A finite-volume module for cloud-resolving simulations of global atmospheric flows

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FVM extended to moist dynamics (with focus on coupling) ➔ synthesis of "dry" ***FVM*** [Piotr Smolarkiewicz, Willem Deconinck, Mats Hamrud, George Mozdzynski, Christian Kühnlein, Joanna Szmelter, Nils Wedi (*JCP*, 2016)] and "moist" schemes advanced in [Wojciech Grabowski & Piotr Smolarkiewicz (*MWR*, 1990, 1996, 2002)]

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Governing (compressible Euler) equations, a physically intuitive form:

$$\frac{d\varrho}{dt} = -\varrho\nabla\cdot\mathbf{u} = 0 \ ,$$

$$\frac{d\mathbf{u}}{dt} = -\frac{\theta(1+q_v/\epsilon)}{\theta_0(1+q_t)}\nabla\varphi - \frac{\mathbf{g}}{\theta_a}\Big(\theta' + \theta_a(\varepsilon q_v' - q_c - q_p)\Big) + \boldsymbol{\mathcal{D}} \ ,$$

$$\frac{d\theta'}{dt} = -\mathbf{u}\cdot\nabla\theta_a + \frac{L\theta}{c_pT}\Big(C_d + E_p\Big) + \mathcal{H} \ ,$$

$$\frac{dq_v}{dt} = -C_d - E_p + D^{q_v} \ ,$$

$$\frac{dq_c}{dt} = C_d - A_p - C_p + D^{q_c} \ ,$$

$$\frac{dq_p}{dt} = A_p + C_p + E_p + D^{q_p} - \frac{1}{\varrho}\nabla\cdot(\varrho\mathbf{u}_{\downarrow}q_p) \ .$$

$$\varphi = c_p\theta_0\left[\left(\frac{R_d}{p_0}\varrho\theta(1+q_v/\epsilon)\right)^{R_d/c_v} - \pi_a\right] \rightarrow$$

$$\frac{d\varphi}{dt} = \xi\phi\nabla\cdot\mathbf{u} - \mathbf{u}\cdot\nabla\phi_a + \xi\phi\left(\frac{\mathcal{R}^\theta}{\theta} + \frac{\mathcal{R}^{q_v}/\epsilon}{1+q_v/\epsilon}\right)$$

$$\mathcal{R}^\theta \equiv \mathrm{RHS(1c)} + \mathbf{u}\cdot\nabla\theta_a$$

notes:

$$q_t := q_v + q_c + q_p \rightarrow \frac{dq_t}{dt} = -\frac{1}{\varrho}\nabla\cdot(\varrho\mathbf{u}_{\downarrow}q_p) + D^{q_t}$$

$$q_l := q_c + q_p \rightarrow \frac{dq_l}{dt} = (C_d + E_p) - \frac{1}{\varrho}\nabla\cdot(\varrho\mathbf{u}_{\downarrow}q_p) + D^{q_l}$$

$$q_t \equiv 0 \rightarrow q_v = q_c = q_p \equiv 0$$

recovers dry dynamics

$$\frac{\theta(1+q_v/\epsilon)}{\theta_0(1+q_t)} = \frac{\theta_\varrho}{\theta_0} := \Theta_\varrho \rightarrow \mathbf{g}(1-\theta_\varrho/\theta_{\varrho a}) \approx \frac{\mathbf{g}}{\theta_a}\left(\theta' + \theta_a(\varepsilon q_v' - q_c - q_p)\right)$$

$$\frac{\theta}{\theta_{\varrho a}} \approx \frac{\theta}{\theta_a} \Longrightarrow \theta' = \theta - \theta_a \text{ while } q_v' = q_v - \frac{\theta}{\theta_a}q_{va}$$

$$0 = -c_p\theta_{\varrho a}\widetilde{\mathbf{G}}\nabla\pi_a - \mathbf{g} - \mathbf{f}\times\mathbf{u}_a + \mathcal{M}(\mathbf{u}_a)$$

$$q_c > 0 \Longrightarrow q_v = q_{vs}$$

$$q_v < q_{vs} \Longrightarrow q_c = 0$$

$$q_{vs} = \frac{\epsilon e_s}{p - e_s} \qquad e_s(T) = e_o\, exp\left[\frac{L}{R_v}\left(\frac{1}{T_o} - \frac{1}{T}\right)\right]$$

