Inducing Tropical Cyclones to Undergo Brownian Motion

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Hodyss et al. 2013: Impact of noisy physics … Mon. Wea. Rev., 141, 4470-4486.
Hodyss et al. 2014: Inducing tropical cyclones … Mon. Wea. Rev., 142, 1982-1966.

Goals

- Present stochastic parametrization for the probabilistic prediction of TC position
- Show how stochastic calculus relates to stochastic parametrization
- Present an example of the importance of understanding stochastic calculus when creating a stochastic parametrization

Common “Stochastic” Methods

- There are many examples of stochastic methods being used in the NWP literature:
  - Buizza et al. 1999
  - Berner et al. 2008, 2011, 2012
  - Charron et al. 2010
  - Reynolds et al. 2011
  - Whitaker and Hamill 2012
  - … and there are many more!

Spatial and Temporal Noise

- Temporal noise defines whether stochastic calculus issues are important
  - Decorrelation time << 4 Δt
  - Δt is model time step

- Spatial noise is part of the motivation for the particular parametrization described here

Amplitude or Phase?

A parametrization with temporal and spatial red noise cannot *explicitly* control the amount of energy that goes into the amplitude or the phase of waves, i.e.

$$u(x,t) \sim A(t) sin\left(kx - \varphi(t)\right)$$

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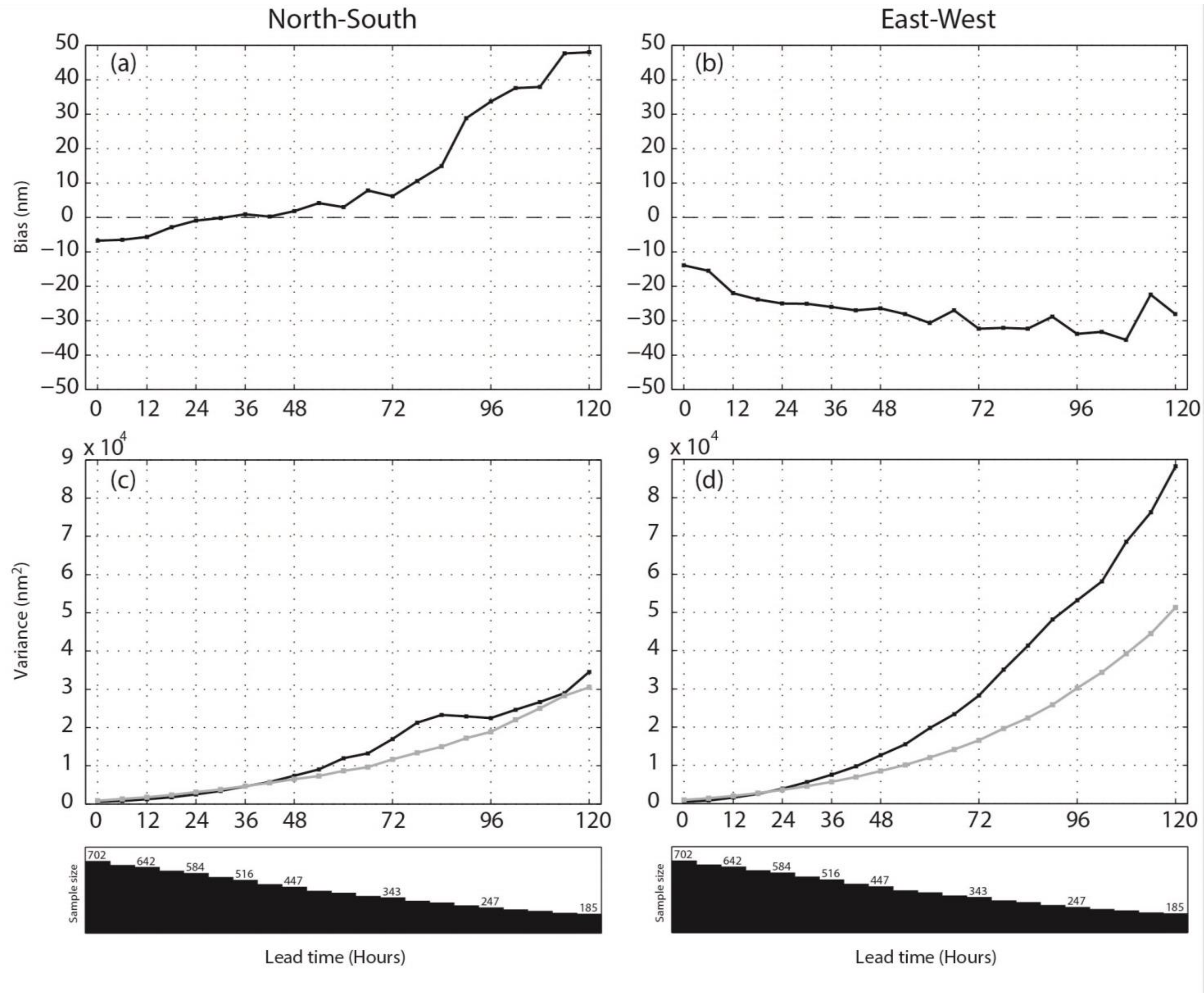
We will describe a method that ***explicitly*** controls the amount of variance added to the amplitude or phase or both.

