Earth System Research Laboratory
Physical Sciences Division

Evaluating methods for representing model error using ensemble data assimilation

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Evaluating model error schemes

- Using an EPS
  - Spread/error consistency, probabilistic scores.
  - Hard to know whether improvement comes simply from reducing spread deficiency.
- Using an EnKF
  - Tougher test if multiplicative inflation used as baseline, since scheme must do more than increase variance.
  - Evolution of all errors in DA cycle (not just model error) must be represented. Model error may not be dominant.

Un(der)-represented error sources in an EnKF ensemble

Model error	$\mathbf{M}\mathbf{x}_\mathbf{a}$
Sampling error	$\frac{1}{N}\sum_{j=1}^{N}(N << \infty)$
Observation error	$\mathbf{R}$
Boundary condition error	$T(z=0) \Rightarrow T_s$
Forward operator error	$\mathbf{H}\mathbf{x}_\mathbf{b}$

Experiences with Env. Canada system

(Houtekamer, Mitchell and Deng, MWR July 2009)

- Operational EnKF tested with
  - Multiple parameterizations
  - SKEB (stochastic kinetic energy backscatter)
  - SPPT (stochastically perturbed physics tend)
  - Additive inflation (isotropic covariance structure)
  - Multi-physics plus additive inflation

Experiences with Env. Canada system

(Houtekamer, Mitchell and Deng, MWR July 2009)

configuration	O-F (energy norm)	Energy spread in ob space
Additive inflation	3.1388	2.0622
Multi-physics	3.2978	1.2773
SKEB	3.4348	1.2671
SPPT	3.3899	1.1670
Multi-physics + add. Infln.	3.0846	2.1335
SKEB + SPPT	3.3352	1.3608
SKEB+SPPT+Mult-physics +rescaled additive infln.	3.0940	2.1092

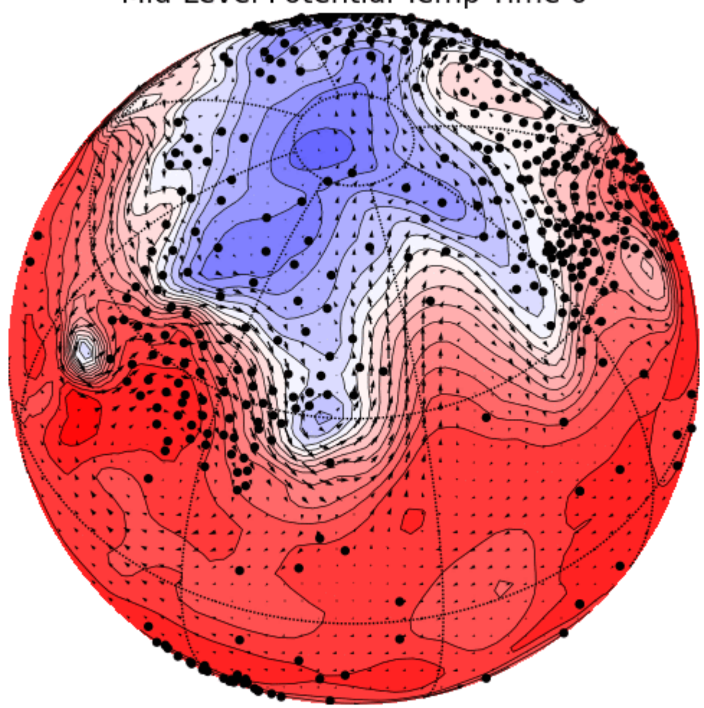
- Biggest impact from ad-hoc additive inflation.
- Addition of multi-physics improves assimilation slightly.
- SPPT and SKEB have less impact (tuned for EPS?, model error not dominant?)

Motivation for simple model expts

- Sources of assimilation error can be controlled (obs. error know perfectly, sampling error can be controlled).
- Access to truth aids in diagnostics, tuning of model error schemes.
- Can run lots of experiments.

2-level PE model on a sphere

- 2-level PE model on a sphere (Lee and Held, 1993 with parameters as in Hamill and Whitaker, 2010).
- 511 12-hourly obs of geopotential height at sonde locations (error = 10 m)
  - 20 member ensemble, serial determinstic (i.e. square-root) EnKF.
  - 1000 assimilation cycles, 3500 km localization (none in vertical)
- Truth from T42 nature run, assimilation with T31 model. Only sources of DA error are model error and sampling error.