Governing equations, actually solved conservation form:

$$\frac{\partial \mathcal{G}\varrho}{\partial t} + \nabla\cdot\left(\mathcal{G}\varrho\mathbf{v}\right) = 0 \ ,$$

$$\frac{\partial \mathcal{G}\varrho\mathbf{u}}{\partial t} + \nabla\cdot\left(\mathcal{G}\varrho\mathbf{v}\otimes\mathbf{u}\right) = \mathcal{G}\varrho\Big(-\Theta_\varrho\widetilde{\mathbf{G}}\nabla\varphi - \frac{\mathbf{g}}{\theta_a}\big(\theta' + \theta_a(\varepsilon q'_v - q_c - q_p)\big) - \mathbf{f}\times\mathbf{u}' + \mathcal{M}' + \boldsymbol{\mathcal{D}}\Big) \ ,$$

$$\frac{\partial \mathcal{G}\varrho\theta'}{\partial t} + \nabla\cdot\left(\mathcal{G}\varrho\mathbf{v}\theta'\right) = \mathcal{G}\varrho\Big(-\widetilde{\mathbf{G}}^T\mathbf{u}\cdot\nabla\theta_a + \frac{L\theta}{c_pT}\big(C_d + E_p\big) + \mathcal{H}\Big) \ ,$$

$$\frac{\partial \mathcal{G}\varrho q_{...}}{\partial t} + \nabla\cdot\left(\mathcal{G}\varrho\mathbf{v}q_{...}\right) = \mathcal{G}\varrho\left(\mathcal{R}^{q_{...}}\right) \ .$$

bulk microphysics

$$\frac{\partial \mathcal{G}\varrho\varphi}{\partial t} + \nabla\cdot(\mathcal{G}\varrho\mathbf{v}\varphi) = \mathcal{G}\varrho\mathcal{R}^\varphi$$

$$\mathcal{R}^\varphi = -\xi\phi\frac{1}{\mathcal{G}}\nabla\cdot(\mathcal{G}\mathbf{v}) - \frac{1}{\mathcal{G}\varrho}\nabla\cdot(\mathcal{G}\varrho\mathbf{v}\phi_a) + \phi_a\frac{1}{\mathcal{G}\varrho}\nabla\cdot(\mathcal{G}\varrho\mathbf{v}) + \xi\phi\Pi$$

$$\Pi \equiv \left(\frac{\mathcal{R}^\theta}{\theta} + \frac{\mathcal{R}^{q_v}/\epsilon}{1+q_v/\epsilon}\right)$$

conservativeness, relative simplicity ,
Eulerian/Lagrangian congruence

$$\frac{d\mathbf{\Phi}}{dt} = \boldsymbol{\mathcal{R}} \equiv \boldsymbol{\mathcal{R}}_c + \boldsymbol{\mathcal{R}}_p$$

$$\mathbf{\Phi} = \begin{bmatrix} \theta \\ q_v \\ q_c \\ q_p \end{bmatrix} ; \quad \boldsymbol{\mathcal{R}}_c = \begin{bmatrix} \frac{L\theta}{c_p T} C_d \\ -C_d \\ C_d \\ 0 \end{bmatrix} ; \quad \boldsymbol{\mathcal{R}}_p = \begin{bmatrix} \frac{L\theta}{c_p T} E_p \\ -E_p \\ -A_p - C_p \\ A_p + C_p + E_p + F_p \end{bmatrix} \quad F_p := -\frac{1}{\varrho}\nabla \cdot (\varrho \mathbf{u}_{\downarrow} q_p)$$

