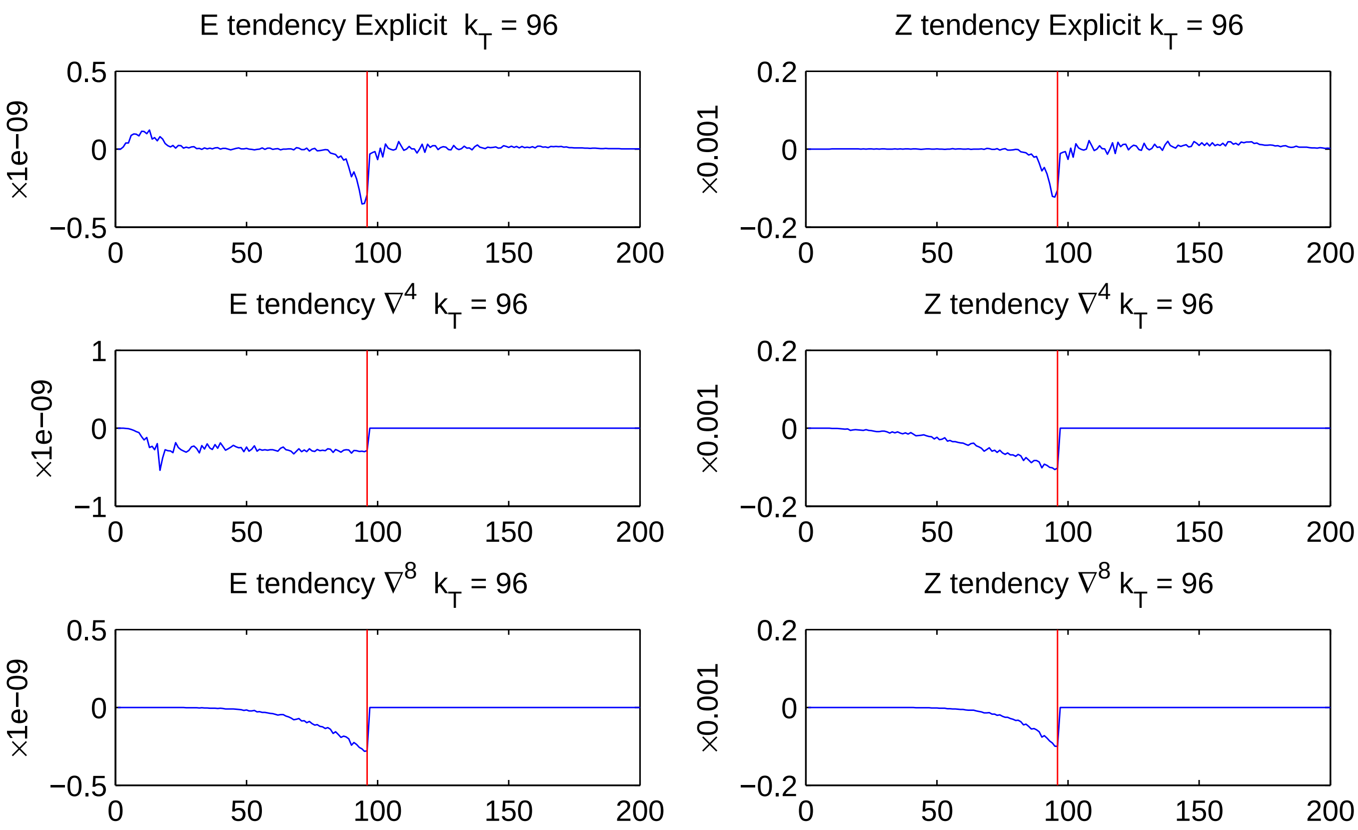
Spectral interactions as represented by ∇^4 and ∇^8