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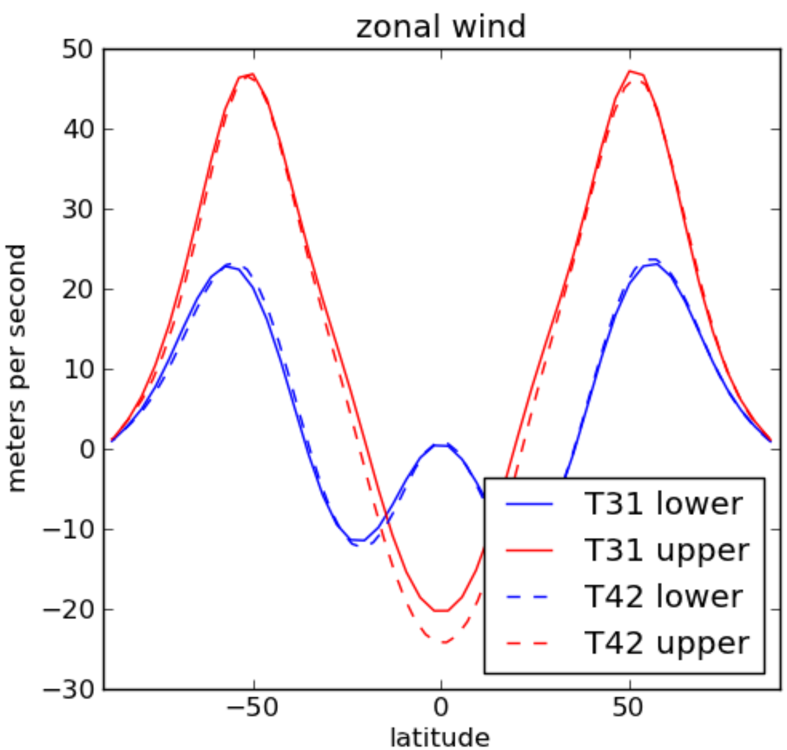
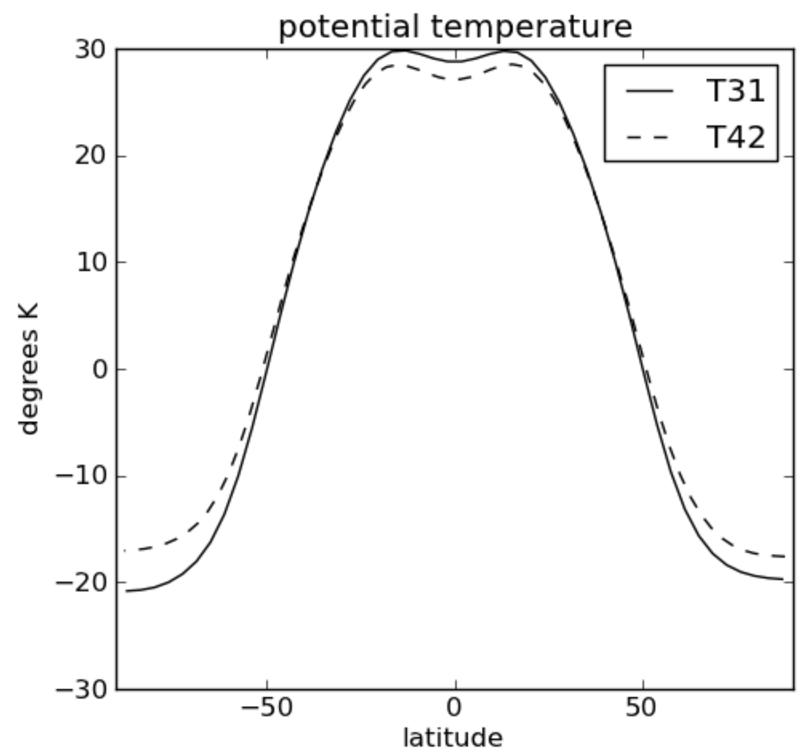




Methods for representing model error
Multiplicative Inflation

- Simple constant inflation not suitable when observing network and dynamics vary in space and/or time.
- Both sampling error and model error are expected to be a larger fraction of the total background error where observations have a larger impact (where σ_b/σ_a is large).
- “relaxation” inflation is stronger where ensemble variance is reduced by the assimilation of observations.

Methods for representing model error *Multiplicative Inflation*

- Relaxation to prior perturbations (RTPP, Zhang et al, 2005) $\mathbf{x}_i'^a \leftarrow (1-\alpha)\mathbf{x}_i'^a + \alpha\mathbf{x}_i'^b$
- Relaxation to prior spread (RTPS)

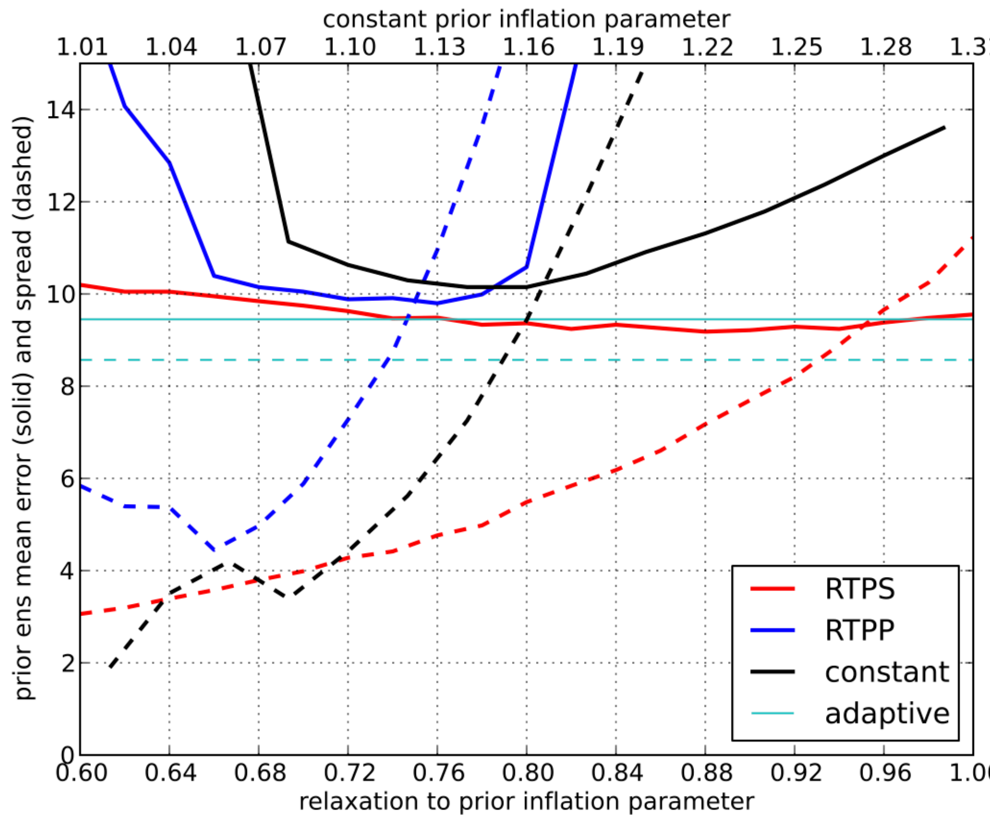
$$\sigma^a \leftarrow (1-\alpha)\sigma^a + \alpha\sigma^b$$

which implies $\mathbf{x}_i'^a \leftarrow \mathbf{x}_i'^a \sqrt{\alpha \frac{\sigma^b - \sigma^a}{\sigma^a} + 1}$

- Both inflate more where observations have a strong tendency to reduce ensemble variance.

Methods for representing model error
Multiplicative Inflation

- Relaxation to prior spread works best, is less sensitive to choice of relaxation parameter.
- Jeff Anderson's Bayesian adaptive inflation method performs similarly.