$$\mathbf{\Phi}(\mathbf{x}, t) = \mathbf{\Phi}(\mathbf{x}_o, t_o) + \int_T (\boldsymbol{\mathcal{R}}_c + \boldsymbol{\mathcal{R}}_p)\ d\tau \quad \rightarrow \quad \mathbf{\Phi}^{n+1} = \left(\mathbf{\Phi} + 0.5\delta t \boldsymbol{\mathcal{R}}_c\right)_o + 0.5\delta t \boldsymbol{\mathcal{R}}_c^{n+1} + \delta t \langle \boldsymbol{\mathcal{R}}_p \rangle_n^{n+1}$$

e.g. $(A_p + C_p)^{n+1} = -\mathcal{R}_p^{q_c}|^{n+1}$ ***and*** $E_p^{n+1} = -\mathcal{R}_p^{q_v}|^{n+1}$ $\rightarrow$ $\mathcal{O}(\delta t)$ estimate

$$\tilde{q_p}^{n+1} = (q_p + \delta t \mathcal{R}^{q_p})_o - \delta t(\widetilde{\mathcal{R}_p^{q_c}} + \widetilde{\mathcal{R}_p^{q_v}})^{n+1}$$

$$(q_c + \delta t R^{q_c})_o + \delta t \mathcal{R}_p^{q_c}|^{n+1} \geq 0 \implies$$
$$-\widetilde{\mathcal{R}_p^{q_c}}|^{n+1} = \min\left(\mathcal{R}_p^{q_c}|^{n+1}, (\delta t^{-1} q_c + R^{q_c})_o\right) ,$$

$$(q_p + \delta t R^{q_p})_o - \delta t(\widetilde{\mathcal{R}_p^{q_c}}|^{n+1} + \mathcal{R}_p^{q_v}|^{n+1}) \geq 0 \implies$$
$$-\widetilde{\mathcal{R}_p^{q_v}}|^{n+1} = \max\left(\mathcal{R}_p^{q_v}|^{n+1}, -(\delta t^{-1} q_p + R^{q_p})_o + \widetilde{\mathcal{R}_p^{q_c}}|^{n+1}\right)$$

furthermore, given the operator $\mathcal{L}(\psi, \boldsymbol{v}, \varrho^*, \delta\tau) := \psi - \frac{\delta\tau}{\varrho^*}\nabla\cdot(\varrho^*\boldsymbol{v}\psi)$

$$\tilde{\tilde{q}}_p = \mathcal{L}\left(\tilde{q}_p^{\,n+1}, \widetilde{\mathbf{G}}^T\mathbf{u}^{\downarrow}|^{n+1}, (\mathcal{G}\varrho)^{n+1}, \delta t\right)$$

$$\widetilde{R_p^{q_p}}|^{n+1} = (\widetilde{\mathcal{R}_p^{q_c}} + \widetilde{\mathcal{R}_p^{q_v}})^{n+1} + (\tilde{\tilde{q}}_p - \tilde{q}_p^{\,n+1})/\delta t$$

completed all explicit integrals → the implicit integrals follow

$$q_v^{n+1} = \widehat{\widehat{q_v}} - \Delta q_c = q_{vs}\left(\widehat{\widehat{\theta}} + \Lambda^\star\Delta q_c,\ \Lambda^\star\right) \ , \quad \text{if} \quad (\widehat{\widehat{q_c}} + \Delta q_c) > 0$$

$$q_c^{n+1} = \widehat{\widehat{q_c}} + \Delta q_c = 0 \ , \quad \text{if} \quad (\widehat{\widehat{q_v}} - \Delta q_c) < q_{vs}\left(\widehat{\widehat{\theta}} + \Lambda^\star\Delta q_c,\ \Lambda^\star\right)$$

