Unstructured Meshes for Atmospheric Simulations

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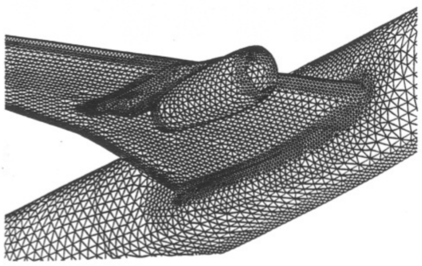
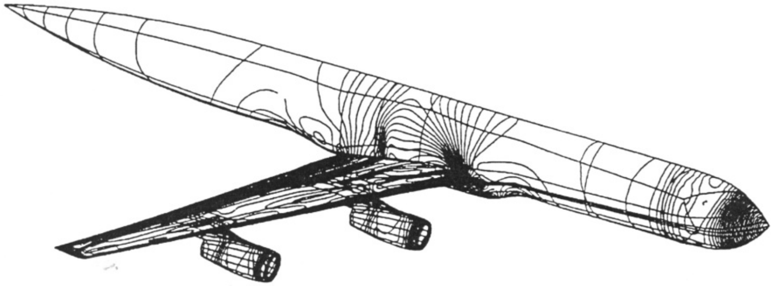
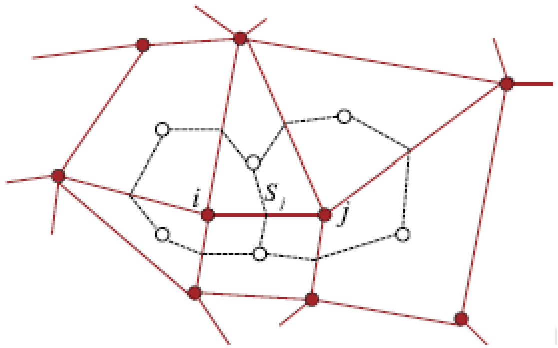
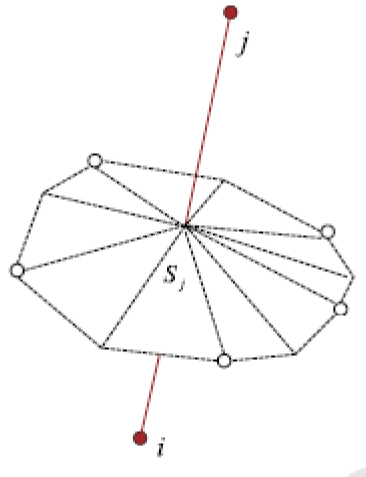


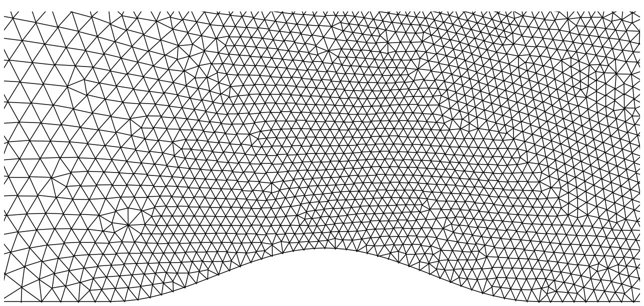


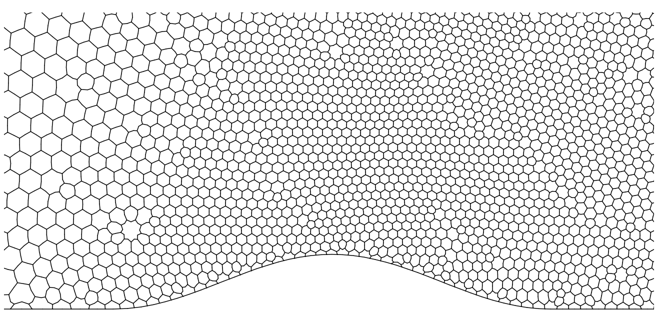
NERC/G004358 award

The Edge Based Finite Volume Discretisation









Edges

Median dual computational mesh
Finite volumes

NFT MPDATA FRAMEWORK

$$\frac{\partial \phi}{\partial t} + \nabla \bullet (\mathbf{V}\phi) = R$$

$$\phi_i^{n+1} = \mathcal{A}_i(\phi^n + 0.5\delta t R,\ \mathbf{V}^{n+1/2}) + 0.5\delta t R^{n+1}$$

Smolarkiewicz 91, Smolarkiewicz & Margolin 93; Mon. Weather Rev.

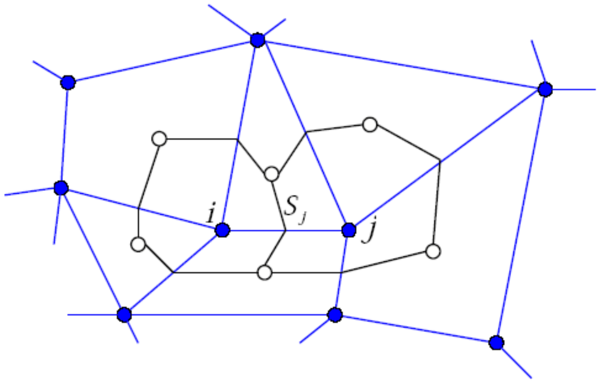
Smolarkiewicz & Szmelter, IJNMF 2008

implicit integration
preconditioned non-symmetric
Krylov-subspace elliptic solver

Advection MPDATA

Notion of MPDATA

Iterative upwind



$$\frac{\partial \Psi}{\partial t} = -\nabla \cdot (\mathbf{v}\Psi)$$

$$\Psi_i^{n+1} = \Psi_i^n - \frac{\delta t}{\mathcal{V}_i} \sum_{j=1}^{l(i)} F_j^{\perp} S_j$$

$$F_j^{\perp} = [v_j^{\perp}]^{+} \Psi_i^n + [v_j^{\perp}]^{-} \Psi_j^n$$

$$[v]^{+} := 0.5(v + |v|) \quad , \quad [v]^{-} := 0.5(v - |v|)$$

FIRST ORDER UPWIND (DONOR CELL)

$$F_j^{\perp} = v_j^{\perp} \Psi|_{s_j}^{n+1/2} + Error \qquad \tilde{v} := -\frac{1}{\Psi} Error$$

compensating velocity

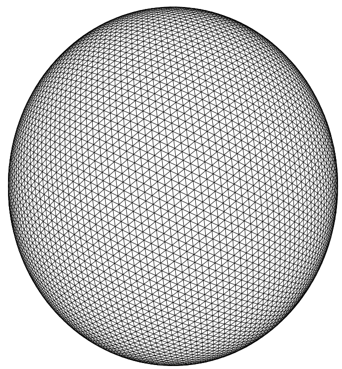
$$Error = -0.5|v_j^{\perp}| \frac{\partial \Psi}{\partial r}|_{s_j}^{*} (r_j - r_i) + 0.5 v_j^{\perp} \frac{\partial \Psi}{\partial r}|_{s_j}^{*} (r_i - 2r_{s_j} + r_j)$$
$$+0.5\delta t v_j^{\perp} (\mathbf{v}\nabla\Psi)|_{s_j}^{*} + 0.5\delta t v_j^{\perp} (\Psi \nabla \cdot \mathbf{v})|_{s_j}^{*} + \mathcal{O}(\delta r^2, \delta t^2, \delta t \delta r)$$

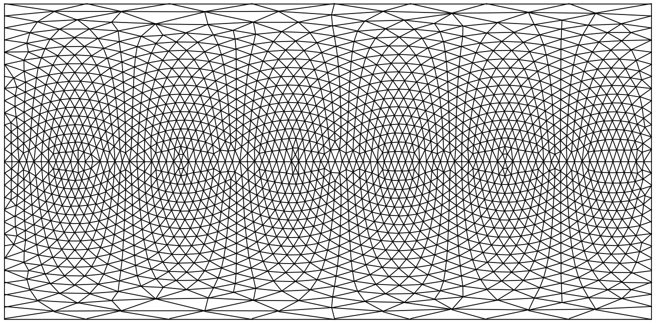
(Smolarkiewicz & Szmelter, J. Comput. Phys. 2005