UTOPIA advection of ζ

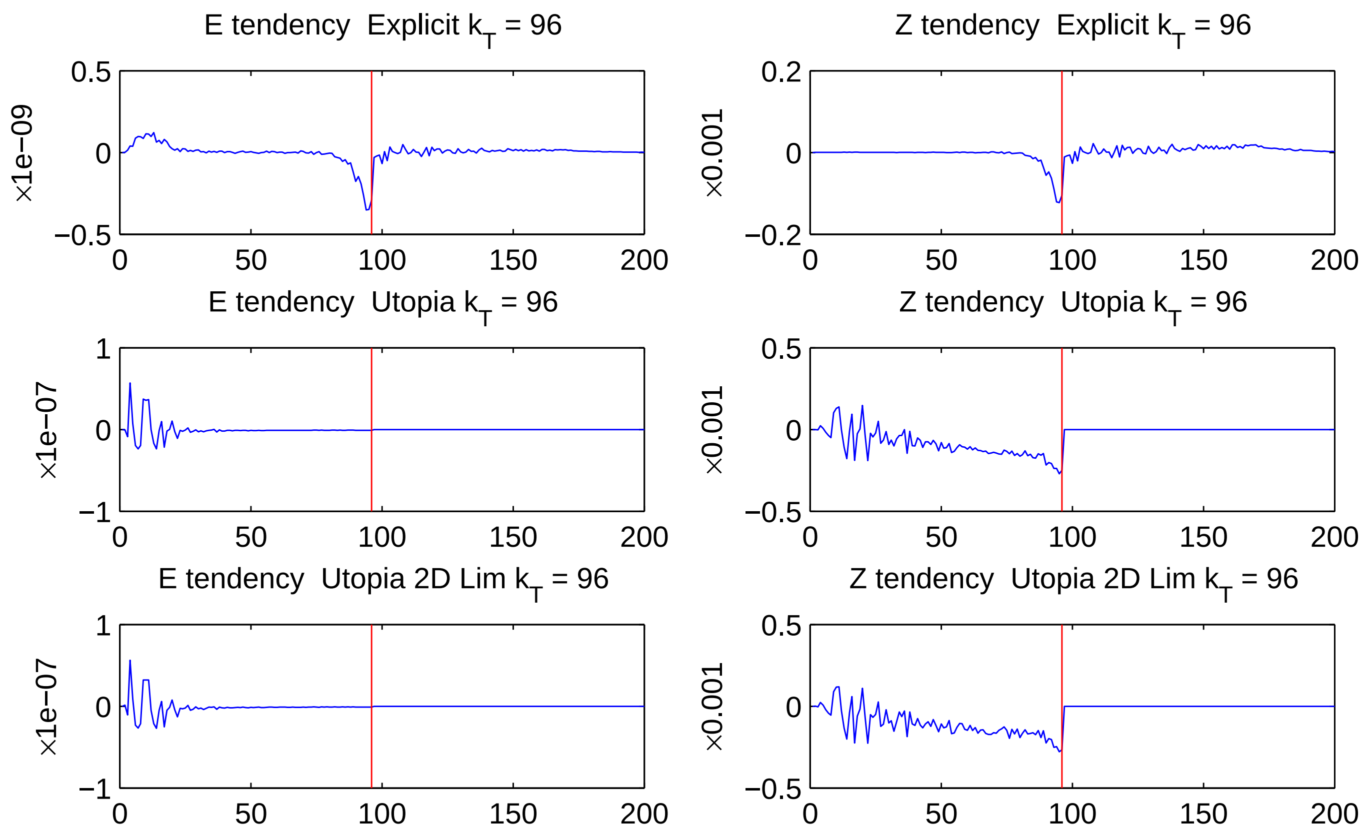
Quasi-third-order upwind scheme.

Inherently dissipative, but more scale-selective than first-order upwind.

Should be comparable to semi-Lagrangian with cubic interpolation.

Can include a flux limiter to prevent over/under-shoots.

Spectral interactions as represented by UTOPIA scheme



Anticipated Potential Vorticity Method

Sadourny and Basdevant (1985).