saturation condition: $\Delta q_c = \widehat{\widehat{q_v}} - q_{vs}\left(\widehat{\widehat{\theta}} + \Lambda^\star\Delta q_c,\ \Lambda^\star\right) \approx \widehat{\widehat{q_v}} - q_{vs}(\widehat{\widehat{\theta}},\ \Lambda^\star) + \left.\frac{dq_{vs}}{d\Delta q_c}\right|_{\Delta q_c=0}\Delta q_c$ →

$$\Delta q_c = \left[\widehat{\widehat{q_v}} - q_{vs}\left(\widehat{\widehat{\theta}},\ \Lambda^\star\right)\right]\left[1 + \left.\frac{dq_{vs}}{d\Delta q_c}\right|_{\Delta q_c=0}\right]^{-1}$$

$$\widehat{\widehat{\theta}}^{+} = \widehat{\widehat{\theta}} + \Lambda^\star\Delta q_c \ , \quad \widehat{\widehat{q_v}}^{+} = \widehat{\widehat{q_v}} - \Delta q_c$$

$$\Delta^+ q_c = \Delta q_c + \mathrm{EQS}_{(38)}\left(\widehat{\widehat{q_v}}^{+},\ \widehat{\widehat{\theta}}^{+}\right)$$

$$\left.\frac{dq_{vs}}{d\Delta q_c}\right|_{\Delta q_c=0} = q_{vs}\left(\widehat{\widehat{\theta}},\ \Lambda^\star\right)\left[1 - e_s(\widehat{\widehat{\theta}}, \Lambda^\star)/p^\star\right]^{-1}\frac{c_p}{R_v}\left(\frac{\Lambda^\star}{\widehat{\widehat{\theta}}}\right)^2$$

$$e_s(\widehat{\widehat{\theta}}, \Lambda^\star) = e_s\left(\widehat{\widehat{T}} = \frac{L}{c_p}\frac{\widehat{\widehat{\theta}}}{\Lambda^*}\right) \ , \quad \text{and} \quad p^\star = p_0\left(\frac{c_p}{L}\Lambda^\star\right)^{R_d/c_p}$$

conservative FV formulation, positivity of advection, and EU/SL congruence enable consistent semi-implicit integration conserving water substance to machine precision