Geospherical framework

$$\frac{\partial G\Phi}{\partial t} + \nabla \cdot (\mathbf{V}\Phi) = GR$$

$$\Phi_i^{n+1} = \mathcal{A}_i(\Phi^n + 0.5\delta t R^n,\ \mathbf{V}^{n+1/2}, G) + 0.5\delta t R_i^{n+1}$$





(Szmelter & Smolarkiewicz, J. Comput. Phys. 2010)

Loughborough University

A global hydrostatic model

$$\frac{\partial G\Phi}{\partial t} + \nabla \cdot (\mathbf{V}\Phi) = GR$$

$$\Phi_i^{n+1} = \mathcal{A}_i(\Phi^n + 0.5\delta t R^n,\ \mathbf{V}^{n+1/2}, G) + 0.5\delta t R_i^{n+1}$$

$$\frac{\partial G\mathcal{D}}{\partial t} + \nabla \cdot (G\mathbf{v}^*\mathcal{D}) = 0\ ,$$

$$\frac{\partial GQ_x}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_x) = G\left(-\frac{1}{h_x}\mathcal{D}\frac{\partial M}{\partial x} + fQ_y - \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_xQ_y\right)\ ,$$

$$\frac{\partial GQ_y}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_y) = G\left(-\frac{1}{h_y}\mathcal{D}\frac{\partial M}{\partial y} - fQ_x + \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_x^2\right)\ ,$$

$$\frac{\partial M}{\partial \zeta} = \Pi\ .$$

$$\mathcal{D}^{n+1} = \partial p^{n+1}/\partial\zeta \quad \downarrow$$

$$\partial M^{n+1}/\partial\zeta = \Pi^{n+1} \quad \uparrow$$

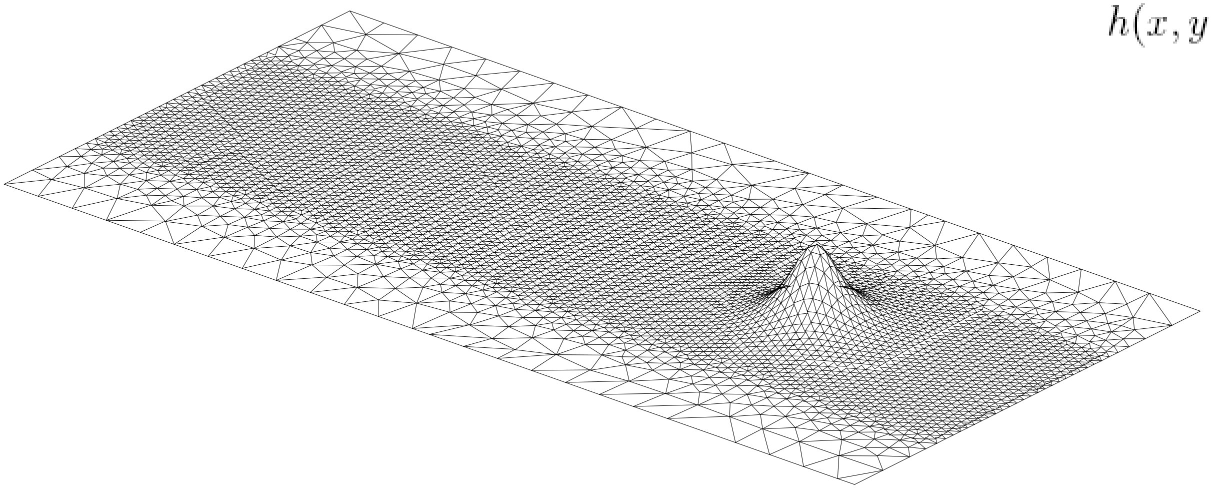
Rotating stratified fluid
An Eulerian-Lagrangian form

Isentropic model

$$\zeta = \theta$$

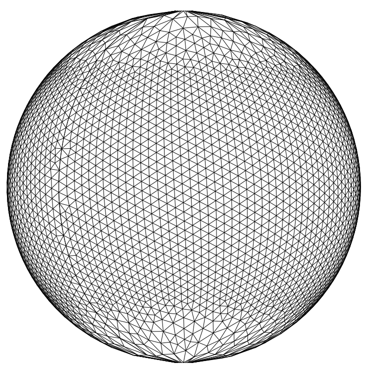
$$\Pi = c_p(p/p_o)^{R_d/c_p}$$

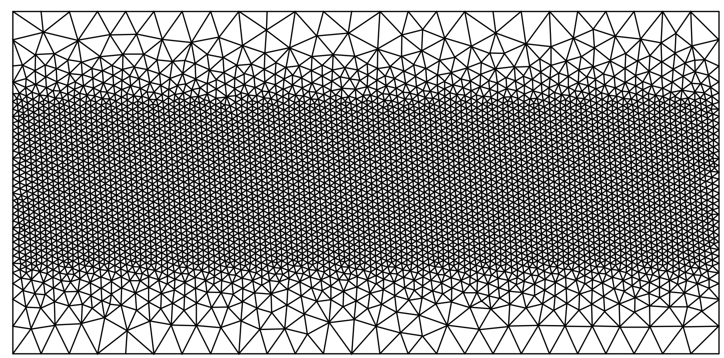
A stratified 3D mesoscale flow past an isolated hill



$$h(x,y) = h_0[1. + (l/\mathcal{L})^2]^{-3/2},$$

(4532 points)