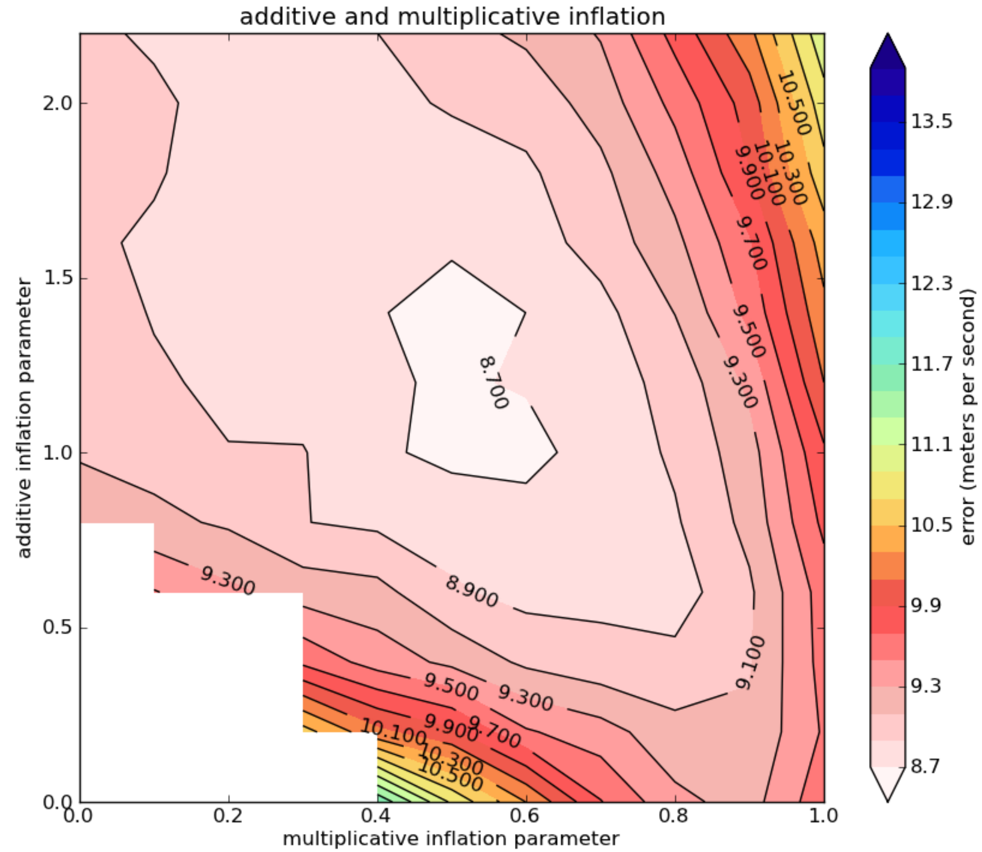
ens mean first-guess error (solid), spread (dashed)

Methods for representing model error
Additive Inflation

- Add random samples from a specified distribution to each ensemble member after the analysis step.
- Env. Canada uses random samples of isotropic 3DVar covariance matrix.
- Here we use a dataset of 12-h forecast errors with the T31 model in which the initial conditions are perfect (T31 truncated states from the T42 nature run).

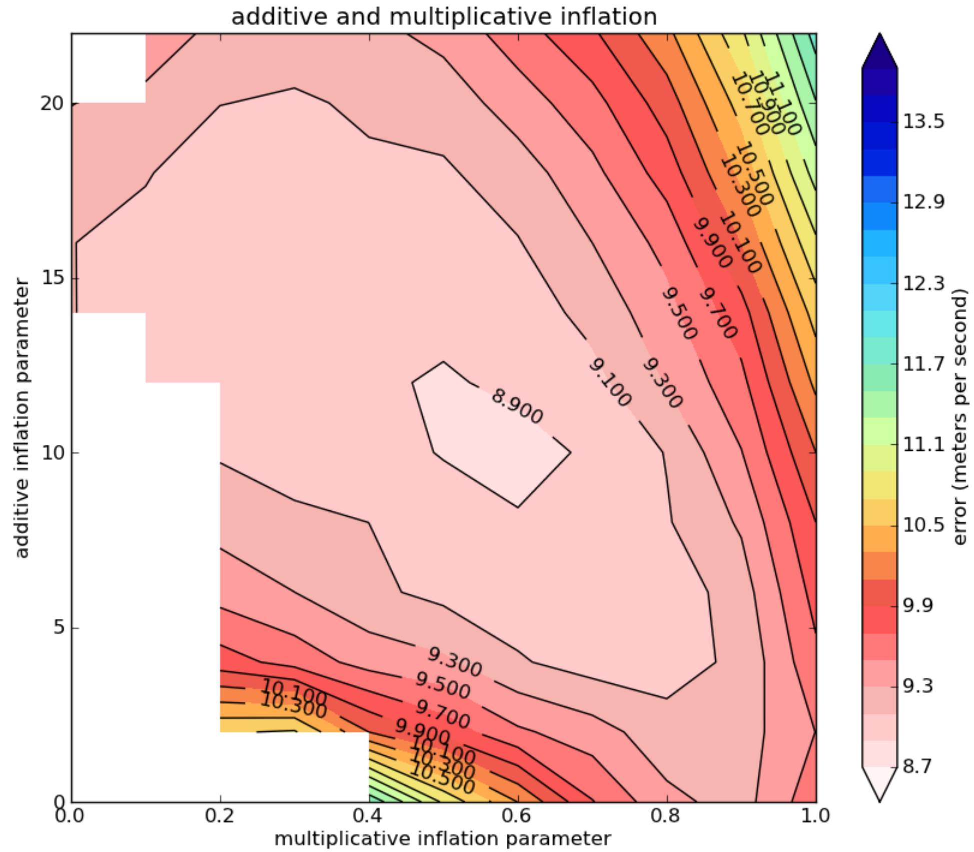
Methods for representing model error
Additive Inflation

- Additive inflation alone outperforms multiplicative inflation alone (compare values y-axis to values along x-axis)
- A combination of both is better than either alone.
- Multiplicative and additive inflation representing different error sources in the DA cycle?