Motivation: Ensemble TC Track Distribution

In (a) and (b) is the bias of the ensemble mean track forecast as a function of forecast lead time.

In (c) and (d) is the error variance about the ensemble mean forecast (black) and the ensemble variance about the ensemble mean (gray).



Parametrization and Stochastic Calculus

- Designing a stochastic parametrization requires careful consideration of stochastic calculus, which means being thoughtful about the numerical method you use.

- There are two standard stochastic calculi in the literature.
  1. Itô (1951) – evaluate the stochastic parameterization at the **start** of each time step (e.g., Forward Euler, Leap-Frog, Adams-Bashforth)
  2. Stratonovich (1966) – evaluate the stochastic parametrization at the **center** of each time step (e.g., Runge-Kutta methods)

Choices because of discretization …

Example Problem:

$$\frac{du}{dt} = \varphi(u) = D(u) + G(u)w$$

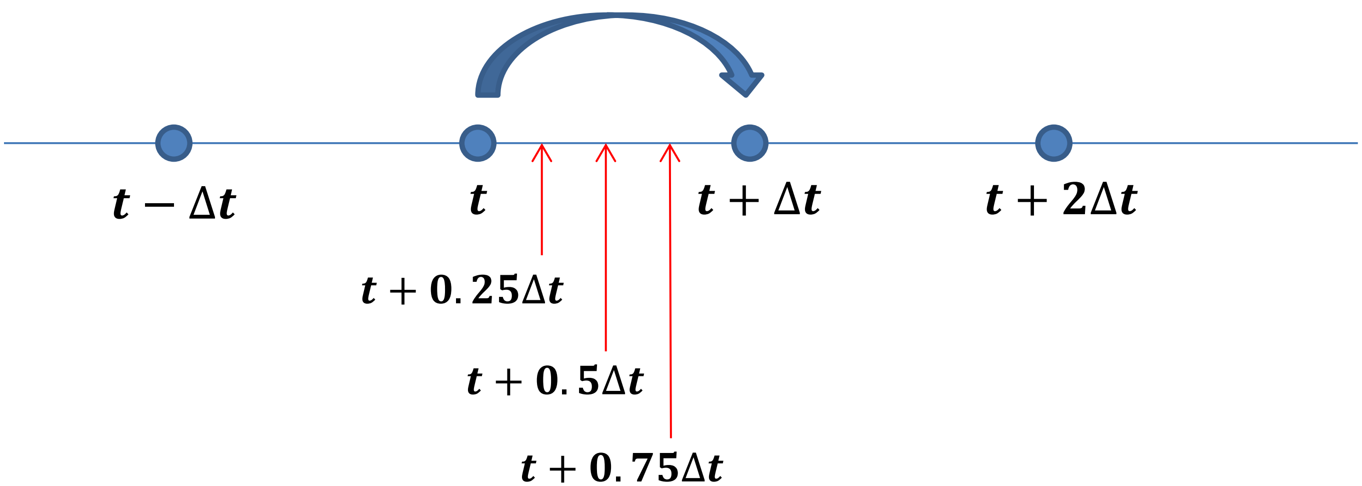
Forward Euler:

$$u(t + \Delta t) = u(t) + \Delta t \varphi(u)$$

2nd Order Runge-Kutta:

$$\tilde{u} = u(t) + \Delta t \varphi(u)$$

$$u\,(t + \Delta t) = u(t) + \frac{\Delta t}{2}[\varphi(u) + \varphi(\tilde{u})]$$



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Why does it matter?

- because the fields are temporally discontinuous the “second derivatives” in the Taylor-series cannot be neglected
- the Itô scheme (FE) provides the solution when the “mask” and the “noise” are uncorrelated
- Stratonovich (RK2) provides the solution for when the “mask” and the “noise” have a particular correlation

Kloeden and Platen, 1999: Numerical solution of SDEs, Springer, New York.
Hodyss et al. 2013: Impact of noisy physics … Mon. Wea. Rev., 141, 4470-4486.

Inducing TCs to undergo Brownian Motion

A simple example of the stochastic parameterization is

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} w$$

***w* is white noise**

↑ Evaluate at the center of each time step

whose solution is a translating wave undergoing Brownian motion

$$u = Aexp[ik(x - ct) + ik\beta_t], \quad \beta_t = \text{Brownian motion}$$

Inducing TCs to undergo Brownian Motion

This same stochastic parametrization from the perspective of Itô reveals

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} w$$

***w* is white noise**

Evaluate at the start of each time step

whose solution is ***not*** the same translating wave undergoing Brownian motion.

$$u = Aexp\left[\frac{1}{2}k^2 t\right] exp[ik(x - ct) + ik\beta_t],$$

Therefore, the theory predicts that if we evaluate the stochastic parametrization according to Itô we will tend to induce the TC to grow!

Inducing TCs to undergo Brownian Motion

To fix this we add the Itô correction, which converts a stochastic parametrization back to Stratonovich,

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}w + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$$

w is white noise

↑ Evaluate at the start of each time step

whose solution is the desired translating wave undergoing Brownian motion.

$$u = Aexp[ik(x - ct) + ik\beta_t],$$

Therefore, the theory predicts that if we evaluate the stochastic parametrization according to Itô we must apply extra diffusion to obtain the desired result.

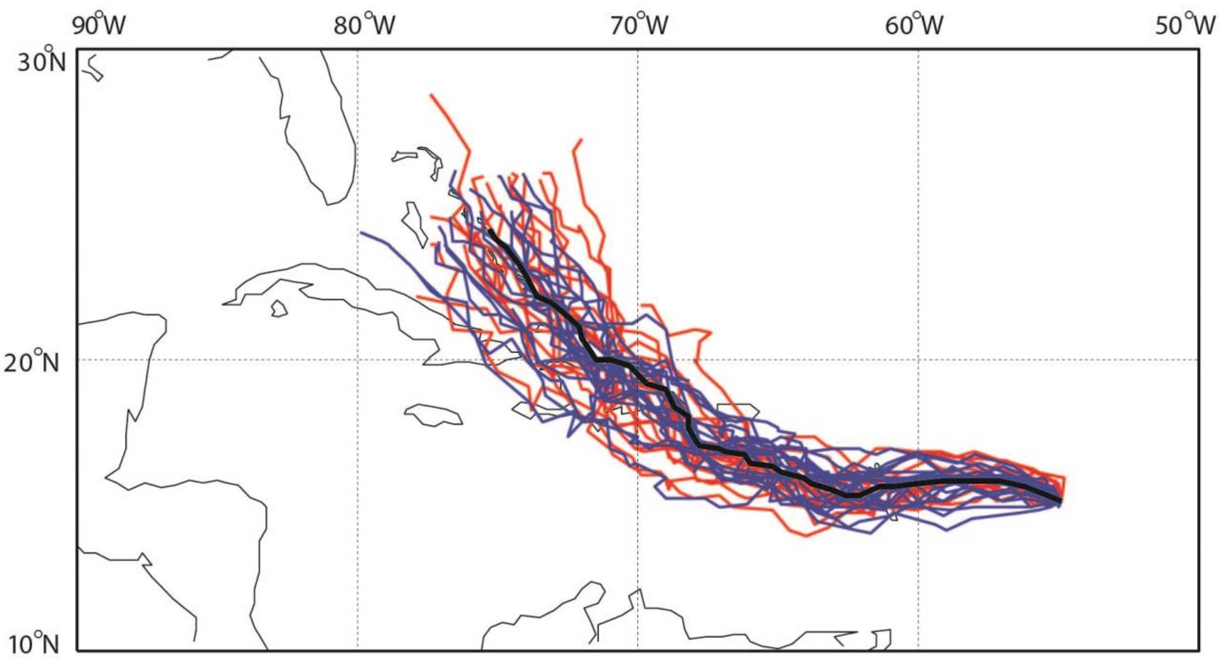
Adding this Stochastic Parametrization to a Global NWP Model