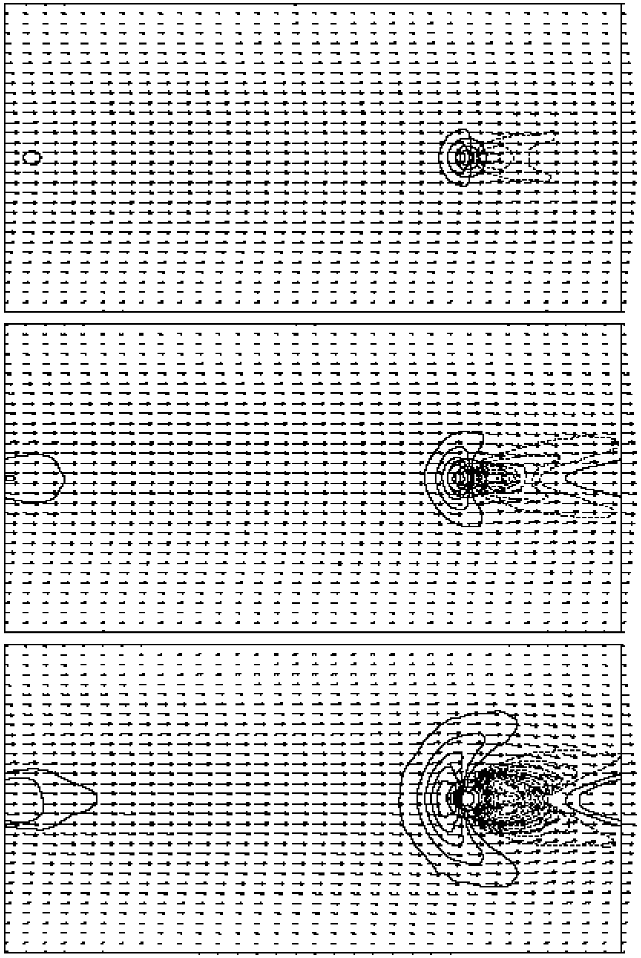


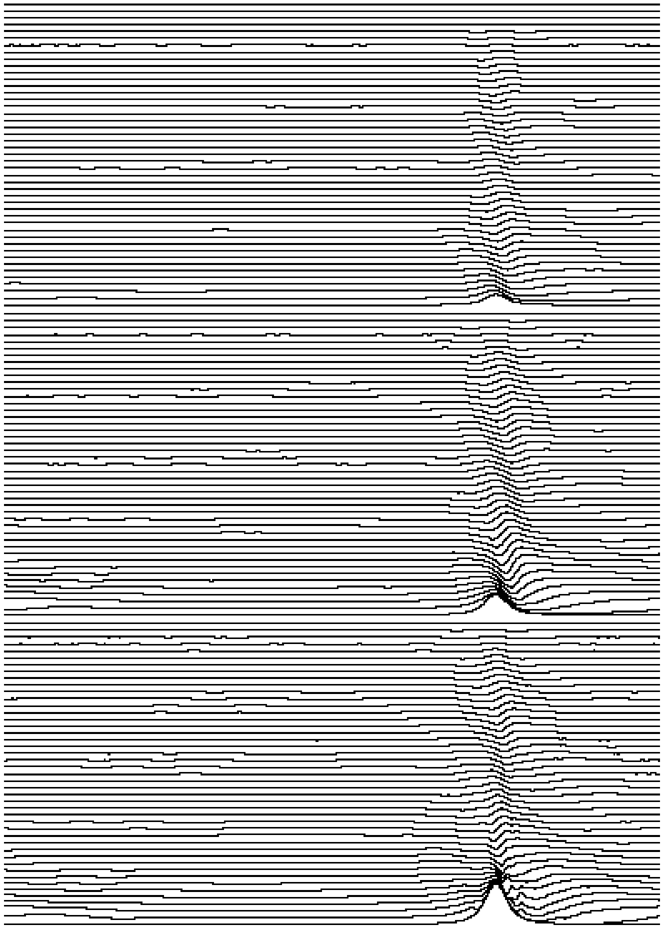


Reduced planets (Wedi & Smolarkiewicz, QJR 2009)

Stratified (mesoscale) flow past an isolated hill on a reduced planet

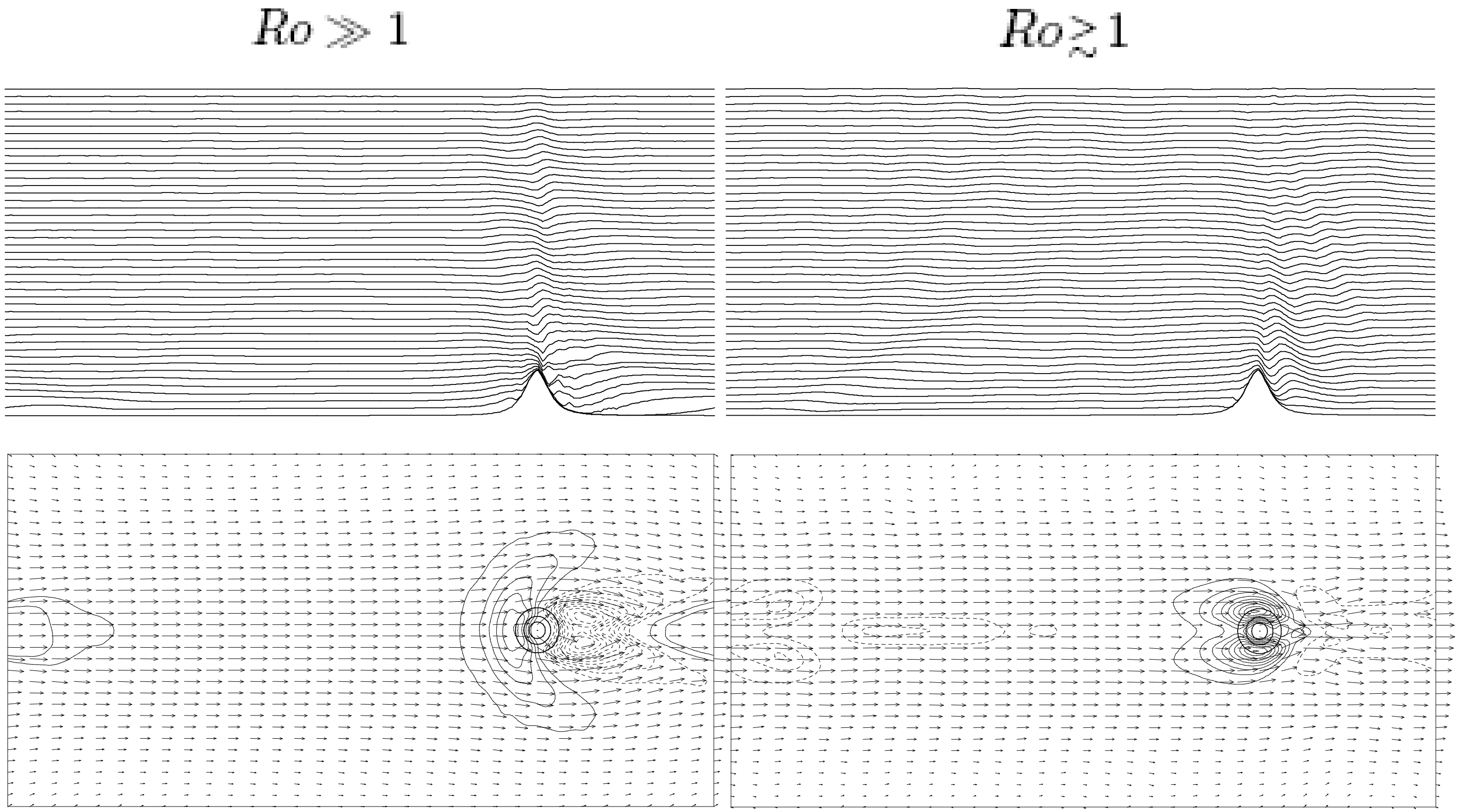
4 hours





Hunt & Snyder J. Fluid Mech. 1980; Smolar. & Rotunno, J. Atmos. Sci. 1989 ; Wedi & Smolar., QJR 2009

Fr=0.5



Smith, Advances in Geophys 1979; Hunt, Olafsson & Bougeault, QJR 2001

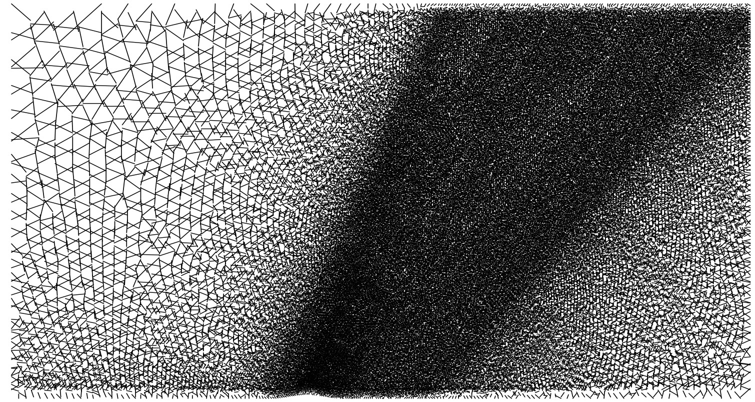
Nonhydrostatic Boussinesq mountain wave

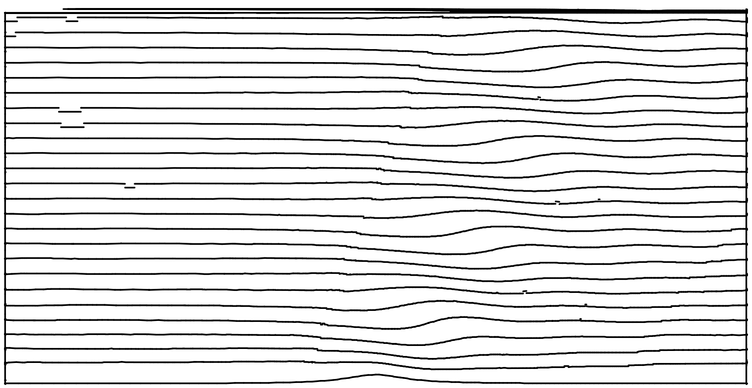
Szmelter & Smolarkiewicz , *Comp. Fluids*, 2011