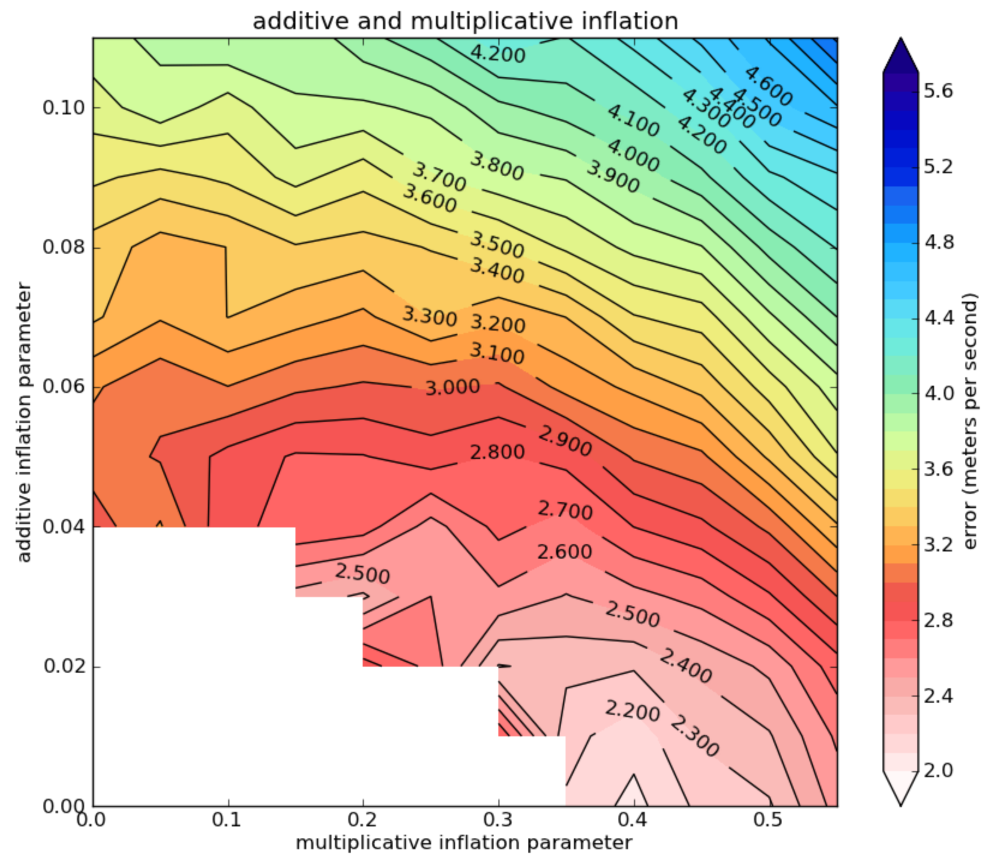
Methods for representing model error
Additive Inflation

- Using random samples of actual model error is unrealistic.
- Instead, try random samples of 12-h differences from T31 run.
- Not quite as good as using actual model error, but still an improvement over multiplicative inflation alone.
- Additive inflation alone still better than multiplicative inflation alone (compare values along x and y axes).



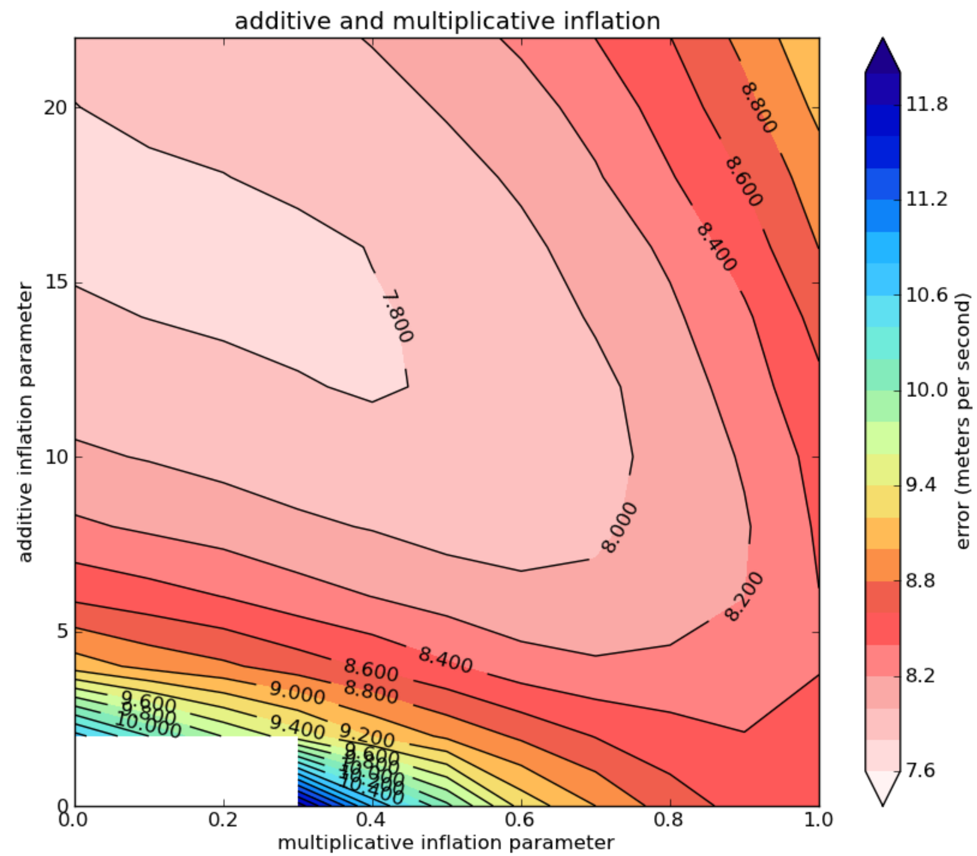
Perfect Model results (Additive + Multiplicative Inflation)

- 6000 km localization, min error 4 times lower.
- When model error is absent, multiplicative inflation alone outperforms combination of add +mult inflation.
- Suggests that multiplicative inflation is better at capturing DA-related (i.e. sampling) error.