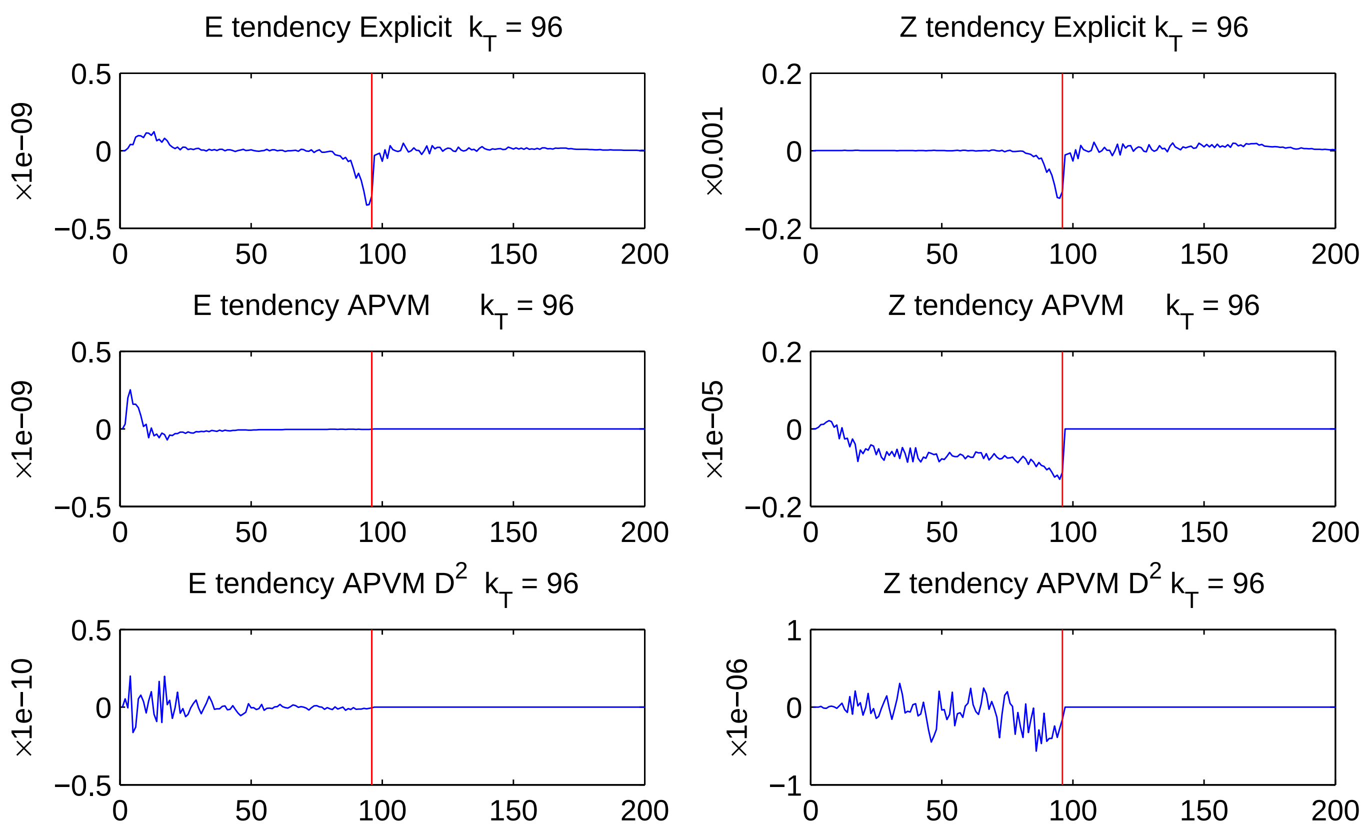
$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta - D)\hat{\mathbf{k}} \times \mathbf{v} + \nabla \left(p + \frac{\mathbf{v}^2}{2} \right) = 0$$

$$\frac{\partial \zeta}{\partial t} + \nabla.(\mathbf{v}\zeta) = \nabla.(\mathbf{v}D)$$