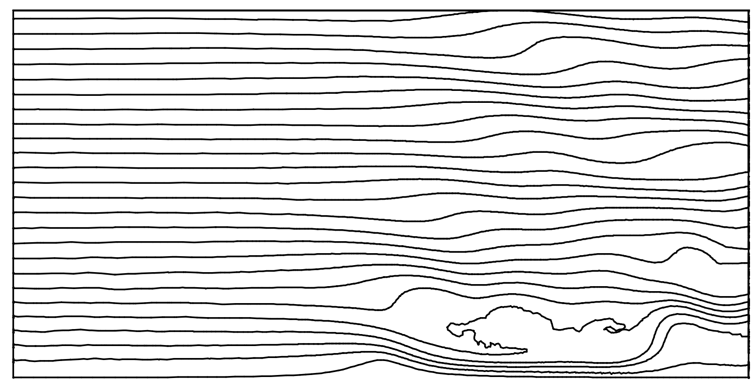
$$\nabla \bullet (\mathbf{V}\rho_o) = 0 \; ,$$

$$\frac{\partial \rho_o V^I}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o V^I) = -\rho_o \frac{\partial \tilde{p}}{\partial x^I} + g\rho_o \frac{\theta'}{\theta_o}\delta_{I2}$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o \theta) = 0 \; .$$







$Fr \lesssim 2$ $\qquad$ $NL/U_o = 2.4$ $\qquad$ $Fr \lesssim 1$

Comparison with the EULAG's (structured mesh) results --- very close with the linear theories (Smith 1979, Durran 2003): over 7 wavelenghts : 3% in wavelength; 8% in propagation angle; wave amplitude loss 7%

$$\frac{\partial \Phi}{\partial t} + \nabla \bullet (\mathbf{V}\Phi) = \mathbf{R}$$

Gravity wave breaking in an isothermal stratosphere

$$\nabla \cdot (\bar{\rho}\mathbf{v}) = 0\,, \quad \frac{D\theta}{Dt} = 0\,, \quad \frac{D\mathbf{v}}{Dt} = -\nabla\Phi' - \mathrm{g}\frac{\theta'}{\bar{\theta}}\,,$$ ***Lipps & Hemler***

$$\nabla \cdot (\bar{\rho}\bar{\theta}\mathbf{v}) = 0\,, \quad \frac{D\theta}{Dt} = 0\,, \quad \frac{D\mathbf{v}}{Dt} = -c_p\theta\nabla\pi' - \mathrm{g}\frac{\theta'}{\bar{\theta}}$$ ***Durran***

$$D\psi/Dt = R$$

by combining $\rho^* \cdot \left(D\psi/Dt = R\right)$ with $\psi \cdot \left(\nabla \rho^* \mathbf{v} = 0\right)$,

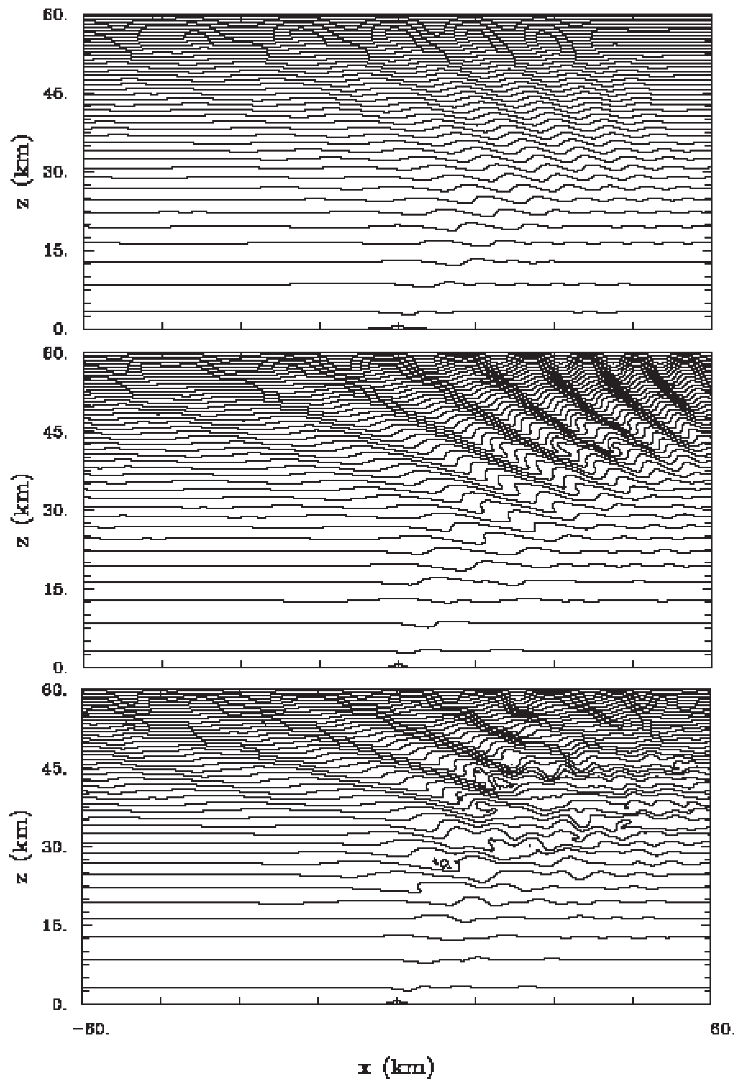
$$\frac{\partial \rho^* \psi}{\partial t} + \nabla \cdot (\rho^* \mathbf{v} \psi) = \rho^* R\,.$$

$$\psi_i^{n+1} = \mathcal{A}_i(\tilde{\psi}, \mathbf{v}^{n+1/2}, \rho^*) + 0.5\delta t R_i^{n+1}$$

$$S_\theta = d \ln \bar{\theta}/dz = 4.4 \cdot 10^{-5}\ \mathrm{m}^{-1}$$

$$\mathbf{v}_e = (u_e, 0) \quad u_e = U = 20\mathrm{ms}^{-1}$$

Isentropes at $t = 60, 90$, and 120 min.

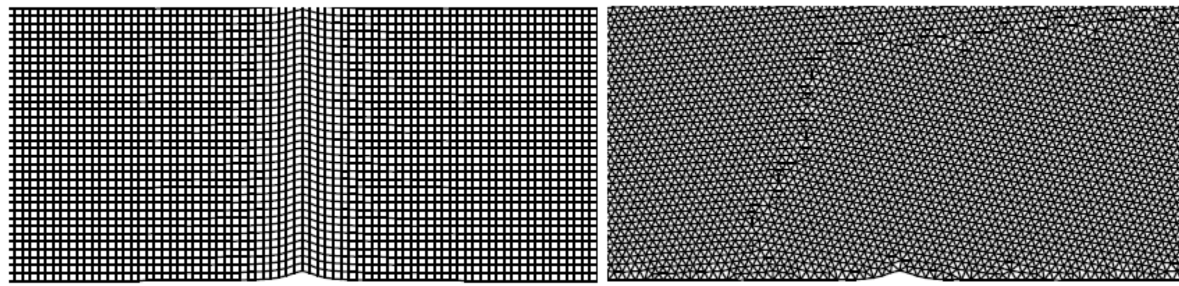


(Prusa et al JAS 1996, Smolarkiewicz & Margolin, Atmos. Ocean 1997 Klein, Ann. Rev. Fluid Dyn., 2010, Smolarkiewicz et al Acta Geoph 2011)