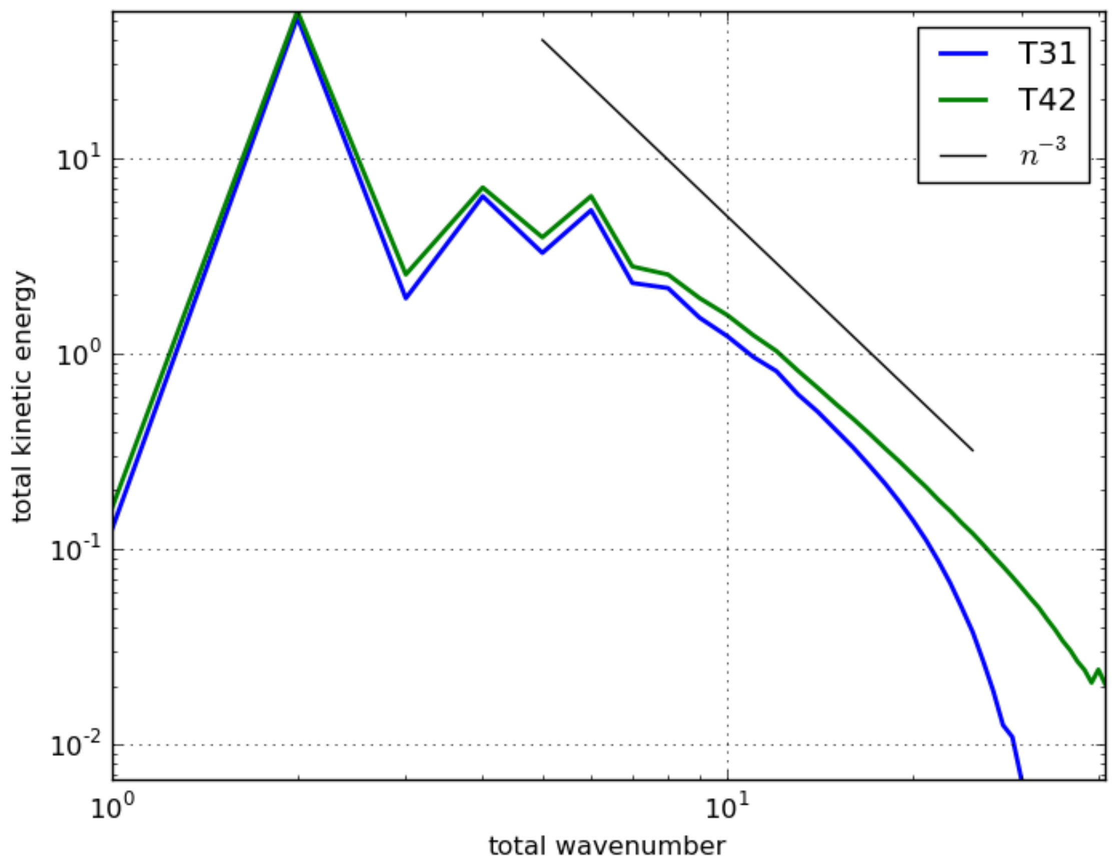
Large ensemble results (Additive + Multiplicative Inflation)

- 200 instead of 20 members, with model error. Min error reduced from 8.7 to 7.7.
- When sampling error is reduced, additive inflation alone outperforms combination of add +mult inflation.
- Suggests that additive inflation is better at capturing model-related errors.



Methods for representing model error *Stochastic Kinetic Energy Backscatter*

- Insufficient resolution causes KE spectra to fall off too rapidly – missing upscale transfer to resolved scales.

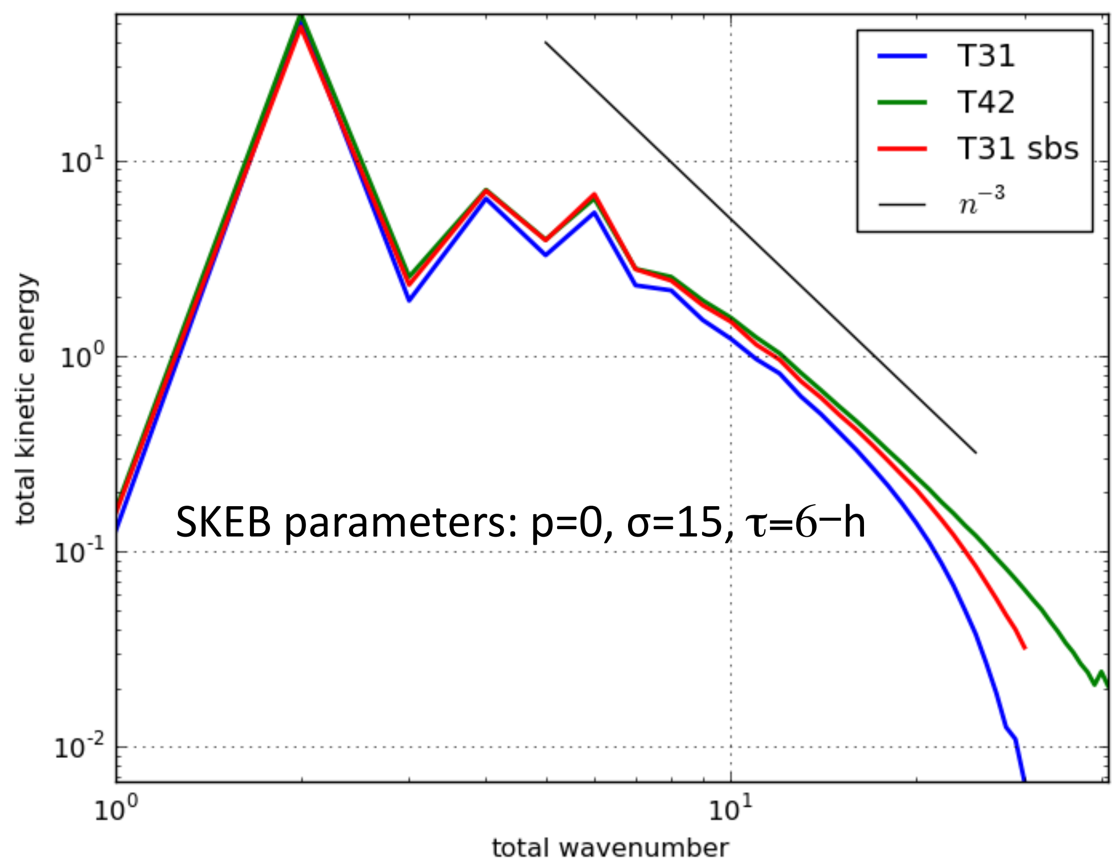


Methods for representing model error
Stochastic Kinetic Energy Backscatter

- Algorithm described in Shutts (2005), Berner et al (2009)
  - Random streamfunction patterns generated by a AR1 process with specified decay timescale (same for all wavenumbers), and variance specified as a function of wavenumber (isotropic spatial correlation).
  - The laplacian of the random pattern multiplied by the hyperdiffusion KE dissipation becomes an extra forcing term in the vorticity equation.
  - Tunable parameters: total variance injected (σ), decay time scale (τ), power law for wavenumber spectrum (p).