Choose $D = \theta \mathcal{L}(\mathbf{v}.\nabla\zeta)$. Here $\mathcal{L} \equiv 1$ or $\mathcal{L} \equiv -\nabla^2$

$$\dot{Z} = -\theta \int (\mathbf{v}.\nabla\zeta)^2 \, dA \quad \text{or} \quad \dot{Z} = -\theta \int \left(\nabla(\mathbf{v}.\nabla\zeta)\right)^2 \, dA$$

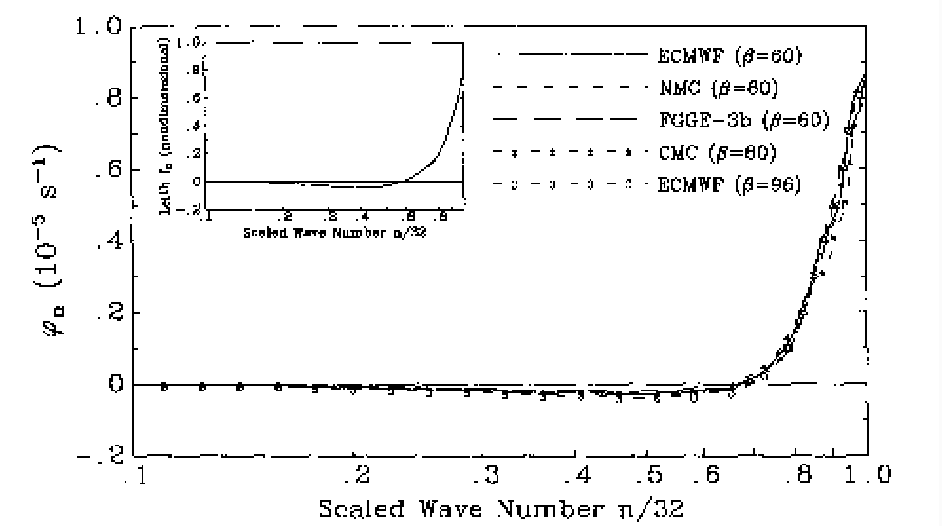
Spectral interactions as represented by APVM



Can we represent the energy backscatter to large scales?

Scale-dependent dissipation/anti-dissipation

Koshyk and Boer (1995)



$$I_n = I_n^{\mathrm{R}} + I_n^{\mathrm{U}}; \qquad I_n^{\mathrm{U}} = -2\phi_n E_n$$

A simple backscatter scheme for BVE

Let $\zeta^* = \mathrm{UTOPIA}(\zeta^n)$

and let $\delta E = E(\zeta^n) - E(\zeta^*)$

Choose a vorticity pattern $\delta\zeta$ and let $\zeta^{n+1} = \zeta^* + \alpha\delta\zeta$.

$$\alpha = -\frac{\delta E}{\int \psi \delta\zeta \, dA}$$

gives energy conservation (to an excellent approximation).

Which vorticity pattern $\delta\zeta$ to use?

E.g.

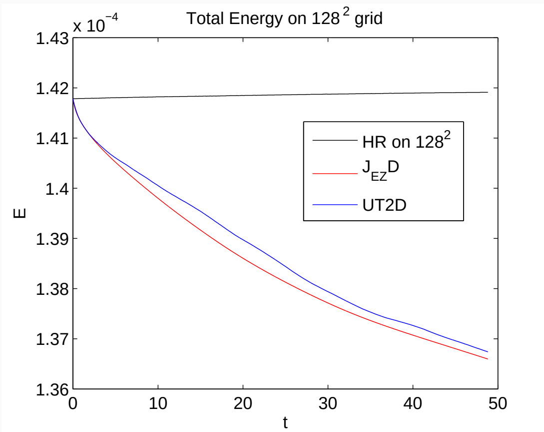
$\delta\zeta_1 = \overline{\zeta}^{4\Delta x}$ (large scales)