Smolarkiewicz et al
Acta Geoph 2011

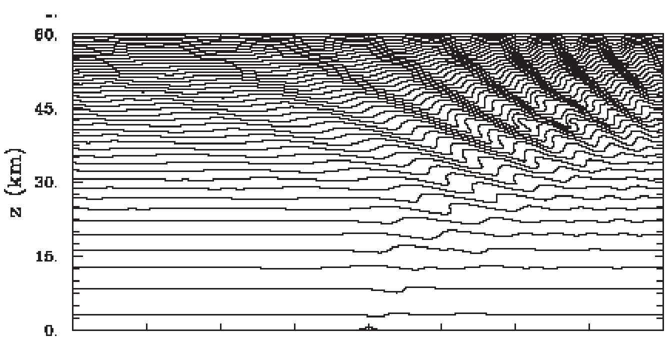
Gravity wave breaking in an isothermal stratosphere

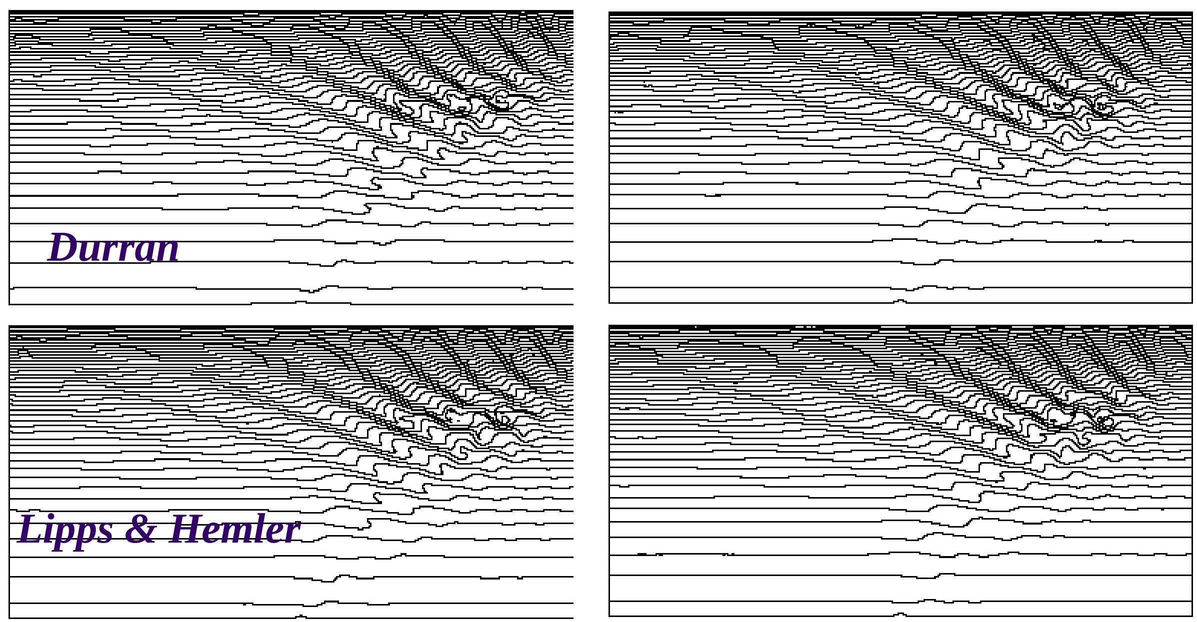
Nonhydristatic Edge-Based NFT



Isentropes at $t = 90$ min

EULAG   CV/GRID





Mesh adaptivity with MPDATA based error indicator

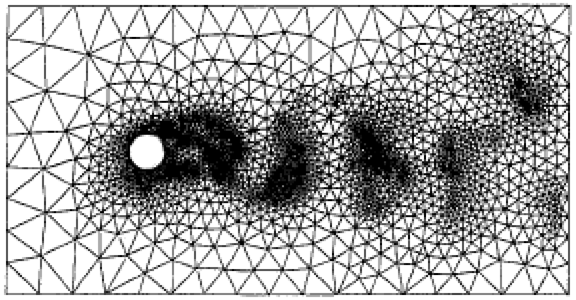
$$F_j^{\perp} = v_j^{\perp}\Psi|_{s_j}^{n+1/2} + Error \qquad \tilde{v} := -\frac{1}{\Psi}Error$$

compensating velocity

$$Error = -0.5|v_j^{\perp}|\frac{\partial\Psi}{\partial r}|_{s_j}^{*}(r_j - r_i) + 0.5v_j^{\perp}\frac{\partial\Psi}{\partial r}|_{s_j}^{*}(r_i - 2r_{s_j} + r_j)$$
$$+0.5\delta t v_j^{\perp}(\mathbf{v}\nabla\Psi)|_{s_j}^{*} + 0.5\delta t v_j^{\perp}(\Psi\nabla\cdot\mathbf{v})|_{s_j}^{*} + \mathcal{O}(\delta r^2, \delta t^2, \delta t\delta r)$$

$$(h_e)^{\text{new}} = h_e/\xi_e^{1/p} \qquad \xi_e = \frac{\| e \|_e}{\bar{e}_m}, \qquad \bar{h}_{\min} \leq h_e \leq \bar{h}_{\max}.$$

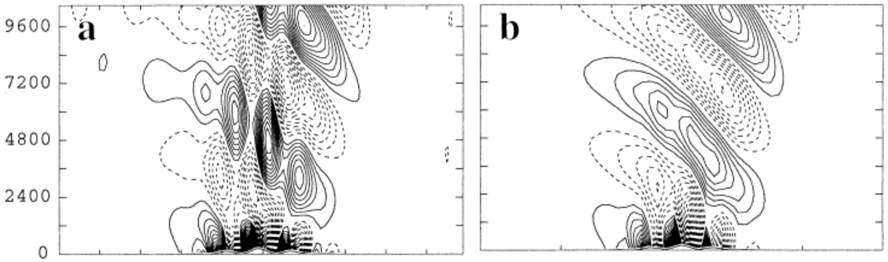
Wu, Zhu, Szmelter & Zienkiewicz , Comp Mech 1990

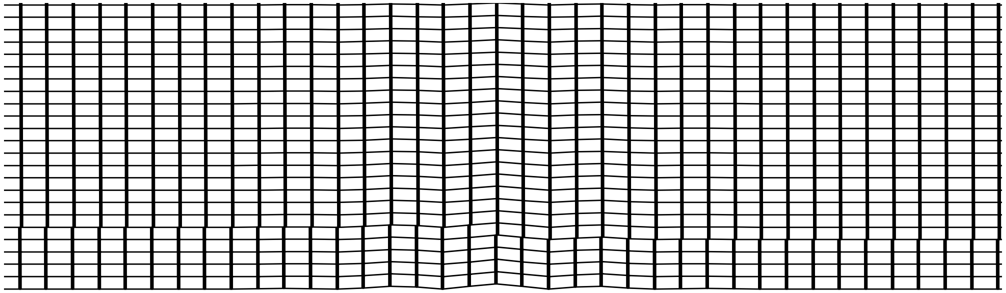


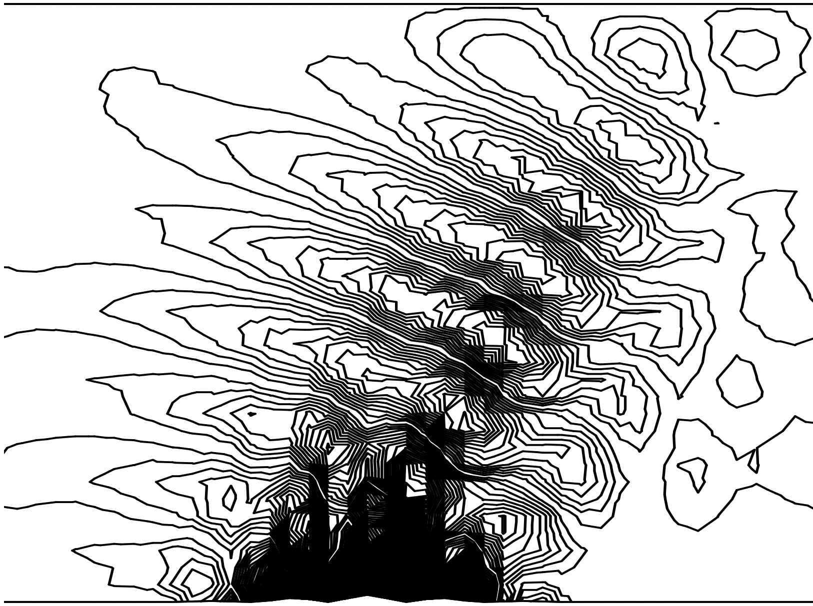
Static mesh adaptivity with MPDATA based error indicator