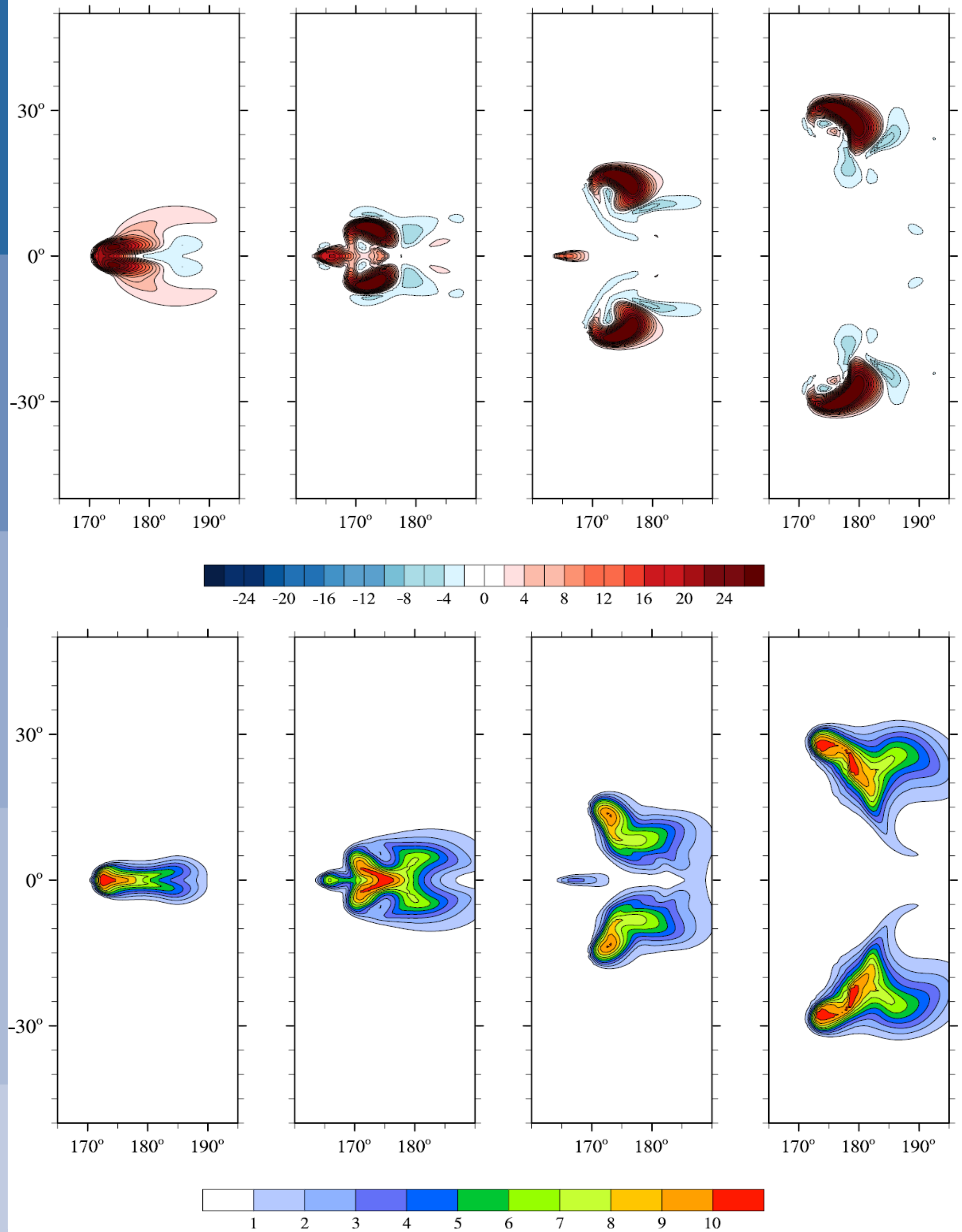
Flow chart for the whole system:

$$\left\{ \varrho_{\mathbf{i}}^{n+1} = \mathcal{A}_{\mathbf{i}}\left(\varrho^{n}, (\mathcal{G}\mathbf{v})^{n+1/2}, \mathcal{G}^{n}, \mathcal{G}^{n+1}\right) \quad \Longrightarrow \quad \mathbf{V}^{n+1/2} = \overline{(\mathcal{G}\varrho\mathbf{v})}^{n+1/2} \right.$$

$$\left\{ \begin{aligned} \widehat{\mathbf{u}}_{\mathbf{i}} &= \mathcal{A}_{\mathbf{i}}\left(\widetilde{\mathbf{u}}, \mathbf{V}^{n+1/2}, \varrho^{*\,n}, \varrho^{*\,n+1}\right) \\ \widehat{\theta'}_{\mathbf{i}} &= \mathcal{A}_{\mathbf{i}}\left(\widetilde{\theta'}, \mathbf{V}^{n+1/2}, \varrho^{*\,n}, \varrho^{*\,n+1}\right) \end{aligned} \right.$$