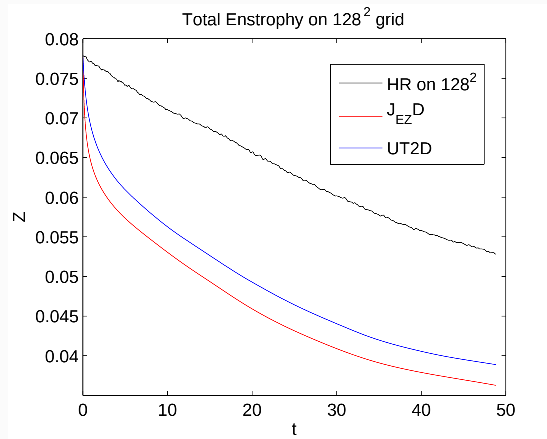
$\delta\zeta_2 = \zeta - \overline{\zeta}^{4\Delta x}$ (small scales)

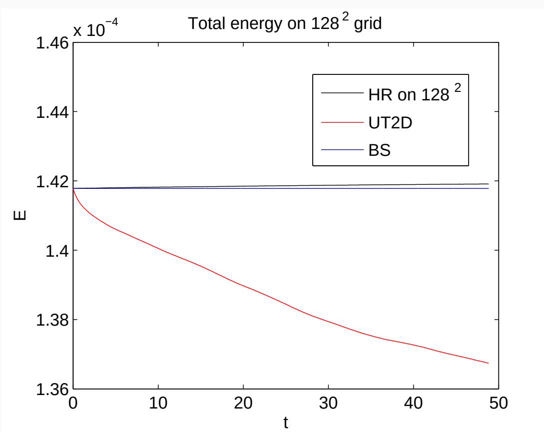
$\delta\zeta_2$ was found to work better in numerical tests, giving better energy statistics and also a small but measurable improvement in l_2 errors.

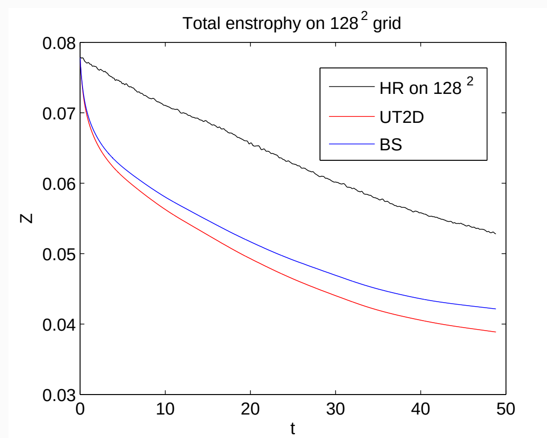
(But this is not really 'backscatter'; more of an energy fixer!)

Decaying turbulence E and Z time series









With and without 'backscatter'

Possible improvements to ‘backscatter’ scheme

- Use scale similarity to derive $\delta\zeta$
- Use spectral dissipation characteristics of basic scheme to derive $\delta\zeta$

Discussion - extension to more complex flows

• The effect of finite Rossby radius;

• Transition to $k^{-5/3}$ energy cascade regime;

• Extension to realistic 3D flow: available vs unavailable energy; fronts; convection; orography; other physical processes...

Conclusions

• For the BVE, explicit calculation of the effects of unresolved scales shows enstrophy removal near the truncation limit and energy input at the most energetic scales. Very robust.