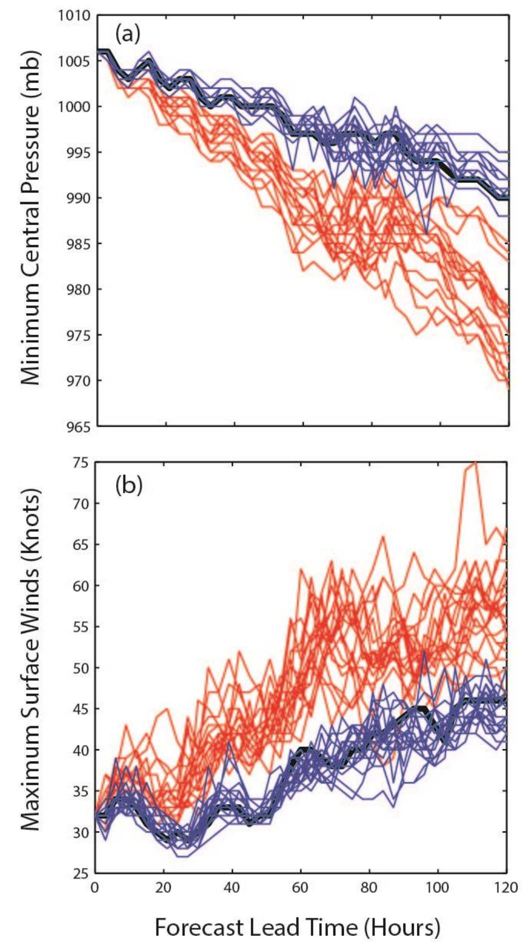
- We used NAVGEM 1.1 (Rev. 1536)
  - T239L50 (approximately 50 km grid spacing); Three time-level semi-Lagrangian scheme
- Converted the formulas on the previous pages to spherical coordinates and add to each prognostic variable's tendency a term like:

$$\frac{\partial u}{\partial x} w \rightarrow \underbrace{M \frac{1}{a cos\theta} \frac{\partial u}{\partial \varphi} \left[c_{\varphi} + \alpha w_1 \right]}_{East-West} + \underbrace{M \frac{1}{a} \frac{\partial u}{\partial \theta} \left[c_{\theta} + \alpha w_2 \right]}_{North-South}$$

- c_{φ}, c_{θ} controls the "bias" of the TC's
- $\alpha w_1, \alpha w_2$ controls the dispersion of the TC's

Hurricane Isaac (2012)