$$\left\{ \begin{aligned} &\widehat{\theta}_{\mathbf{i}} = \mathcal{A}_{\mathbf{i}}\left(\widetilde{\theta}, \mathbf{V}^{n+1/2}, \varrho^{*\,n}, \varrho^{*\,n+1}\right) \quad \textbf{\textit{and}} \quad \widehat{R^{\theta}}_{\mathbf{i}} = \mathcal{A}_{\mathbf{i}}\left(\mathcal{R}^{\theta}|^{n} + \mathcal{H}(q_{...})|^{n}, \mathbf{V}^{n+1/2}, \varrho^{*\,n}, \varrho^{*\,n+1}\right), \\ &\widehat{q_{...}}_{\mathbf{i}} = \mathcal{A}_{\mathbf{i}}\left(\widetilde{q_{...}}, \mathbf{V}^{n+1/2}, \varrho^{*\,n}, \varrho^{*\,n+1}\right) \qquad\qquad \widehat{R^{q_{...}}}_{\mathbf{i}} = \mathcal{A}_{\mathbf{i}}\left(\mathcal{R}^{q_{...}}|^{n} + D(q_{...})|^{n}, \mathbf{V}^{n+1/2}, \varrho^{*\,n}, \varrho^{*\,n+1}\right) \\ &\mathcal{PCM}\left(\widehat{\theta}, \widehat{q_{...}}, \widehat{R^{\theta}}, \widehat{R^{q_{...}}}\right) \Longrightarrow \left[\theta, q_{...}, \mathcal{R}^{\theta}, \mathcal{R}^{q_{...}}\right]^{n+1} \quad \textbf{\textit{and}} \quad \mathcal{B}^{q} := \theta_a\left(\varepsilon q_v' - q_c - q_p\right) \end{aligned} \right.$$

$$\left| \begin{aligned} &\theta'_{\mathbf{i}} = \widehat{\widehat{\theta}}'_{\mathbf{i}} - 0.5\delta t\left(\widetilde{\mathbf{G}}^{T}\mathbf{u}\cdot\nabla\theta_a\right)_{\mathbf{i}} \\ &\mathbf{u}_{\mathbf{i}} = \widehat{\mathbf{u}}_{\mathbf{i}} - 0.5\delta t\left(\Theta_{\varrho}^{\star}\widetilde{\mathbf{G}}\nabla\varphi + \frac{\mathbf{g}}{\theta_a}\left[\theta' + \mathcal{B}^{q,\,n+1}\right]\right)_{\mathbf{i}} \\ &\qquad -0.5\delta t\left(\mathbf{f}\times\left[\mathbf{u} - \frac{\theta^{\star}}{\theta_a}\mathbf{u}_a\right] - \mathcal{M}'\left(\mathbf{u}^{\star}, \mathbf{u}_a, \frac{\theta^{\star}}{\theta_a}\right)\right)_{\mathbf{i}} \\ &\mathbf{L}\,\mathbf{u} = \widehat{\widehat{\mathbf{u}}} - 0.5\delta t\,\Theta_{\varrho}^{\star}\widetilde{\mathbf{G}}\nabla\varphi \;\rightarrow\; \mathbf{u} = \check{\check{\mathbf{u}}} - \mathbf{C}\nabla\varphi \;\rightarrow\; \mathbf{v} = \check{\check{\mathbf{v}}} - \widetilde{\mathbf{G}}^{T}\mathbf{C}\nabla\varphi \;\rightarrow \\ &\qquad 0 = -\sum_{\ell=1}^{3}\left(\frac{A_{\ell}^{\star}}{\zeta_{\ell}}\nabla\cdot\zeta_{\ell}(\check{\check{\mathbf{v}}} - \widetilde{\mathbf{G}}^{\mathrm{T}}\mathbf{C}\nabla\varphi)\right) - B^{\star}(\varphi - \widehat{\varphi}) \end{aligned} \right.$$