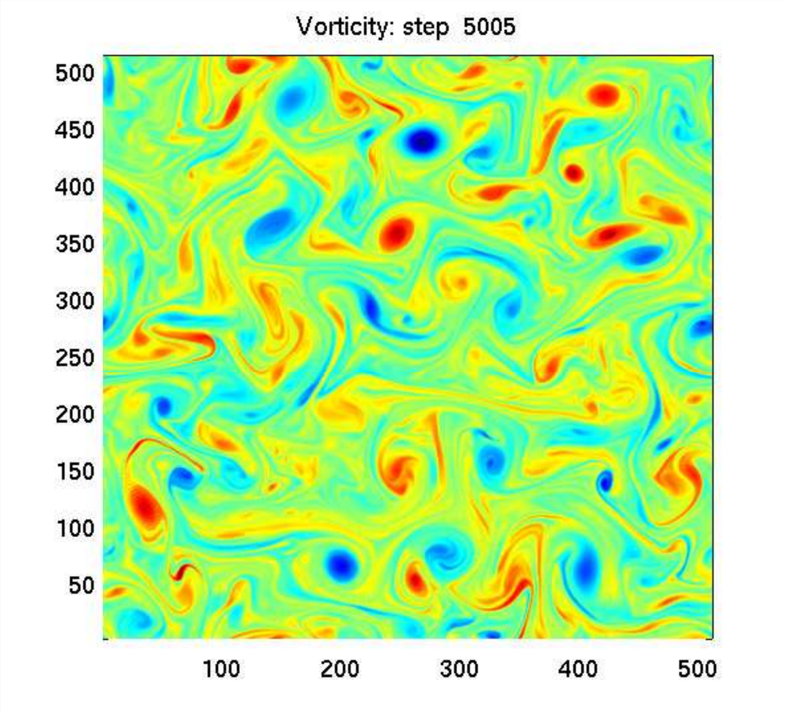
• Both ILES schemes and simple explicit dissipation schemes can remove enstrophy at small scales (but are typically not scale-selective enough

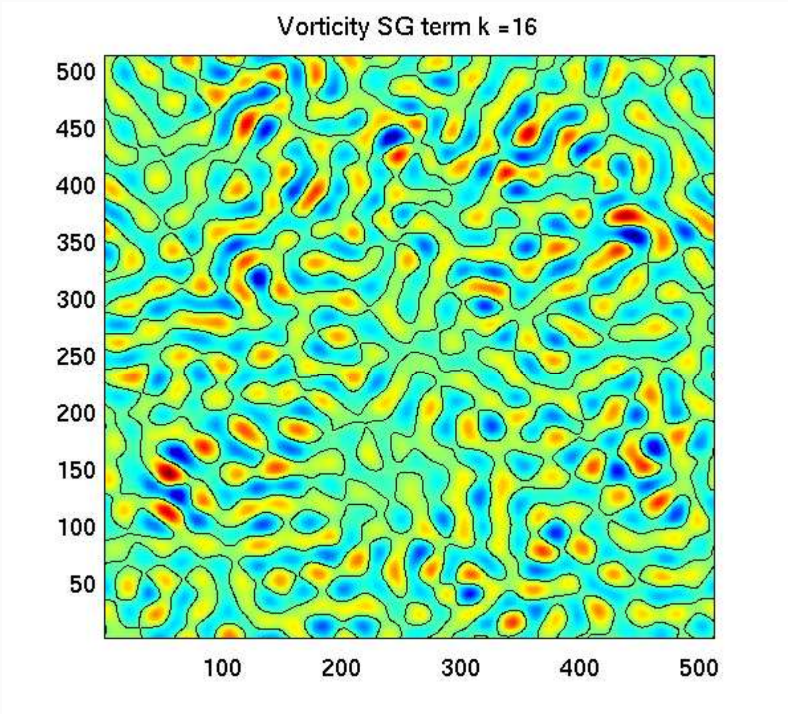
• Neither ILES schemes nor standard SG models capture the energy backscatter.

• A simple 'backscatter' model can improve energy statistics and l_2 errors (but it's really an energy fixer).

• It should be possible to extend this approach to more complex flow.

Subgrid forcing of vorticity





$$\partial_t \overline{\zeta} + \partial_j \left(\overline{v_j} \overline{\zeta} \right) = \mathrm{SG} = \partial_j \left(\overline{v_j} \overline{\zeta} - \overline{v_j \zeta} \right)$$

Scale similarity of backscatter?