Schär Mon. Wea. Rev. 2002

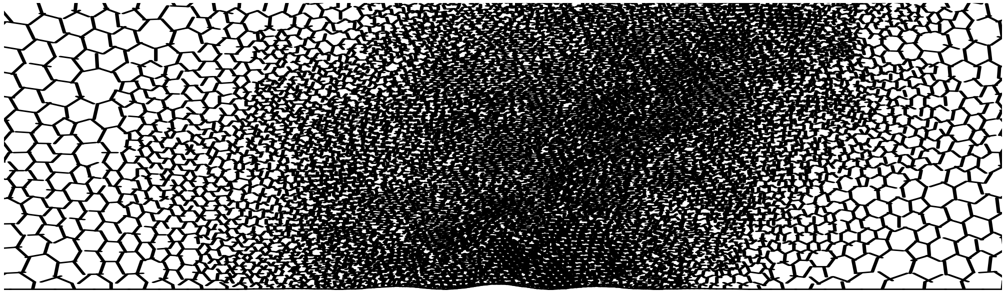
(Recommended mesh ca10000 points)

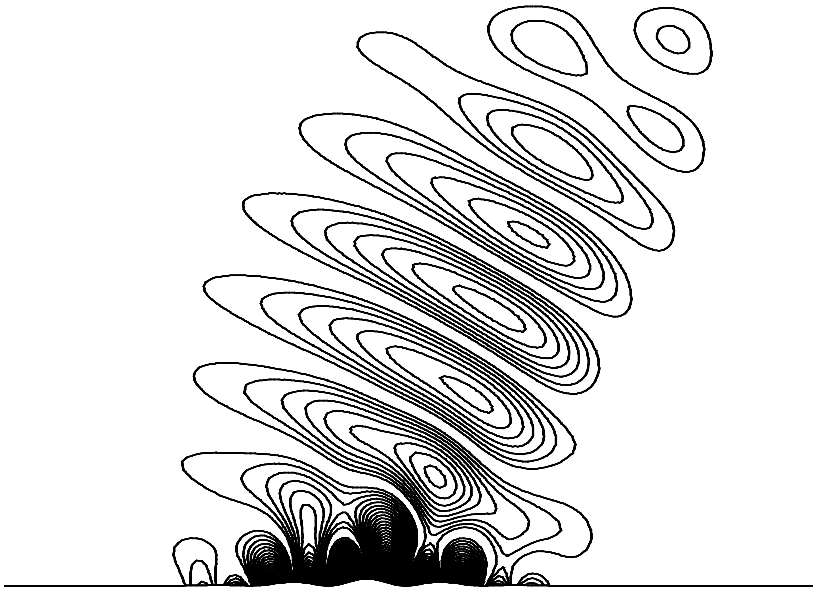






Coarse initial mesh 80x45 =3600 points and solution

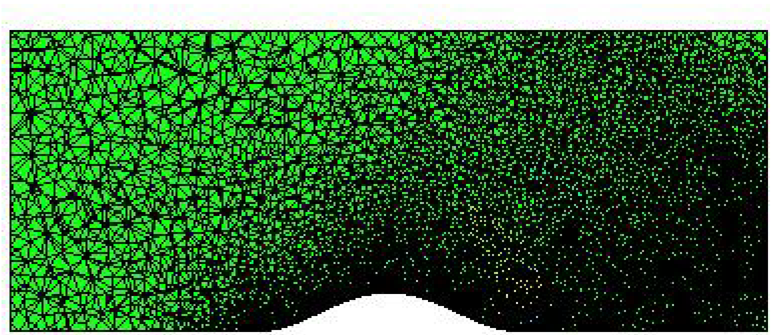


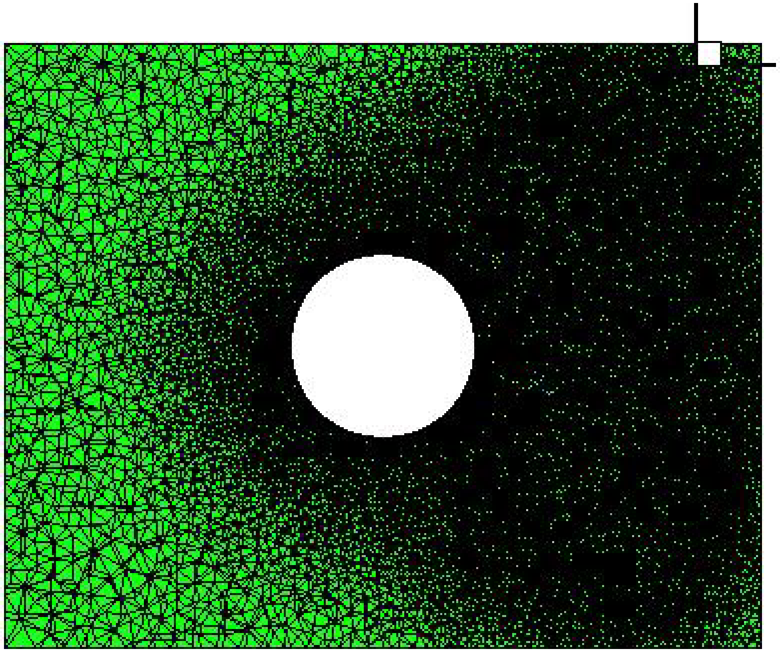


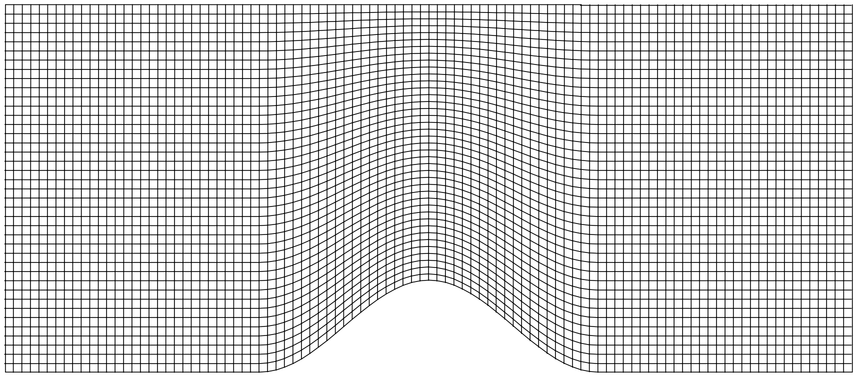
Adapted mesh 8662 points and solution

Stratified flow past a steep isolated hill

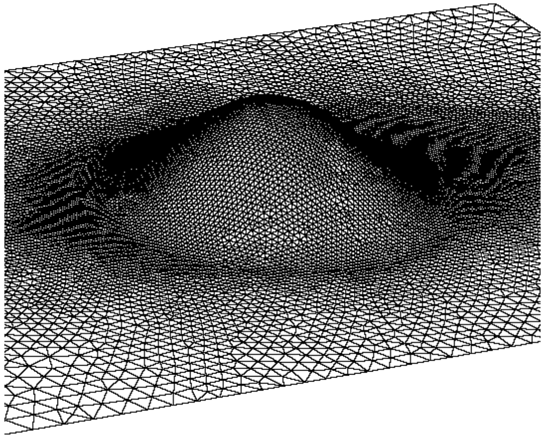
1121192 Cartesian dx=100
692533 Distorted prisms dx=100-400
441645 tetra dx=50 -450