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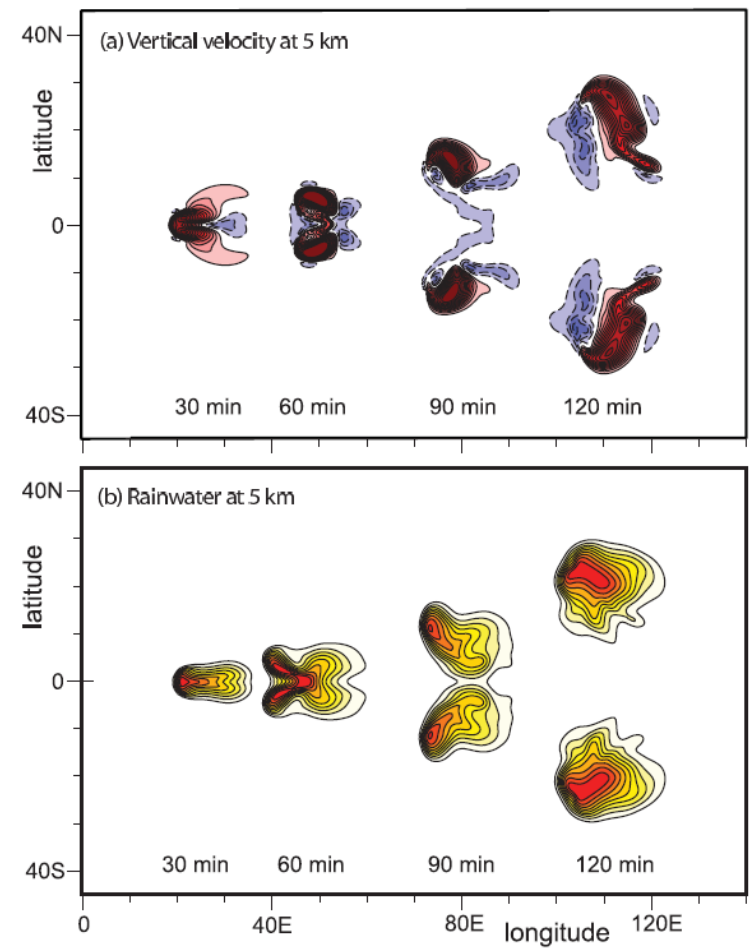

Figure 11. Horizontal cross sections for the simulation with $X = 120$ and $\Delta \sim 500$ m at 5 km at 30 min intervals for (a) vertical velocity (c.i. = 2 m s^{-1}) and (b) rainwater (c.i = 1 g kg^{-1}). Here the longitudinal positions are displayed in the ground-relative framework.

conservative FV formulation + positivity of advection + Eulerian/Lagrangian congruence enable effective integrations of all-scale PDEs of moist-precipitating atmospheric dynamics, while conserving H_2O substance to round-off error