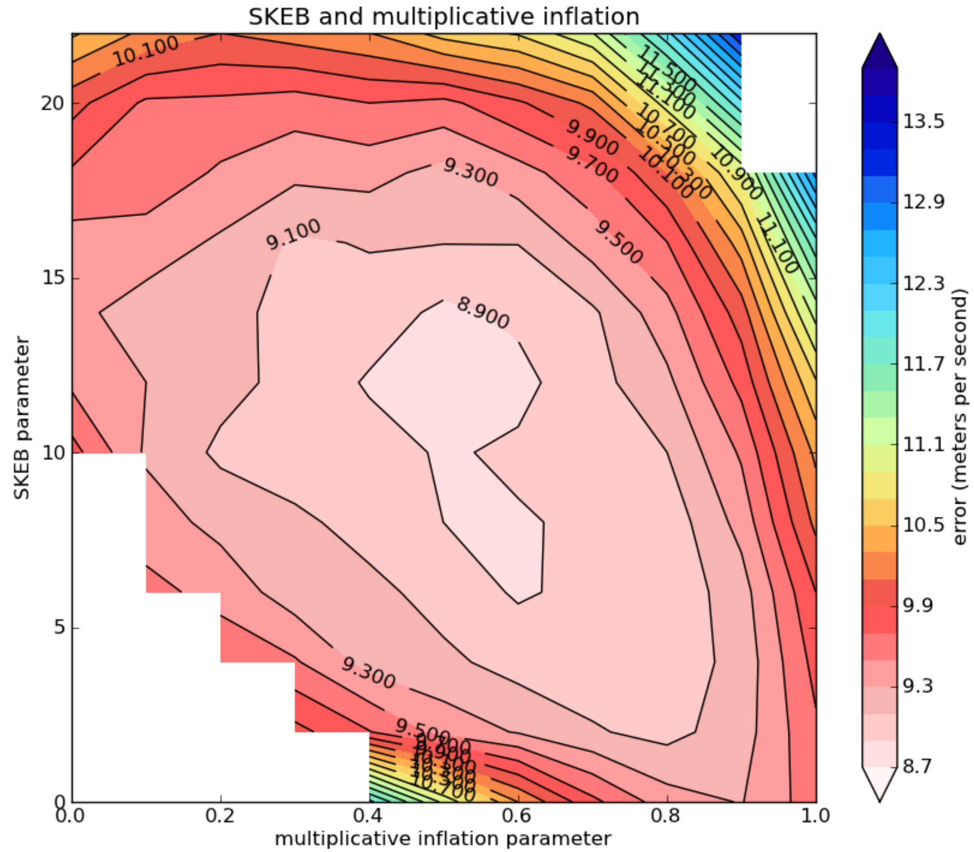
Methods for representing model error
Stochastic Kinetic Energy Backscatter

- Adding SKEB to T31 model makes KE spectrum look similar to T42 model.



Methods for representing model error
Stochastic Kinetic Energy Backscatter

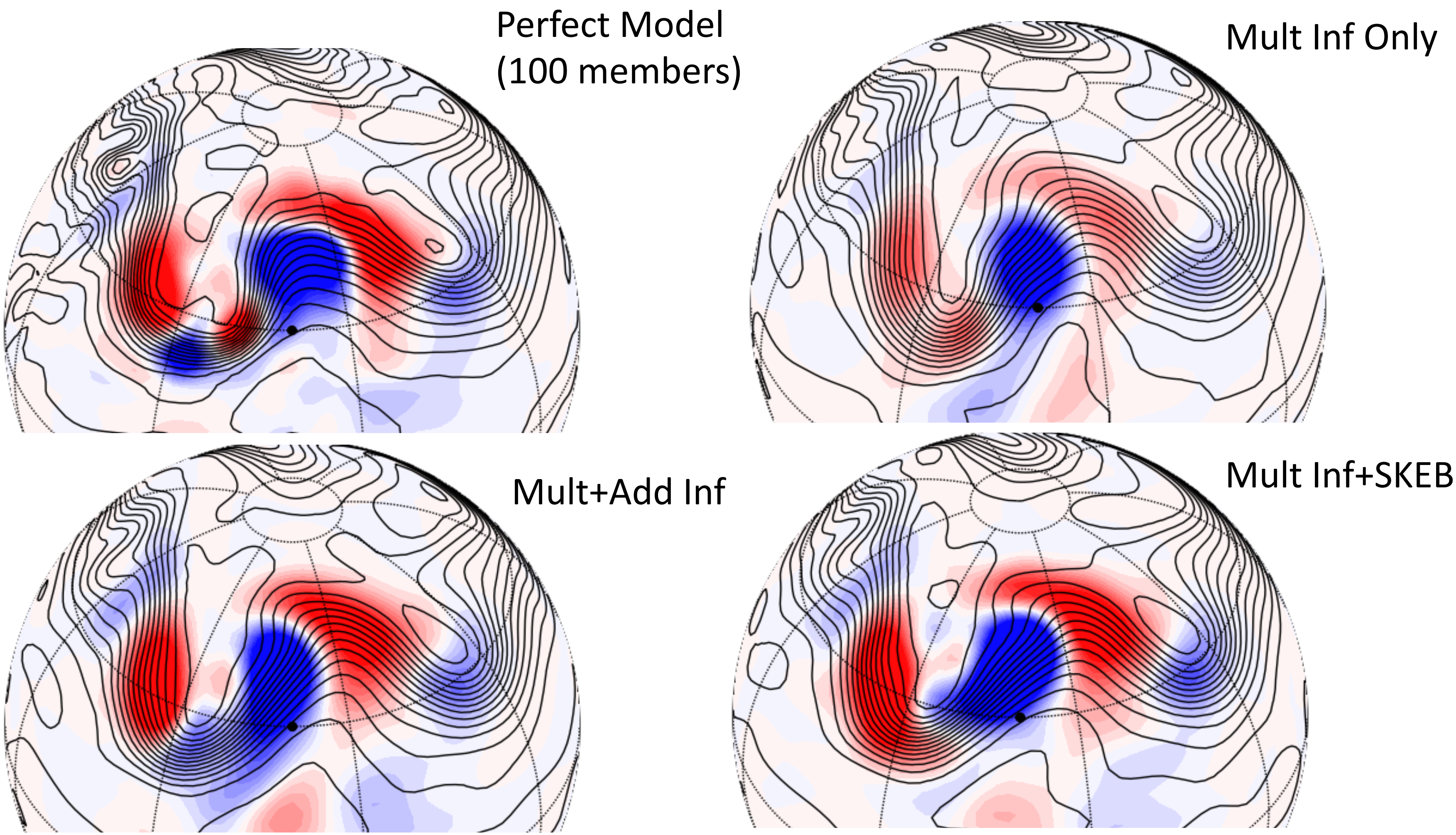
- A combination of SKEB and multiplicative inflation works better than either alone.
- SKEB alone comparable to multiplicative inflation alone (compare values along x and y axes).
- Results are slightly inferior to those obtained using additive + multiplicative inflation.
- y-axis is amplitude of random pattern (σ) – results do not change much if p (power law) or time-scale (τ) are varied.