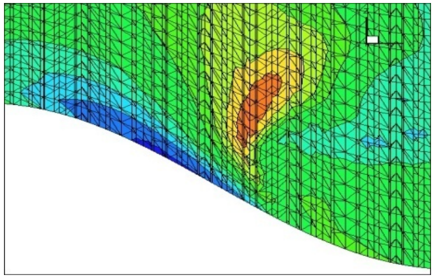






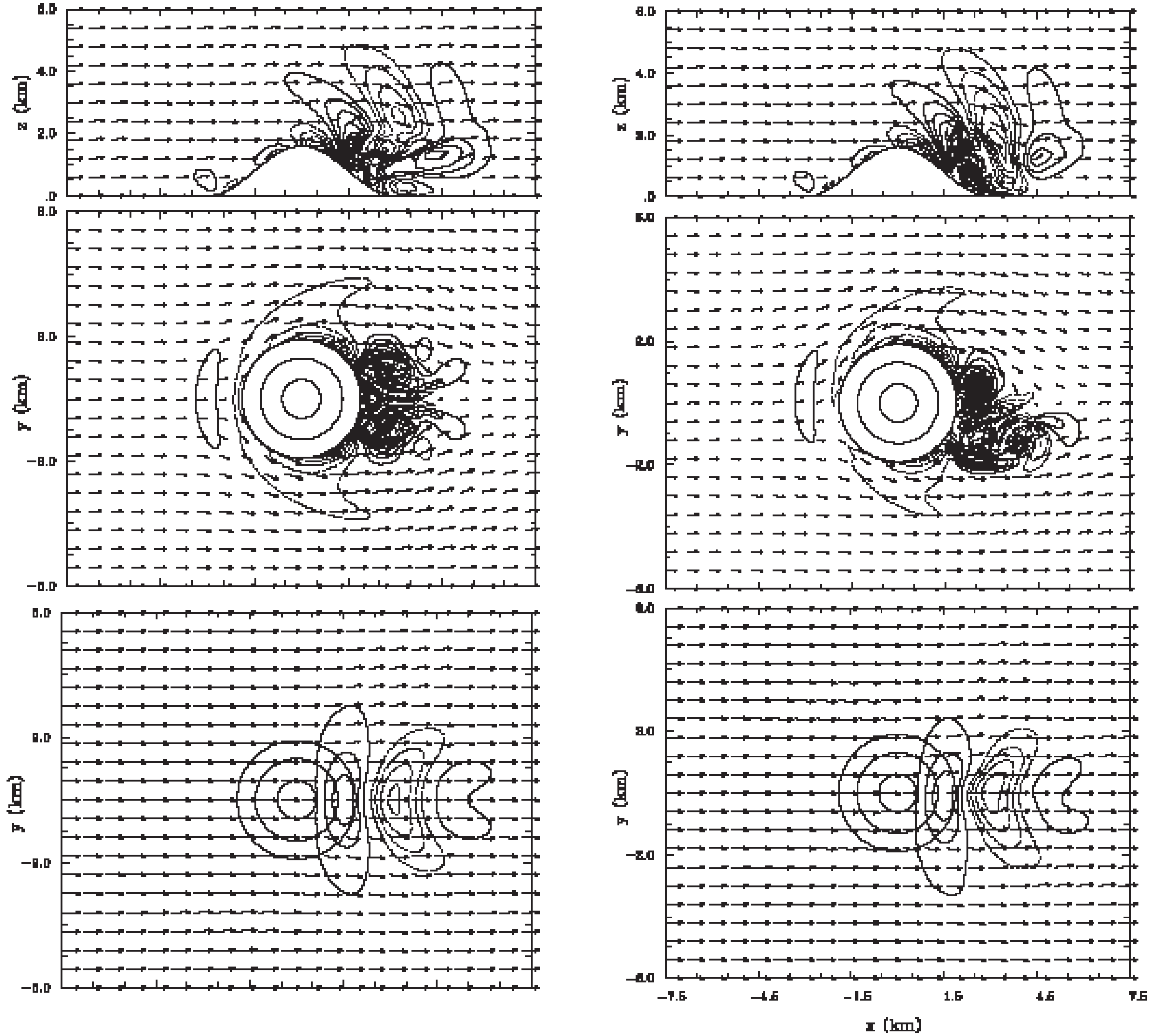
$$z_{i,k} = (k-1)\delta z \left(1 - \frac{h_i}{H}\right) + h_i \ ,$$





Stratified flow past a steep isolated hill

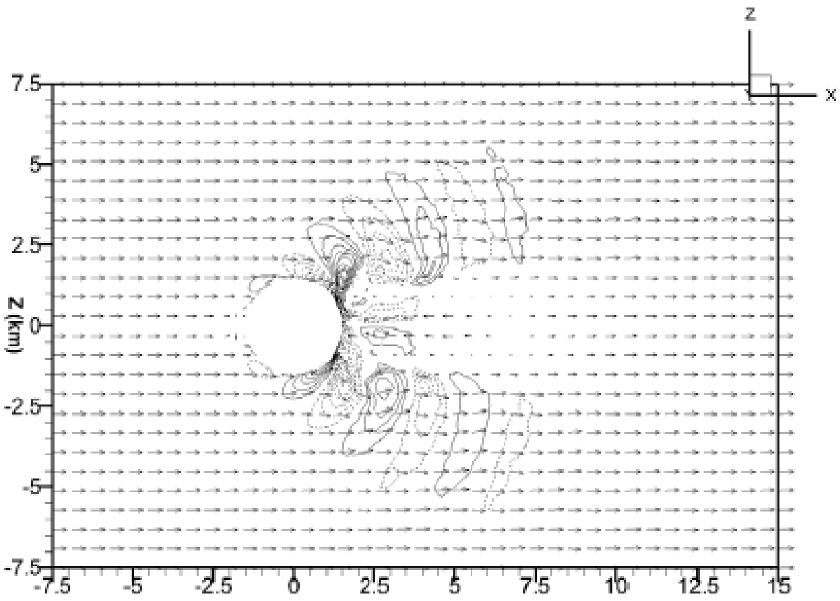
Hunt & Snyder JFM 1980
Smolarkiewicz & Rotuno JAS 1989

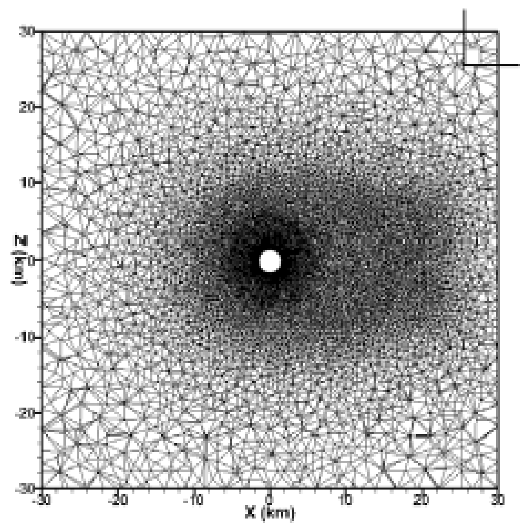


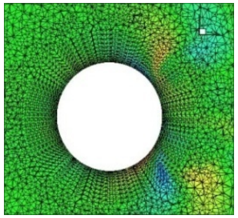
$Fr = 1/3, \ Ro \nearrow \infty$ $\qquad$ $Fr = 1/3, \ Ro \approx 3$