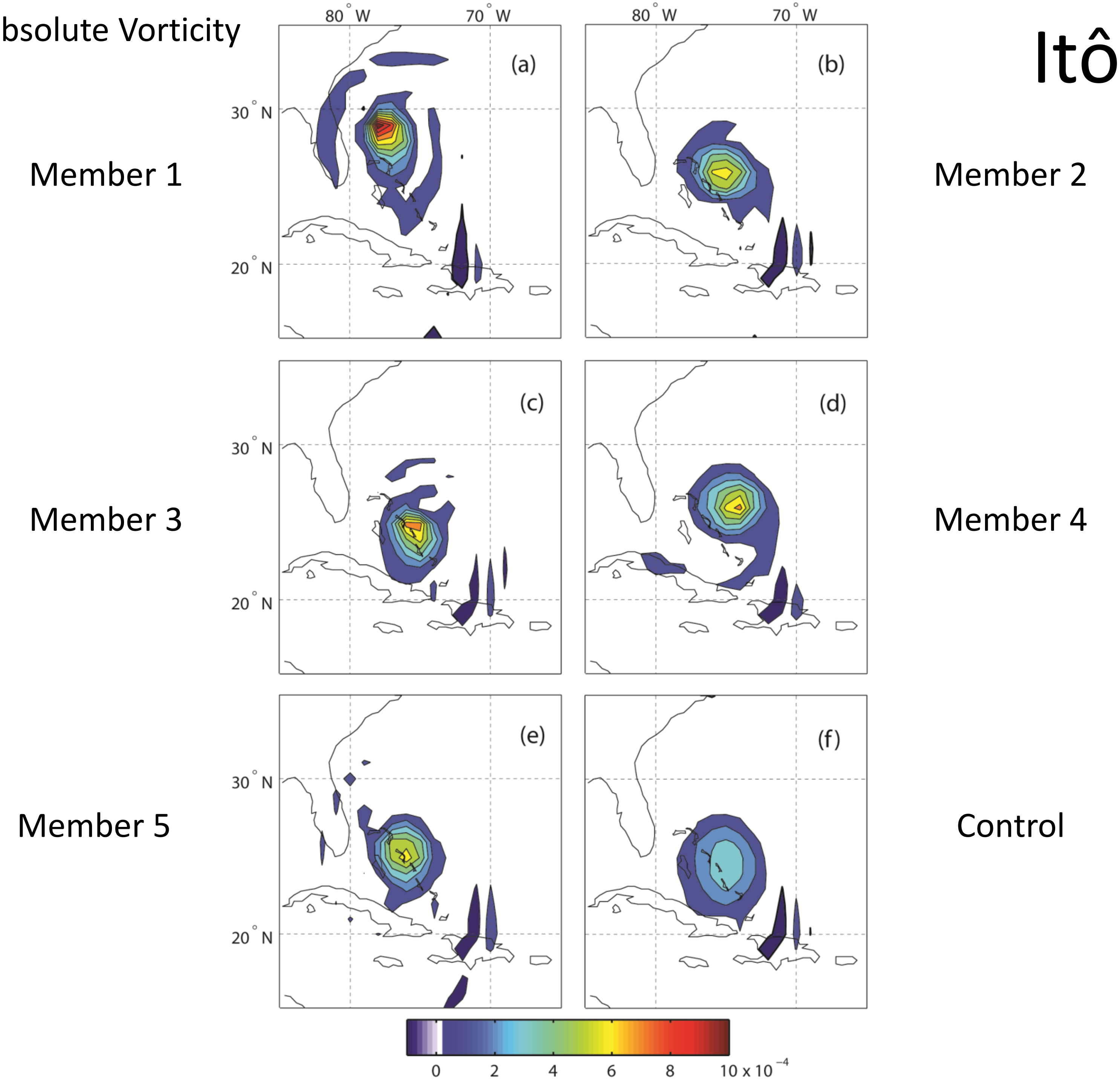
Red - Itô **Blue - Stratonovich**

Black - Control

The intensity of the TCs are too strong from the Itô algorithm.