Single-Ob Increments

Background Mean – solid black, T increment for T ob at black dot – colors

10,000 km localization



Conclusions

- Improving background-error covariances in an EnKF is a tough test for a model error scheme.
- Multiplicative inflation and stochastic physics/additive inflation sample different sources of error in the DA
  - One samples model error, is not sensitive to the observation network.
  - One is sensitive to the observation network, samples other errors in the DA (sampling error, mis-specification of obs error, errors in forward operator etc.)
  - Combination of both works better than either alone (when there are both sources of error).
- Any of these methods can only do so much – improving the forecast model will usually have a larger impact on the data assimilation.

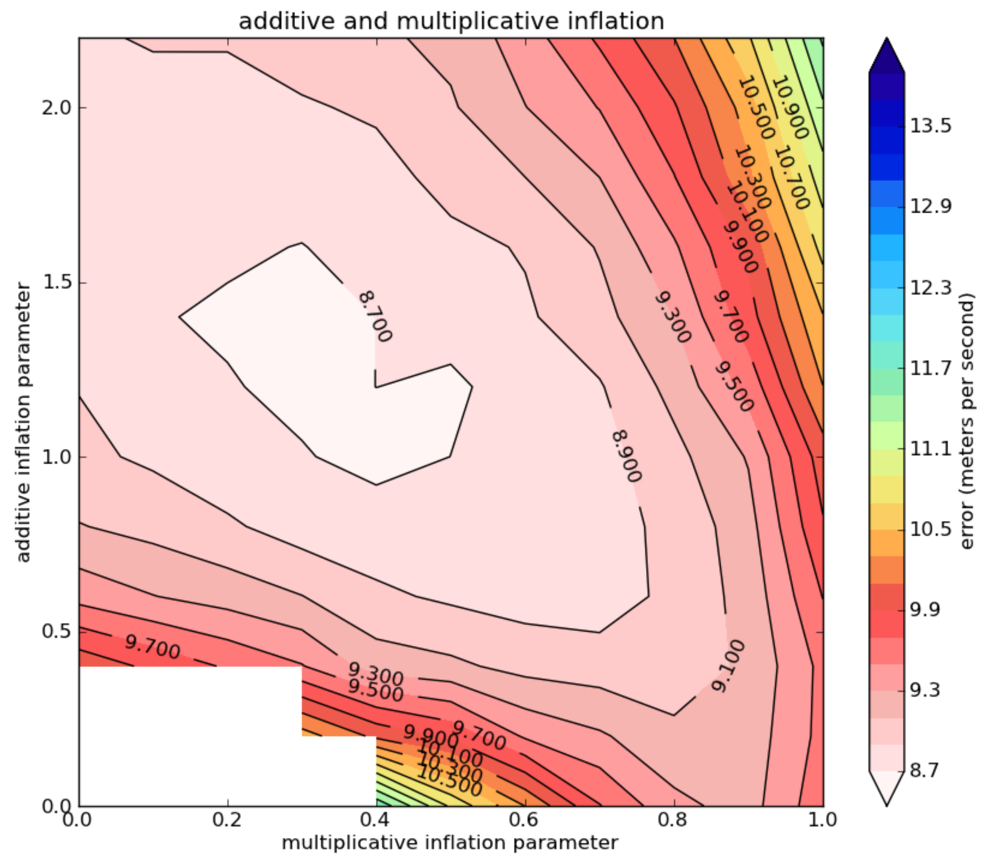
Extra Slides

Reference

- sampling error largest where σ_b/σ_a is large (Sacher and Bartello 2008 MWR, 1640-1654).
- model error is a larger fraction of background error in regions of dense/accurate obs where σ_b/σ_a is large (Daley and Menard 1993 MWR, 1554-1565).
- adaptively estimated inflation looks like σ_b/σ_a (Anderson et al 2009 BAMS, 1283-1296, Fig. 13).
- Additive inflation does most of the work in the Env Canada EnKF (Houtekamer, Mitchell and Deng, MWR July 2009).