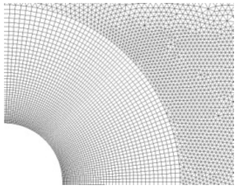
Hanazaki , JFM 1989

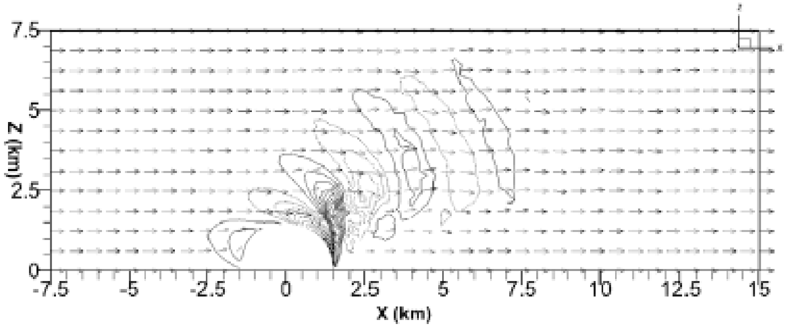
Low Froude Number Fr=0.3 Flow Past a Sphere and Hemi-Sphere

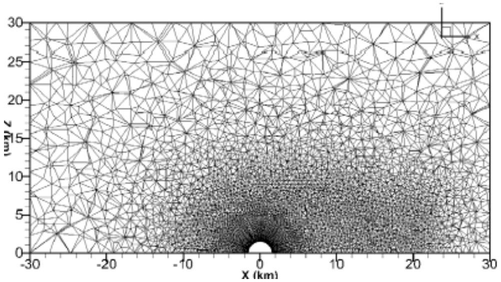










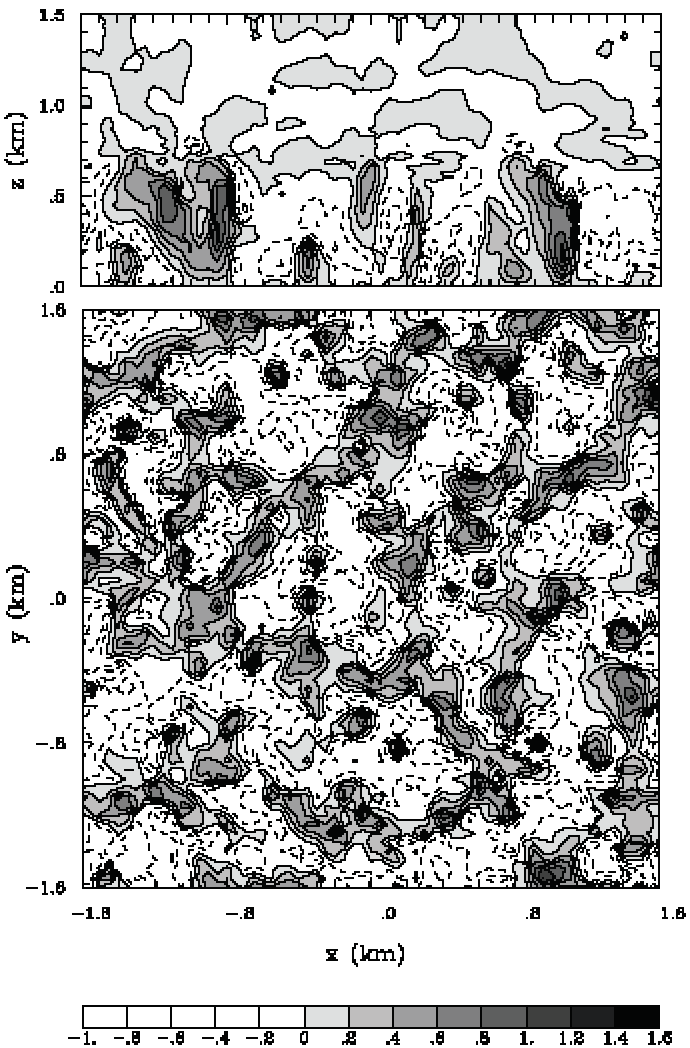


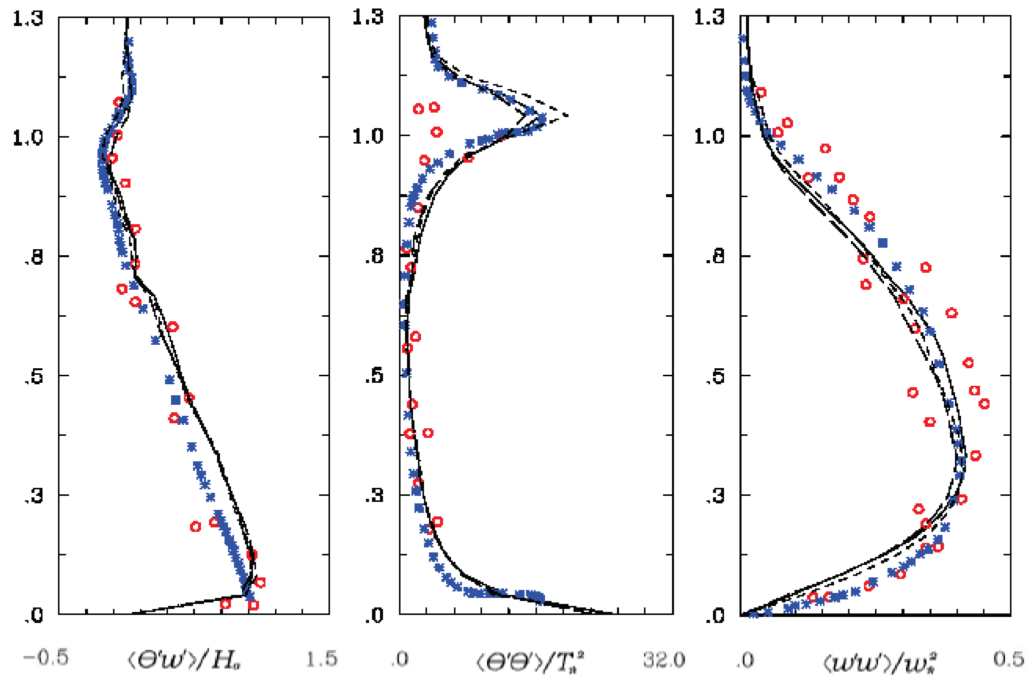
Hybrid mesh

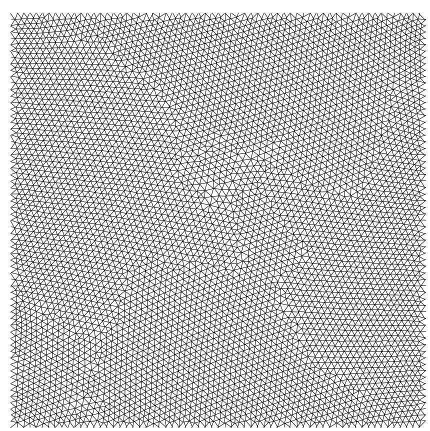
Convective Planetary Boundary Layer

Schmidt & Schumann JFM 1989

Smolarkiewicz et al JCP 2013







Edge-based T

Edge-based C

EULAG

SS LES

Observation

64x64x51

Conclusions:

• Unstructured-mesh discretization sustains the accuracy of structured-grid discretization and offers full flexibility in spatial resolution.

• NFT MPDATA solvers proved to provide a convenient general framework for atmospheric model development.

•

• It appears that future atmospheric models will likely blend various equations and numerical methods. Flexible meshes combined with a differential manifolds formulation are well suited for this purpose.