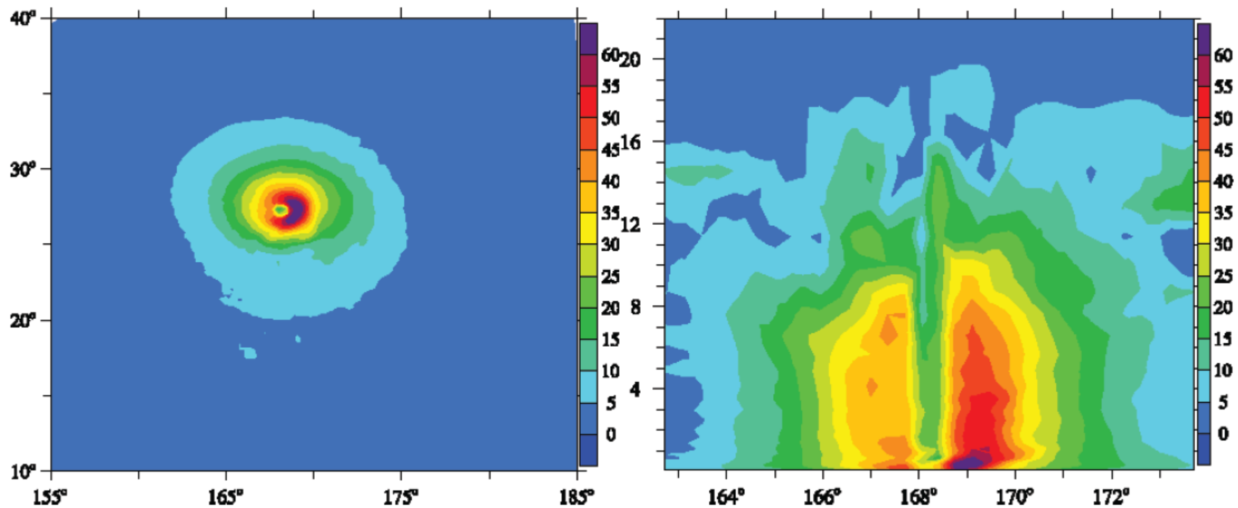

Fig. 3. FVM results for the tropical cyclone experiment of DCMIP (experiment-id: 162) after 10 days of simulation. Horizontal wind magnitude [m/s] in the horizontal (lef) and zonal-height cross sections. Longitude [deg] marks abscissas of both panels, whereas latidude [deg] and altitude [km], marks ordinates of, respectively, right and left panel.

The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP7/2012/ERC Grant agreement no. 320375)

Further reading:

TECHNICAL MEMORANDUM

760

The modelling infrastructure of the Integrated Forecasting System: Recent advances and future challenges

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November 2015

Special topic paper presented at the 44th session of ECMWF's Scientific Advisory Committee, Reading,UK

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European Centre for Medium-Range Weather Forecasts
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Centre européen pour les prévisions météorologiques à moyen

Journal of Computational Physics 314 (2016) 287–304

Contents lists available at ScienceDirect

Journal of Computational Physics

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A finite-volume module for simulating global all-scale atmospheric flows

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ARTICLE INFO

Article history:
Received 11 August 2015
Received in revised form 7 March 2016
Accepted 8 March 2016
Available online 10 March 2016

Keywords:
Atmospheric models
Hierarchical modelling
Non-oscillatory forward-in-time schemes
Gravity waves
Numerical weather prediction

ABSTRACT

The paper documents the development of a global nonhydrostatic finite-volume module designed to enhance an established spectral-transform based numerical weather prediction (NWP) model. The module adheres to NWP standards, with formulation of the governing equations based on the classical meteorological latitude–longitude spherical framework. In the horizontal, a bespoke unstructured mesh with finite-volumes built about the reduced Gaussian grid of the existing NWP model circumvents the notorious stiffness in the polar regions of the spherical framework. All dependent variables are co-located, accommodating both spectral-transform and grid-point solutions at the same physical locations. In the vertical, a uniform finite-difference discretisation facilitates the solution of intricate elliptic problems in thin spherical shells, while the pliancy of the physical vertical coordinate is delegated to generalised continuous transformations between computational and physical space. The newly developed module assumes the compressible Euler equations as default, but includes reduced soundproof PDEs as an option. Furthermore, it employs semi-implicit forward-in-time integrators of the governing PDE systems, akin to but more general than those used in the NWP model. The module shares the equal regions parallelisation scheme with the NWP model, with multiple layers of parallelism hybridising MPI tasks and OpenMP threads. The efficacy of the developed nonhydrostatic module is illustrated with benchmarks of idealised global weather.

1. Introduction

Numerical weather prediction (NWP) has achieved high proficiency over the past 30 years. This owes much to advancements in computer hardware, observational networks and data assimilation techniques as well as numerical methods for integrating hydrostatic primitive equations (HPE). One particular numerical approach embraced widely by NWP combines semi-implicit time stepping with semi-Lagrangian advection (SISL) and with spectral-transform spatial discretisation of the governing HPE [46]. The SISL time stepping enables integrations with Courant numbers of the fluid flow and wave motions much larger than unity, whereas the spectral-transform discretisation facilitates the efficient solution of elliptic equations induced by the SISL approach. Moreover, it circumvents the computational expense of the latitude–longitude (lat–lon) co-

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http://dx.doi.org/10.1016/j.jcp.2016.03.015
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