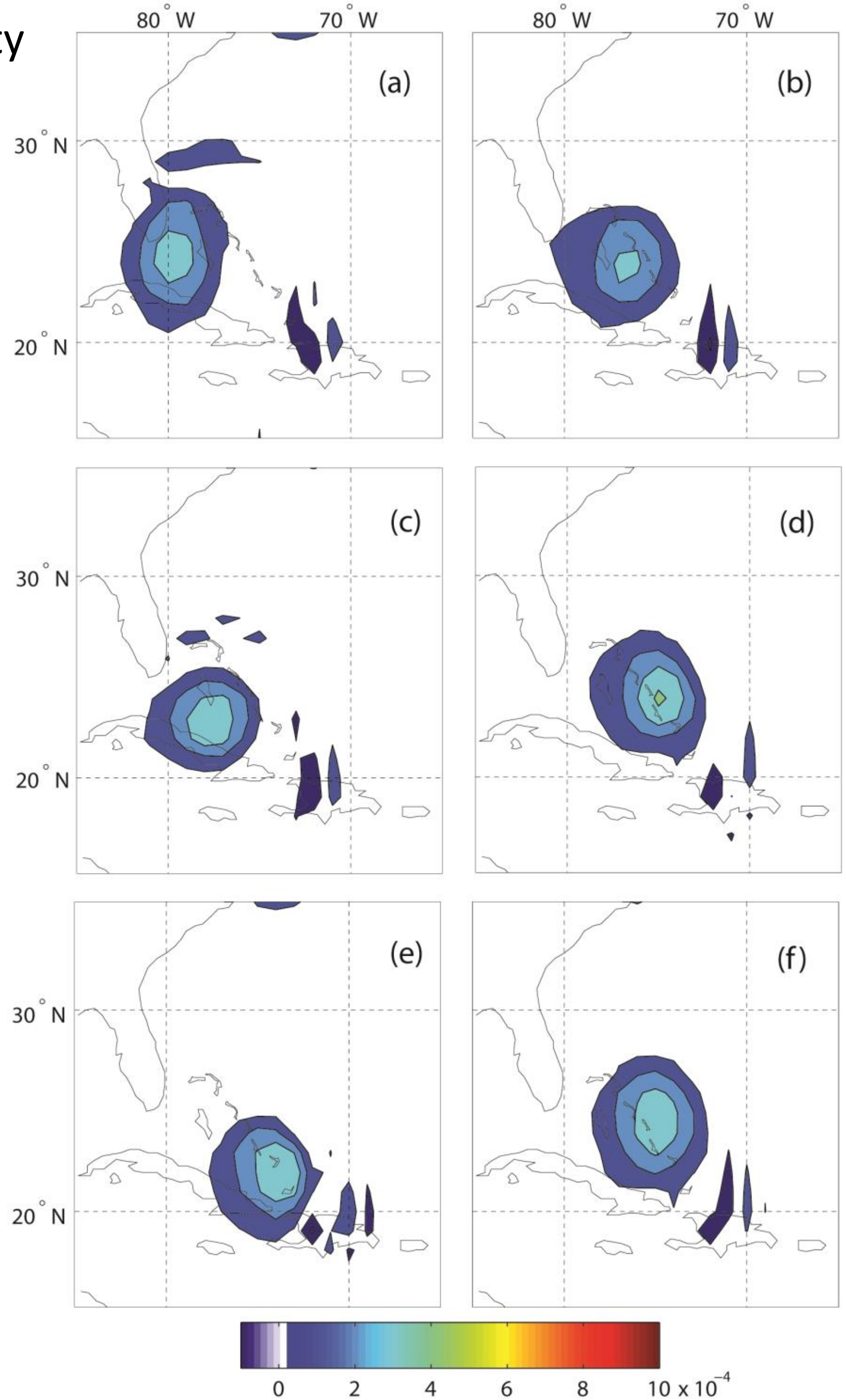
850 mb Absolute Vorticity

Itô



850 mb Absolute Vorticity

Stratonovich



Member 1

Member 2

Member 3

Member 4

Member 5

Control

Summary

- The numerical method you use to integrate your stochastic parameterization matters when you use white noise.
- A stochastic parameterization was described that allows control over the mean and variance of the TC track distribution.
  - Itô correction is required for NAVGEM!

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