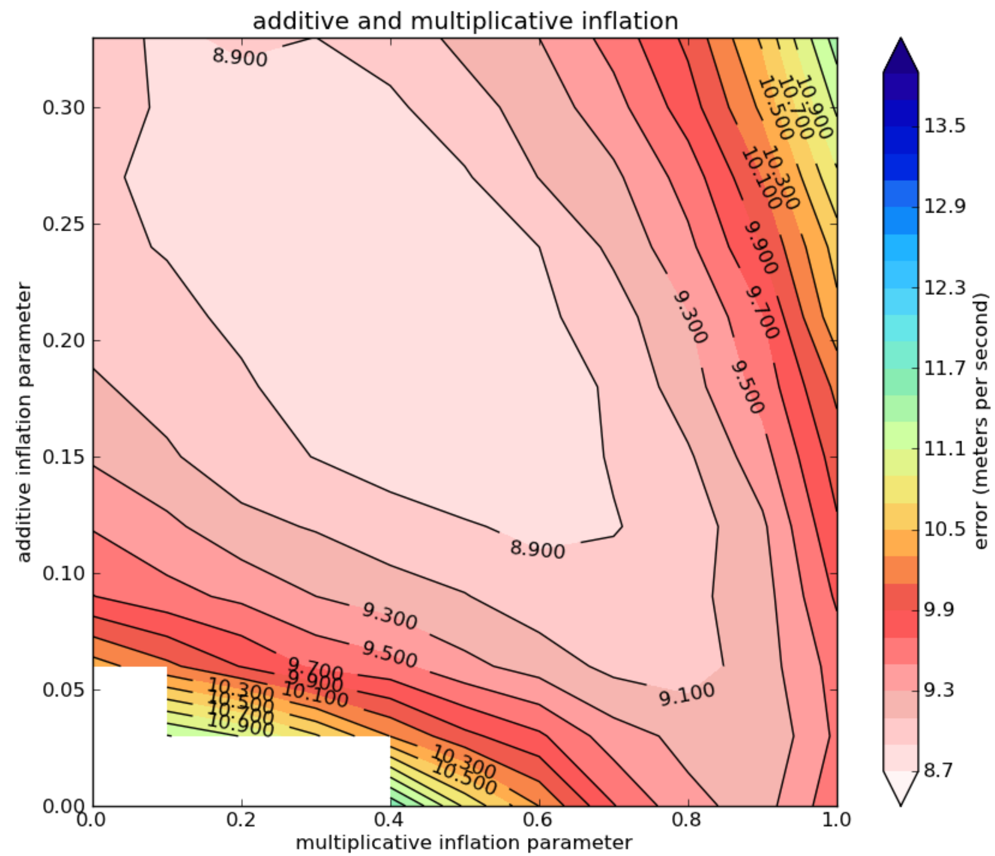
Methods for representing model error
Evolved Additive Inflation

- Adding the additive noise to the previous ensemble mean analysis, evolving forward 12-h, then add the resulting perturbations to the current analysis is an improvement (following Hamill and Whitaker, 2010, MWR, 117-131)
- Flow-dependence gained by "conditioning" the perturbations to the currenty dynamics helps a little.



Methods for representing model error
Evolved Additive Inflation

- Using random samples of actual model error is unrealistic.
- Instead, try random samples of 12-h differences from T31 run.
- Samples are added to the previous analysis mean, then evolved forward 12-h, and recentered around the current analysis mean(following Hamill and Whitaker, 2010, MWR, 117-131)
- Not quite as good as using actual model error, but still an improvement over multiplicative inflation alone.
- Additive inflation alone still better than multiplicative inflation alone (compare values along x and y axes).



Experiences with Env. Canada system
(Houtekamer, Mitchell and Deng, MWR July 2009)

- Most of spread comes from additive inflation.
- Multi-physics adds some variance, esp in lower trop.
- SPPT and SKEB do not provide much variability to background ensemble.
- Are other sources of assimilation error (non-model) dominant, or do stochastic schemes need to be developed/tuned specifically for DA systems?

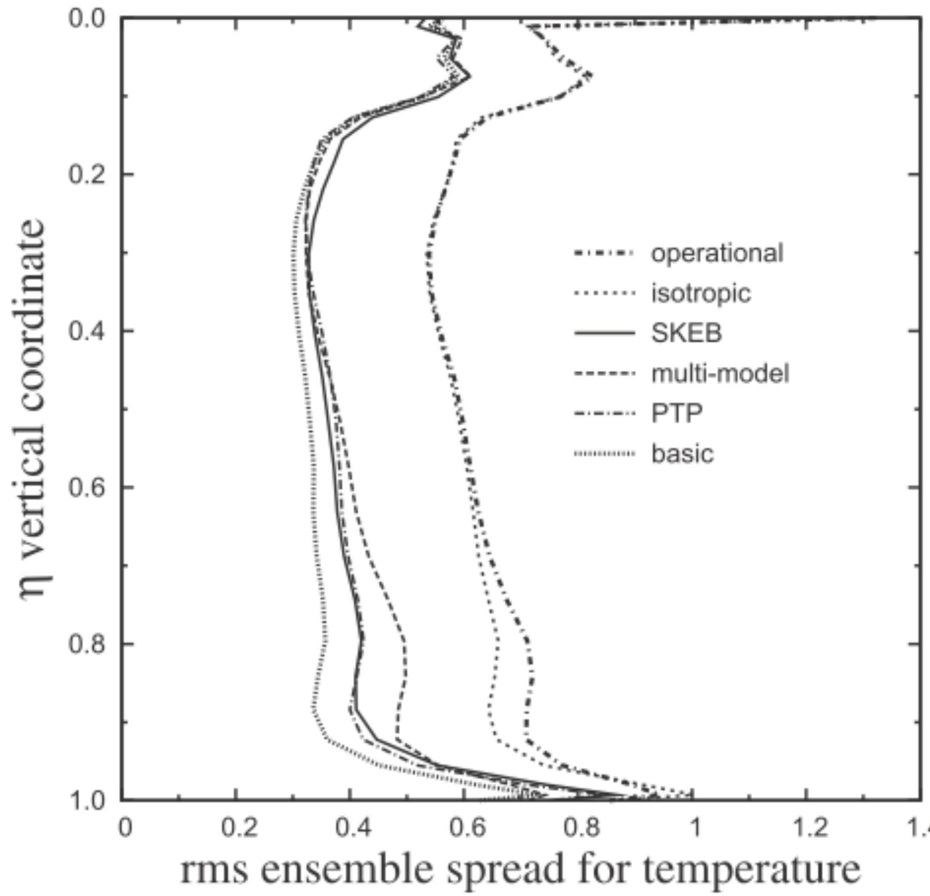


FIG. 10. Vertical profiles of the rms ensemble spread for temperature (K) for different EnKF configurations. The global rms values have been computed for the analyses valid between 0000 UTC 1 Jul 2006 and 1200 UTC 10 Jul 2006.